1999 Algebra I State Finals
Solutions Manual

1. Cross multiplying gives $2x - 2y = 5x + 5y$.

Simplify to $-7y = 3x$.

Divide by $3y$: $\mathbf{-\frac{7}{3}} = \frac{x}{y}$

2. Using the slope - point formula, $m(-4-2) = (2-(-7))$

$m = -\frac{3}{2}$

$-7 = -\frac{3}{2}(2) + b$

$b = -4$

This eliminates choice A. After making a quick sketch, it becomes apparent that since the slope and y - intercept are both negative, no portion of the graph of the line lies in the first quadrant, so choice E is true. Choices B and D are true, so the only possible answer is **C**.

3. let $w =$ the number of weeks worked

Choices A and B are true. Choices C, D, and E ask about which job is better for a certain period of time worked. So figuring out when the two jobs, pay the same,

$-45 + (20)(4.50)w = (20)(3.50)w$

$w = 2.25$

Working less than 2.25 weeks (45 hours) means that Kelly's Car Wash pays more, so when $40 < w < 45$, **choice E** is not true.

4. $x\#7 = \frac{x^2 - 7^2}{x+7} = 3.$

$x^2 - 49 = 3x + 21.$

$x^2 - 3x - 70 = 0.$

$(x-10)(x+7) = 0$

$x = \mathbf{7}, -10$

5. $2^n + 2^n + 2^n + 2^n$

$= 4(2^n)$

$= 2^2(2^n)$

$= 2^{n+2}$

In choice E, $\mathbf{16^{\frac{n+2}{4}}} = 16^{\frac{1}{4}(n+2)} = (16^{\frac{1}{4}})^{(n+2)} = 2^{n+2}$

6. The sum of two real numbers is another real number. The absolute value of any number is greater than or equal to 0. Therefore, **all b < 0** is impossible, and would cause $|x+a|$ to have no solutions.

7. Ramona is at her doorstep at the beginning and end. The graph showing $d = 0$ at the beginning and end, while showing that Ramona stops as long as it took her to travel a particular distance, is graph **D.**

8. The only way it is possible to buy more books and still spend less money is if more books are sold at a price that is cheap enough. We need to compare all values n of books sold at a higher price to the smallest n sold at a lower price. To buy 25 books costs $C(25) = 275$.

$C(24) = 288,\ C(23) = 276$

$C(25) < C(\mathbf{23})$

$C(25) < C(\mathbf{24})$

Even though 49 books are priced cheaper, per book, than 24 books, you are required to buy too many to save money. To buy 49 books costs $C(49) = 490$.

$C(49) < C(\mathbf{45})$

$C(49) < C(\mathbf{46})$

$C(49) < C(\mathbf{47})$

$C(49) < C(\mathbf{48})$

Thus, there are only 6 values where buying more books is actually cheaper.

9. $m = \frac{11-(-2)}{-2-1} = -\frac{13}{3}$

$m = \frac{\text{rise}}{\text{run}}$

$\text{rise} = (m)(\text{run}) = (-\frac{13}{3})(2) = \mathbf{-\frac{26}{3}}$

10. Let $C(x)$ = cost of wiring in dollars

Let $(200 - x)$ = length of wiring on land

Let $\sqrt{60^2 + x^2}$ = length of wiring in the river

$C(x) = 25(\sqrt{60^2 + x^2}) + 20(200 - x)$

Don't misread the problem; it doesn't ask what the exact, best possible value for x is. Instead, it just basically asks "if you were to try these values, which turns out to have the lowest cost?".

Using a calculator, the answer is when x = **80**.

11. Two inequalities are going on...

$7x + 2 \le 23$ $\qquad$ $3x - 5 \ge 1$

$x \le 3$ $\qquad$ $x \ge 2$

The word "and" indicates a junction, so $2 \le x \le 3$.

The integers 2 and 3 are the only **2** values to satisfy the inequality.

12.

$$\begin{array}{r}
x+1 \\
x^2 - x + 1\overline{\big)x^3 + 0 - 3x + 7} \\
\underline{x^3 - x^2 + x} \\
x^2 - 4x + 7 \\
\underline{x^2 - x + 1} \\
\mathbf{-3x - 6}
\end{array}$$

13. Just by trying simple guesses using multiplication and addition/subtraction, the * function comes out to be doubling the first number and adding the second number.

$11 * 20$ = **42**.

14.

S	X
0	5
5	7
12	program terminates

15. This could get confusing, so take a simple problem that you know. Say perhaps that 52 (dividend) is divided by 8 (divisor). The result is 6 (quotient) with 4 as a remainder. Putting it together:

$(8 \times 6) + 4 = 52$

(divisor × quotient) + remainder = dividend

Applying the above generalization to the problem,

$(x-2)(3x^2 - x - 5) + (-1) = P(x)$

$P(-1) = \mathbf{2}$

16. For each statement, try to disprove or prove it, but it's not necessary to do the work on paper.

I. Finding examples for p and r that make $\frac{p}{r} \geq 1$, $p = 101$, $r = 11$. This statement, therefore, is not one that must be true.

II. Since all primes greater than 2 are odd, p and r are greater than 9, and the sum of two odds are even, then: p and r are odd, $p + r$ is even, and since even numbers greater than 2 (or 9 or 18) are not prime, then $p + r$ is not prime.

III. The prime factorization of pr is $p^1 r^1$. The number of factors is the product of one more than each exponent of the prime numbers. In this case, there are (1+1)(1+1)= 4 total factors. One of these factors is 1. The other three are p, r, and pr. Since p and r are greater than 9, we are sure that these 3 factors are greater than 1, and this statement is true.

Only statements **II** and **III** must be true.

17. $r = \frac{d}{t}$

$d = \frac{1}{3}(90) = 30$ miles

$t = \frac{1}{5}(2) = \frac{2}{5}$ hours

$r = \frac{30}{\frac{2}{5}} = \mathbf{75\ mph}$

18.

$\boxed{6} = 21$

$\boxed{5} = 15$

$\boxed{6} - \boxed{5} = 6$

$\boxed{\mathbf{3}} = 6$

19. For a statement to be false, only one counter-example is necessary.
In statement C, two irrational numbers' product must always be irrational for the set of irrationals to be closed under multiplication. However, $\sqrt{2}\times\sqrt{2} = 2$. So statement **C** is false.

20. 3(6)-(-1)=**19**

21. $r = 4\text{R}$

$$e = \frac{\text{E}}{3}$$

$$e + r = \frac{\text{E}}{3} + 4\text{R} = \frac{\mathbf{E+12R}}{\mathbf{3}}$$

22. $r = 7a \qquad w = \frac{7}{5}a$

$$r - w = 7a - \frac{7}{5}a$$

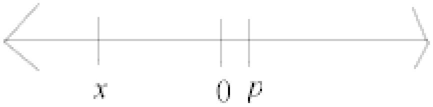
$$= \frac{\mathbf{28}}{\mathbf{5}}\mathbf{a}$$

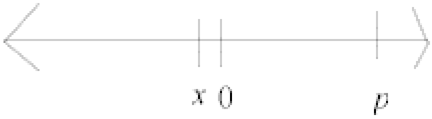
23.

I. If $p > q$, then subtracting p gives $0 > q - p$.

II. $qy < 0$ since a positive time a negative gives a negative

III.





Since nothing in the problem clarifies, the statement "must be less than zero" is too extreme to make. The number lines above are examples to this effect. Only statements **I** and **II** must be true.

24. Let x = the smallest of the 5 integers

Let s = the sum of the next 5 integers

$w = x + (x+1) + (x+2) + (x+3) + (x+4) = 5x + 10$

$s = (x+5) + (x+6) + (x+7) + (x+8) + (x+9)$

$= 5x + 35$

$= (5x + 10) + 25$

$=$ **w + 25**

25. For simplicity's sake, seperate the 2 portions within the largest parenthases.

$M(a, m(b, c)) = M(a, b) = b$

$m(d, m(a, e)) = m(d, a) = a$

$M(b, a) =$ **b**

26.

$p^2 = odd \times odd = odd$

$n \cdot p = odd$ when n is odd

$n \cdot p = even$ when n is even

$(p^2 + n \cdot p) = odd + odd = even$ when n is odd

$(p^2 + n \times p) = odd + even =$ **odd when n is even**

27. Nothing can be determined until the bases of the two sides are the same.

Then, we know that if $x^a = x^b$, then $a = b$.

$16^{x^2+x+4} = 32^{x^2+2x}$

$(2^4)^{x^2+x+4} = (2^5)^{x^2+2x}$

$2^{4(x^2+x+4)} = 2^{5(x^2+2x)}$

$4x^2 + 4x + 16 = 5x^2 + 10x$

$0 = x^2 + 6x - 16$

$0 = (x+8)(x-2)$

$x = -8, 2$

product of solutions $= (-8)(2) =$ **-16**

28. adding both equations yields

$$cx + x = 5$$

$$x = \frac{5}{c+1}$$

substituting x into the first equation yields

$$\frac{5}{c+1} - y = 2$$

$$y = \frac{5}{c+1} - \frac{2(c+1)}{c+1}$$

$$= \frac{3-2c}{c+1}$$

Both x and y must be >0 to be in Quadrant I, so...

$\frac{5}{c+1} > 0$	$\frac{3-2c}{c+1} > 0$
$5 > 0?$	$3 - 2c > 0$
	$c < \frac{3}{2}$

The second equation gives us one boundary for values of c, but the other equation must be solved more indirectly. The value $\frac{5}{c+1}$ must be positive, which means that $c + 1 > 0$ and $c > -1$. Since both coordinates must be greater than 0, this is a junction, and $\mathbf{-1 < c < \frac{3}{2}}$.

29. The function must be exponential since the percentage of remaining dice is multiplied by itself for however many times the dice are rolled. Also, the percentage of remaining dice must be a decimal less than 1 (dice just don't come from nowhere when they're rolled), so the only choice not eliminated is **D**.

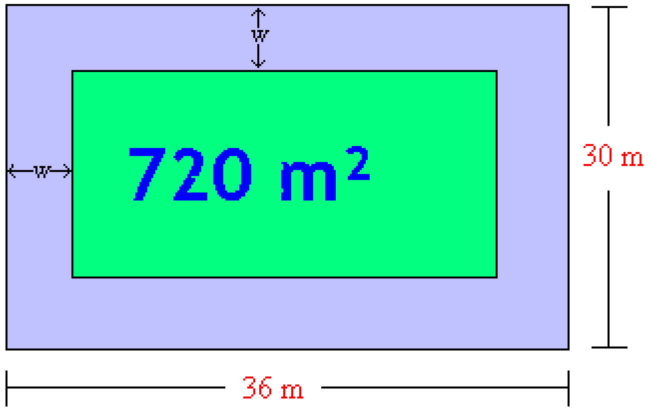
30. Since $1 \le a < b \le 9$, the prime factorization will contain 2 1-digit primes and 1 2-digit prime. Also, the problem implies none of the primes are raised to any power (which excludes 250 and 525 since their prime factorizations include 5^2).

$$378 = 2 * 3^2 * 7$$

$$1995 = 3 * 5 * 7 * 19$$

It's easy to see that **none of these (E)** are the solution.

31.



Let A = area of the garden

$A = 720 = (36 - 2x)(30 - 2x)$

$180 = (18 - x)(15 - x)$

$x^2 - 33x + 90 = 0$

$(x - 30)(x - 3) = 0$

$x = \mathbf{3},\ 30\text{(extraneous)}$

32. Intuitively, 9 rolls should indicate the odds, but systematically:

Let $P(x)$ = probability of and odd being rolled

$P(1) = P(3) = P(5) = x$

$P(2) = P(4) = P(6) = 2x$

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$x = \frac{1}{9}$

$P(6) = \mathbf{\frac{2}{9}}$

33. Order of operations says to simplify the expression within parenthases, and then evaluate the output when plugged into $f(x)$.

$(x^2 + 1)^2 + (x^2 - 1)^2$

$= (x^4 + 2x^2 + 1) + (x^4 - 2x^2 + 1)$

$= 2x^4 + 2$

$= 2(x^4 + 1) \qquad 2 * number = even\ number$

$f(2x^4 + 2) = \mathbf{2x^4 + 1}$

34. If $x^4 + y^3 = 1$, then we know $(-x, y)$ is on the graph since $(-x)^4 + y^3 =$ $x^4 + y^3 = 1$. And in general, when the coordinate is raised to an even power every single time in the equation, then symmetry by the other axis occurs. Since y is raised to an odd number, then x-axis and origin symmetry are ruled out. Symmetry about $y = x$ is another story, but since $x^4 + y^3 = x^3 + y^4$ is not necessarily true, it is ruled out. The only symmetry is about the **y - axis**.

35. Suppose R is just a rectangle whose 4 vertices are (1,2),(1,-2),(-1,2), and (-1-2). The x-axis and y-axis symmetries in the problem are satisfied, but the point **(2,1)** is not contained in R.

36. If $a < 0$, then $(1-a) > 1$.

$f(1-a) = [(1-a)-1]^2$

$= \mathbf{a^2}$

37. The line containing $(a,0)$ and $(0,b)$ is $y = -\frac{b}{a}x + b$. The slope of the second line must be $\frac{a}{b}$, and using the slope-point formula,

$\frac{a}{b}(x-b) = (y-0)$

$y = \frac{a}{b}(x-b) = a\frac{(x-b)}{b} = \mathbf{a(\frac{x}{b} - 1)}$

38. If $x < 1$, then x must be made positive so that $f(x)$ is not undefined for those values of x. If $|x|$ is the input for $f(x)$, then it would be defined for all values x where $|x| \geq 1$. But not all values are included (since $f(x)$ is still undefined for $0 \leq x < 1$), so to get the radicand of $f(x)$ always non-negative, it's input will have to be $|x| + 1$. This means the input will be $|x| + 1$, and since $g(f(value)) = value$, then $\mathbf{g(f(|x| + 1)) = |x| + 1}$.

39. bathtubs filled = time $\times$ (fill rate)

hot water fill rate $=\frac{1}{H}$, cold water fill rate $=\frac{1}{C}$

Rates have to be added, not times.

tubs filled after 1 min $=1(\frac{1}{H})$

tubs remaining after 1 min $= 1 - \frac{1}{H}$

time needed after 1 min $= \dfrac{1-\frac{1}{H}}{\frac{1}{C}+\frac{1}{H}} = \dfrac{C(H-1)}{(H+C)}$

total time needed $=1+\dfrac{C(H-1)}{(H+C)} = \dfrac{\mathbf{H(C+1)}}{\mathbf{(H+C)}}$

40. To get the average to 60, the 3rd test score must be less than 60. This makes the highest and "middle" scores equal to 70.

Let s = 3rd test score

$(0.50)(70)+(0.30)(70)+(0.20)x \geq 60$

$x \geq \mathbf{20}$