

ALGEBRA 1
STATE MATHEMATICS CONTEST FINALS
APRIL 28, 2011

1. What is the ones digit of 7^{2011} ?

- a. 1 b. 7 c. 9 d. 3 e. 5

2. Jane is keeping a journal in a notebook. She numbers each page of the journal by hand. When she finishes numbering the pages she realizes it took her twice as many digits as the number of the last page to do so. How many pages does the journal contain?

- a. 108 b. 120 c. 79 d. 80 e. 55

3. How many points (x, y) with integer coordinates lie inside or along the boundary of the circle of radius 5 centered at $(0, 0)$?

- a. 75 b. 78 c. 81 d. 82 e. 90

4. If $f(x) = 3x - 7$, then $f^{-1}(f^{-1}(-1))$ is which of the following?

- a. -10 b. -37 c. -4 d. 3 e. $-\frac{44}{3}$

5. Suppose that the number $11A1B$ is divisible by 3, where A and B represent **distinct** digits from $\{0, 1, 2, \dots, 9\}$. How many values of $A + B$ are possible?

- a. 5 b. 8 c. 6 d. 7 e. 3

6. Six people are sitting in a room. When an 80-year old man enters the room, the average age of the people in the room doubles. What was the average age of the six people originally sitting in the room?

- a. 15 b. 40 c. 30 d. 20 e. 10

7. Working together, Gary and his dad were able to paint their living room in h hours. From experience, Gary's dad knows that it takes him d hours to do this task alone. How long would it take Gary to paint the living room alone?

- a. $\frac{d-h}{dh}$ hours b. $\frac{h-d}{dh}$ hours c. $\frac{dh}{h-d}$ hours d. $\frac{dh}{d+h}$ hours e. $\frac{dh}{d-h}$ hours

8. If $f(x) = x^2 + 1$ and $g(x) = x^3$, which of the following statements is/are true?

I. $f(g(-x)) = -f(g(x))$.

II. $|f(x)| = f(x)$.

III. $g(-x) = |g(x)|$.

- a. I only b. I & II c. I & III d. II only e. II & III

9. If $-3x^2 + kx + 18$ has two integer roots that are five units apart, then k could be which of the following?

- a. -3 b. 9 c. 1 d. -1 e. -9

10. A Kakuro puzzle asks the solver to enter a digit from 1 to 9 inclusive in each blank so that the sum of the number in each row or column matches the clue and so that no digit is repeated in the sum. For example, the sum for a clue 6 with two blanks could only be $6 = 1 + 5$ or $6 = 2 + 4$.

Which number(s) can be a clue in the position marked x ?

	7	8	4
x			
6			

- a. 11 or 15 b. 11 only c. 13 only d. 13 or 15 e. 15 only

11. A beginning algebra student mistakenly thinks that the equation

$$(x - 2y)^2 = x^2 - 4y^2$$

holds for all values of x and y . What can be said of the ratio $\frac{x}{y}$ if this equation holds and y is not zero?

- a. $\frac{x}{y} < 0$.
 b. $\frac{x}{y}$ is positive and irrational.
 c. $\frac{x}{y}$ is positive and rational, but not an integer.
 d. $\frac{x}{y}$ is a prime number.
 e. $\frac{x}{y}$ is positive and a perfect cube.

12. For how many pairs of positive integers (m, n) is

$$\frac{2}{m} + \frac{3}{n}$$

an integer?

- a. 5 b. 9 c. 6 d. 7 e. 11

13. Bill starts walking along a 6-mile trail from the beginning (the trailhead). Some time later his brother Phil rides his bike along the trail in the same direction starting at the trailhead. Phil can ride three times as fast as Bill can walk. When he passes Bill, Phil (on the bike) continues to the end of the trail and Bill turns around and heads back towards the trailhead. When Phil gets to the end of the trail, he turns his bike around and goes back along the trail arriving at the trailhead at the same time Bill does. How far from the trailhead did Phil pass Bill?

- a. 1 mi. b. 3 mi. c. 5 mi. d. 4 mi. e. 2 mi.

14. The parabola $ax^2 - 6x + c$ has only one root. Determine the product ac .

- a. -6 b. 2 c. 9 d. 2 e. 6

15. Suppose that $x + y = 8$ and $x^2 + y^2 = 80$. Find the product xy .

- a. 16 b. -8 c. 8 d. -16 e. -32

16. The graphs of $f(x) = -4x^2 + 6x + 7$ and $g(x) = \frac{1}{2}x^2 + \frac{3}{2}x - 2$ meet in two points. Find the slope of the line containing these two points.

- a. 3 b. $\frac{1}{3}$ c. $\frac{2}{3}$ d. $\frac{3}{2}$ e. 2

17. A standard coin is tossed four times. What is the probability that heads comes up exactly twice?

- a. $\frac{5}{8}$ b. $\frac{7}{16}$ c. $\frac{3}{8}$ d. $\frac{5}{16}$ e. $\frac{1}{2}$

18. A drawer contains 3 blue socks and 4 black socks. It is too dark to tell which is which. If you choose two socks at random, the probability that you have a matching pair is

- a. $\frac{24}{49}$ b. $\frac{3}{7}$ c. $\frac{2}{7}$ d. $\frac{8}{49}$ e. $\frac{18}{49}$

19. The function $f(x) = x^2 - 4x + c$ has an x -intercept -3 . Find the y -coordinate of its vertex.

- a. 25 b. -5 c. 5 d. $\frac{5}{2}$ e. -25

20. What is **false** concerning the line containing $(4, 3)$ and $(-2, -6)$?

- a. It has slope greater than its x -intercept.
b. It is parallel to $y = \frac{3}{2}x + 10$.
c. It is perpendicular to $2x + 3y = 5$.
d. It contains the point $(1, -\frac{3}{2})$.
e. It has y intercept -3 .

21. Simplify

$$\frac{\frac{1}{a^2} - \frac{1}{ab}}{\frac{1}{ab^3} - \frac{1}{b^2}}$$

- a. $\frac{b(b-a)}{a^2(ab-1)}$ b. $\frac{b^2(a-b)}{a(a^2b-1)}$ c. $\frac{b(b-a)}{a(1-a^2b)}$ d. $\frac{b(b-a)}{a(ab-1)}$ e. $\frac{b^2(a-b)}{a(1-a^2b)}$

22. If $x \# y$ is defined for all **integers** by

$$x \# y = \frac{1}{\frac{1}{x} + \frac{1}{y}}$$

and if $x \# y$ is 7, what is a possible value of x ?

- a. 14 b. 7 c. 1 d. 49 e. 3

23. If the equation $|x + a| = |x - b|$ has two distinct solutions, what can be said about the numbers a and b ?

- a. $\frac{a}{b} = -1$ or $a = b = 0$ b. $\frac{a}{b} = 1$ c. $\frac{a}{b} = 1$ or $a = b = 0$ d. $a = b = 0$ e. $\frac{a}{b} = -1$

24. Consider the polynomial $2x^2 - Cx + 12$. If the roots of this polynomial are a and b and it is known that $a = 1$, find $b + C$.

- a. 4 b. 7 c. 20 d. 28 e. 14

25. Suppose that p and q are distinct prime numbers and that $2 < p$, $2 < q$. Which of the following is/are true?

- I. pq has 4 positive integer factors.
- II. $p + q$ must be composite.
- III. $p - q$ must be composite.

- a. I. only b. All are true c. I and III only d. II and III only e. I and II only

26. For all numbers x and y , the operation $x \diamond y$ is given by $x \diamond y = ax + by + c$. If $2 \diamond 3 = 3$, $7 \diamond 4 = 17$ and $4 \diamond 10 = 2$ solve $3 \diamond y = 0$.

- a. 3 b. 2 c. 5 d. 0 e. 9

27. When the polynomial $P(x)$ is divided by $x - 3$ the quotient is $2x^2 - 5x + 1$ and the remainder is 4. Find $P(4)$.

- a. 17 b. 13 c. 1 d. 4 e. 9

28. Let x and y be integers so that $3x + 7y = 1$. There are many solutions to this equation. Which of the following is the smallest positive value of $x + y$.

- a. 5 b. 7 c. 4 d. 3 e. 1

29. How many positive integers less than 60 have no prime factors in common with 60? (Note that 1 is not a prime number.)

- a. 17 b. 15 c. 16 d. 19 e. 18

30. Let $f(x) = 4x - 8$ and let $g(x) = 5x + 1$. Find $f^{-1}(g^{-1}(1))$.

- a. 16 b. $-\frac{3}{22}$ c. 2 d. $\frac{1}{16}$ e. -8

31. Find the line through $(3, 0)$ perpendicular to the line through $(0, 3)$ and $(3, 5)$.

- a. $3x - 2y = 9$ b. $9x - 2y = 3$ c. $3x + 2y = 9$ d. $2x + 3y = 9$ e. $2x + 9y = 3$

32. A fair die and a weighted die are tossed. The weighted die is constructed so that the 6 shows up twice as often as each of the other numbers. What is the probability of rolling a sum less than 6?

- a. $\frac{5}{14}$ b. $\frac{1}{9}$ c. $\frac{1}{6}$ d. $\frac{3}{8}$ e. $\frac{5}{21}$

33. Connie wants to mail a package. She has twenty 12¢ stamps and twenty 18¢ stamps. The post office weighs her package and tells her the cost of the postage. If she is able to use her stamps to pay the exact postage, it must be which of the following?

- a. \$1.52 b. \$2.12 c. \$1.72 d. \$2.32 e. \$1.92

34. Of the 96 participants in a math contest, 63 participate in the math club, 58 participate in the engineering club, 45 participate in orchestra, and 27 do all three. If 33 are in the math club and the orchestra, 36 are in the engineering club and the orchestra and 18 are in the engineering club and the math club but not the orchestra, how many math contest participants are involved in none of these three activities?

- a. 11 b. 7 c. 14 d. 17 e. 4

35. Find the sum of all solutions to $x + 10 = 7\sqrt{x}$.

- a. 29 b. 7 c. 100 d. 17 e. 49

36. Which number is double 4^{30} ?

- a. 2^{31} b. 2^{61} c. 8^{30} d. 4^{31} e. 4^{60}

37. The base-5 number 1034 is the same as the base- b number 220. Find b .

- a. 10 b. 7 c. 8 d. 9 e. 11

38. For a positive integer n , define the number $n! = 1 \cdot 2 \cdot 3 \cdots (n - 1) \cdot n$. The number $30!$ ends with how many zeros?

- a. 7 b. 8 c. 5 d. 6 e. 3

39. How many distinct 6-digit numbers can be made using the digits 1, 1, 1, 2, 2, 3?

- a. 108 b. 60 c. 220 d. 216 e. 729

40. A pea weighs as much as three crumbs; five crumbs weigh as much as a pea and a kernel; a bean weighs as much as a pea and a crumb. If a kernel weighs 6 grams (g), how much does a bean weigh?

- a. 8g b. 16g c. 12g d. 9g e. 10g

Algebra I Key
State Mathematics Contest Finals

April 28, 2011

Answer Key:	1. D	11. D	21. B	31. C
	2. A	12. B	22. A	32. E
	3. C	13. B	23. A	33. E
	4. D	14. C	24. C	34. D
	5. A	15. B	25. E	35. A
	6. E	16. E	26. E	36. B
	7. E	17. C	27. A	37. C
	8. D	18. B	28. D	38. A
	9. A	19. E	29. C	39. B
	10. C	20. A	30. C	40. C

Best of Three: 22, 24, 28

Sudden Death: 29, 12, 33, 30, 40