1999 Algebra II State Finals
Solutions Manual

1. (B) The original equation is $y = \frac{4}{3}x - 4$. The new equation will of the form

$$y = -\frac{3}{4}x + b$$
$$3 = -\frac{3}{4}(2) + b$$
$$b = \frac{9}{2}$$
$$y = -0.75x + 4.5$$

2. (D) The triangle is a right triangle with right angle at the origin and its other two vertices as the x-intercept and y-intercept of $y = -0.75x + 4.5$. In that equation, when $y = 0$, $x = 6$. When $x = 0$, $y = \frac{9}{2}$. The area of the triangle is

$$A = \frac{6 \cdot \frac{9}{2}}{2} = 13.5$$

3. (C) The distance from a line $Ax + By + C = 0$ to a point x_0, y_0 is given by the formula

$$\left|\frac{Ax_0 + Bx_0 + C}{\sqrt{A^2 + B^2}}\right|$$

Plugging in values,

$$\left|\frac{(4)(0) + (-3)(0) - 12}{\sqrt{4^2 + 3^2}}\right| = 2.4$$

4. (B) Any line which passes through the circle's center, $(-3, -2)$, must contain a diameter of the circle. If the slope of the equation is m, then such a line can be described as

$$y = m(x + 2) - 3$$
$$y = mx + 2m - 3$$

5. (D)

$$x^2 - xc + (c - 1) = 0$$

Using the quadratic formula,

$$x = \frac{c \pm \sqrt{c^2 - 4(c-1)}}{2}$$
$$= \frac{c \pm (c-2)}{2}$$
$$= c - 1, 1$$

6. (C) In a quadratic Ax^2+Bx+C, only one solution exists if $B^2-4AC = 0$. For this equation,

$$c^2 - 4(c-1) = 0$$
$$(c-2)^2 = 0$$
$$c = 2$$

7. (A)

$$5x + 15y = 5a$$
$$\underline{+(-6x - 15y = -3b)}$$
$$-x = 5a - 3b$$
$$x = -5a + 3b$$

8. (D) For this quadratic, we want the following equation to be true

$$\frac{-B + \sqrt{B^2 - 4AC}}{2A} - \frac{-B - \sqrt{B^2 - 4AC}}{2A} = 5$$
$$\frac{\sqrt{B^2 - 4AC}}{A} = 5$$
$$\frac{\sqrt{m^2 + 144}}{3} = 5$$
$$m = \pm 9$$

9. (B) Initially, let $f(x) = ax^2 + bx + c$.

$$f(0) = c = 3$$
$$f(8) = 64a + 8b + c = 3$$
$$64a + 8b = 0$$
$$b = -8a$$
$$f(x) = ax^2 + (-8a)x + 3$$
$$= a(x^2 - 8x) + 3$$

10. (A) Knowing the 2 roots and the constant mulitplier, we can say that the form of the quadratic function is $f(x) = a(x-2)(x-10)$ This expands to $ax^2 - 12ax + 20a$. The x-value of the vertex is $-\frac{b}{2a}$, which is $-\frac{(-12a)}{2a} = 6$. $f(6) = -16a$, making the vertex $(6, 16a)$.

11. (B) Intersections occurs when

$$x + b = x^2 - 3x + 5$$
$$x^2 - 4x + (5 - b) = 0$$

There is only one intersection point when the above quadratic has only one solution, i.e. it is a perfect square. This occurs when

$$(-4)^2 - 4(5 - b) = 0$$
$$b = 1$$

12. (A) Let the quadratic have the form $a_1x^2 + b_1x + c_1$ and the cubic have the form $a_2x^3 + b_2x^2 + c_2x + d$. The x-values of all intersection points will satisfy the equation

$$a_2x^3 + b_2x^2 + c_2x + d_2 = a_1x^2 + b_1x + c_1$$
$$a_2x^3 + (b_2 - a_1)x^2 + (c_2 - b_1)x + (d_2 - c_1) = 0$$

This is a cubic, and the Fundamental Theorem of Algebra guarantees that there are no more than 3 solutions for a 3rd degree equation. However, a cubic and a quadratic must intersect since the cubic has a steeper graph for larger and smaller values of x. As the values of x increase (or decrease, depending on the direction of both functions), the graph of the cubic will "overtake" the graph of the quadratic.

13. (C) The baseball will reach its maximum height when $t = \frac{-49}{2(-4.9)} = 5$.

$$h(5) = 122.5$$

14. (E) Let $y = \frac{t-1}{t+1}$. If the y's and the t's are switched, then solving for y arrives at the inverse.

$$t = \frac{y' - 1}{y' + 1}$$

$$f^{-1}(t) = y' = \frac{t+1}{1-t}$$

$$f^{-1}(\frac{1}{c} + 1) = \frac{\frac{1}{c} + 2}{-\frac{1}{c}}$$

$$= -1 - 2c$$

15. (A)

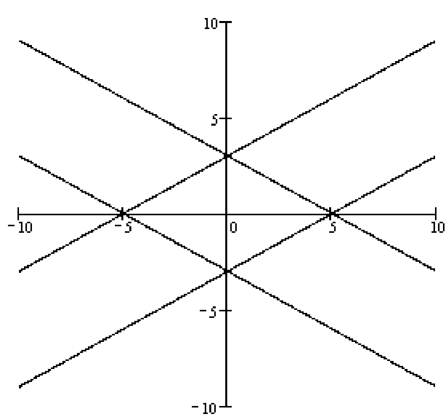
$$|y| = \frac{3}{5}|x| - 3$$

This equation represents four equations which can be expressed as

$$y = \pm\frac{3}{5}x \pm 3$$

The enclosed area is a rhombus whose diagonals have length 10 and 6, so the resulting area is

$$\frac{6 \cdot 10}{2} = 30$$



16. (A)

$$x - y = 100 \text{ computers}$$
$$\text{empty truck} = x - 4(100 \text{ computers})$$
$$= x - 4(x - y)$$
$$= 4y - 3x$$

17. (B)

$$\text{Let } x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{\ldots}}}}}$$
$$x^2 - 6 = x$$
$$(x - 3)(x + 2) = 0$$
$$x = 3 \text{ since } x > 0$$

18. (E) $2^{x-2} = 2^x - 2$. Let $y = 2^{x-2}$.

$$y = 4y - 2$$
$$y = 2^{x-2} = \frac{2}{3}$$
$$x = 2 + \log_2(\frac{2}{3})$$
$$= 2 + \log_2 2 - \log_2 3$$
$$= 3 - \log_3$$

19. (D)

$$\log_7(5a) - \log_7(a - 4) = 1$$
$$\log_7 \frac{5a}{a - 4} = 1$$
$$\frac{5a}{a - 4} = 7$$
$$a = 14$$

20. (E) $(e^b)^2 + 5(e^b) = 14$. Let $x = e^b$.

$$x^2 + 5b - 14 = 0$$

$$(x+7)(x-2) = 0$$

$x = 2$ beause $e^b > 0$ for all b.

$$x = e^b = 2$$

$$b = \ln 2$$

21. (C)

$$e^{bx} = e^{(\ln 2)\cdot x} = 2^x$$

$$2^x = 2(x+1)^2$$

No numerical method can be provided to solve the equation, but graphing the functions reveals that the functions do intersect. Testing the solutions provided, $x = 7$ works.

22. (A) Let $b =$ the length of the base.

$$42 = \frac{1}{2}b(b-6)$$

$$b^2 - 6b - 84 = 0$$

$$b = \frac{6 \pm \sqrt{36 + 4(84)}}{2}$$

$$b = 3 + \sqrt{93}$$

23. (B) The probability that the first card is an ace is $\frac{4}{52}$. After the first card is drawn and is an ace, the probability that the next card is also an ace is $\frac{3}{51}$. Then the probability that the next 3 cards are not aces is $\frac{48}{50} \cdot \frac{47}{49} \cdot \frac{46}{48}$.
There are $\frac{5!}{2!3!} = 10$ combinations of drawing the 2 aces within the 5 card set. Thus the probability is 10 times greater, or

$$10\left(\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{48}{50} \cdot \frac{47}{49} \cdot \frac{46}{48}\right)$$

$$= \frac{2162}{54145}$$

24. (E)

$$x^3 - 7x + 6 \leq 0$$

$$(x-1)(x+3)(x-2) \leq 0$$

We can use a graph of $f(x) = (x-1)(x+3)(x-2)$ to predict that $f(x) \leq 0$ when $x \leq -3$ or $1 \leq x \leq 2$. As a check, test the binomial factors $(x-1)$, $(x+3)$, and $(x-2)$ to see if they are positive, 0, or negative in the ranges $x \leq -3$, $-3 \leq x \leq 1$, $1 \leq x \leq 2$, and $2 \leq x$.

25. (C)

$$\left(\sqrt{3\sqrt{x} - \sqrt{7x + \sqrt{4x-1}}} \cdot \sqrt{3\sqrt{x} + \sqrt{7x + \sqrt{4x-1}}}\right) \cdot \sqrt{2x + \sqrt{4x-1}}$$

$$= \left(\sqrt{9x - (7x + \sqrt{4x-1})}\right) \cdot \sqrt{2x + \sqrt{4x-1}}$$

$$= \sqrt{(2x)^2 - (4x-1)} = 13$$

$$2x - 1 = 13$$

$$x = 7$$

26. (C) Long division will find the answer, but since $q(x)$ is a linear term, we can use a shortcut. Let

$$p(x) = q(x)h(x) + r$$

where r is the remainder and $h(x)$ is the quotient polynomial produced by long division. Let $x = 2$.

$$p(2) = q(2)h(2) + r$$

Since $q(2) = 0$,

$$r = p(2) = 17$$

27. (E)

$$z(2+i) = 1$$

$$= \frac{1}{2+i} \cdot \frac{2-i}{2-i}$$

$$= \frac{2-i}{5}$$

None of the given choices represent z.

28. (C)

$$f(f(x)) = f(x^2 - c^2)$$
$$= (x^2 - c^2)^2 - c^2 = 0$$

Let $y = x^2$ and $b = c^2$.

$$y^2 - 2yb + (b^2 - b) = 0$$
$$y = \frac{2b \pm \sqrt{(2b)^2 - 4(b^2 - b)}}{2}$$
$$= b \pm \sqrt{b}$$
$$x^2 = c^2 \pm \sqrt{c^2}$$
$$x = \pm\sqrt{c^2 \pm c}$$

29. (A)

$$x\log_7 3^2 = \log_7 7 \cdot 4$$
$$x = \frac{\log_7 7 + \log_7 4}{2\log_7 3}$$
$$x = \frac{1 + b}{2a}$$

30. (B) Assuming $|x| < 1$,

$$x + x^2 + x^3 + x^4 + \ldots = \frac{x}{1 - x} = \frac{3}{2}$$
$$x = \frac{3}{5}, \text{ which agrees with our assumption.}$$

31. (D)

$$f \otimes g(x) = (2x + 1)^2 - 1 - \left(2(x^2 - 1) + 1\right)$$
$$= 4x^2 + 4x + 1 - 1 - (2x^2 - 2 + 1)$$
$$= 2x^2 + 4x + 1$$

32. (A)

$$p \otimes r(x) = 5(at + b) - 2 - (a(5t - 2) + b) = 0$$
$$5b - 2 - (-2a + b) = 0$$
$$a + 2b = 1$$

33. (B) In the first 60 miles, R.C. has driven $\frac{60}{50} = \frac{6}{5}$ hours. In the next 60 miles, R.C. has only $\frac{4}{5}$ hours to drive, so he must go $\frac{60}{\frac{4}{5}} = 75$ mph.

34. (A) The input of $f(x)$ will be the output of $f^{-1}(x)$ and the output of $f(x)$ will become the input of f^{-1}. This means that $x = f^{-1}(t) = 0.2$ and $t = f(x) = f(0.2) = -4.792$

35. (C)

$$V_m = V_0 + \frac{1}{t}(V_0 t + \frac{1}{2}at^2)$$

$$\frac{2}{a}(V_m - 2V_0) = t$$

36. (B) Let f = the connection fee, and r = the charger per kwh. There is a system of equations:

$$f + 980r = \$80.36$$

$$f + 910r = \$75.53$$

$$70r = \$4.83$$

$$r = \$0.069 = 6.9 \text{ cents}$$

37. (D)

$$2^{x+2} = 4^{3x-4}$$

$$2^{x+2} = \left(2^2\right)^{3x-4}$$

$$2^{x+2} = 2^{2(3x-4)}$$

$$x + 2 = 6x - 8$$

$$x = 2,\ y = 16$$

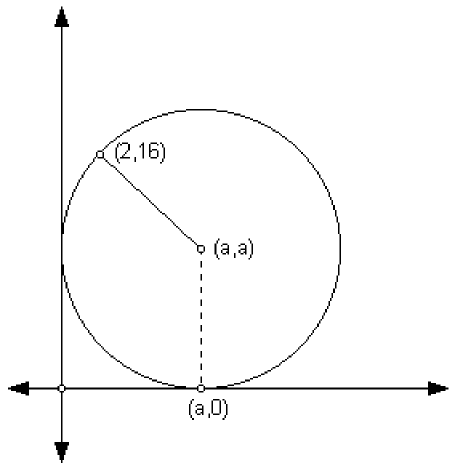
38. (C) Since all radii are congruent, the radius extending from the center, (a, a) to $(2, 16)$ and the radius extending from (a, a) to one of the axes are congruent.

$$\sqrt{(a-2)^2 + (a-16)^2} = a$$

$$a^2 - 36a + 260 = 0$$

$$(a - 10)(a - 26) = 0$$

There is more than one circle that is tangent to the axes and contains the point $(2, 16)$, but the smaller of the 2 circles has a radius of 10 and is centered at $(10, 10)$.



39. (B) The line connecting the point and $(4, 0)$ will be perpendicular to $y = 2x - 3$. It has the equation $y = -\frac{1}{2}x + 2$. The point is located at the intersection of the two lines.

$$2x - 3 = -\frac{1}{2}x + 2$$

$$x = 2, \; y = 1$$

40. (D) We can find the area of the triangle made by 3 points, $(2, 16), (10, 10), (2, 1)$, in the coordinate plane using the formula

$$A = \left| \frac{1}{2} \begin{vmatrix} 2 & 16 & 1 \\ 10 & 10 & 1 \\ 2 & 1 & 1 \end{vmatrix} \right| = \frac{1}{2} |-120| = 60$$

where each row corresponds to a point. The first column contains x-coordinates, the second column contains y-coordinates, and the third column is 1.