## 2.3 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

**In Exercises 1–6, use the Product Rule to differentiate the function.**

**1.** $g(x) = (x^2 + 3)(x^2 - 4x)$

**2.** $f(x) = (6x + 5)(x^3 - 2)$

**3.** $h(t) = \sqrt{t}(1 - t^2)$

**4.** $g(s) = \sqrt{s}(s^2 + 8)$

**5.** $f(x) = x^3 \cos x$

**6.** $g(x) = \sqrt{x} \sin x$

**In Exercises 7–12, use the Quotient Rule to differentiate the function.**

**7.** $f(x) = \dfrac{x}{x^2 + 1}$

**8.** $g(t) = \dfrac{t^2 + 4}{5t - 3}$

**9.** $h(x) = \dfrac{\sqrt{x}}{x^3 + 1}$

**10.** $h(s) = \dfrac{s}{\sqrt{s} - 1}$

**11.** $g(x) = \dfrac{\sin x}{x^2}$

**12.** $f(t) = \dfrac{\cos t}{t^3}$

**In Exercises 13–18, find $f'(x)$ and $f'(c)$.**

| | *Function* | *Value of c* |
|---|---|---|
| **13.** | $f(x) = (x^3 + 4x)(3x^2 + 2x - 5)$ | $c = 0$ |
| **14.** | $f(x) = (x^2 - 2x + 1)(x^3 - 1)$ | $c = 1$ |
| **15.** | $f(x) = \dfrac{x^2 - 4}{x - 3}$ | $c = 1$ |
| **16.** | $f(x) = \dfrac{x + 5}{x - 5}$ | $c = 4$ |
| **17.** | $f(x) = x \cos x$ | $c = \dfrac{\pi}{4}$ |
| **18.** | $f(x) = \dfrac{\sin x}{x}$ | $c = \dfrac{\pi}{6}$ |

**In Exercises 19–24, complete the table without using the Quotient Rule.**

| | *Function* | *Rewrite* | *Differentiate* | *Simplify* |
|---|---|---|---|---|
| **19.** | $y = \dfrac{x^2 + 3x}{7}$ | | | |
| **20.** | $y = \dfrac{5x^2 - 3}{4}$ | | | |
| **21.** | $y = \dfrac{6}{7x^2}$ | | | |
| **22.** | $y = \dfrac{10}{3x^3}$ | | | |
| **23.** | $y = \dfrac{4x^{3/2}}{x}$ | | | |
| **24.** | $y = \dfrac{5x^2 - 8}{11}$ | | | |

**In Exercises 25–38, find the derivative of the algebraic function.**

**25.** $f(x) = \dfrac{4 - 3x - x^2}{x^2 - 1}$

**26.** $f(x) = \dfrac{x^3 + 5x + 3}{x^2 - 1}$

**27.** $f(x) = x\left(1 - \dfrac{4}{x + 3}\right)$

**28.** $f(x) = x^4\left(1 - \dfrac{2}{x + 1}\right)$

**29.** $f(x) = \dfrac{3x - 1}{\sqrt{x}}$

**30.** $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

**31.** $h(s) = (s^3 - 2)^2$

**32.** $h(x) = (x^2 - 1)^2$

**33.** $f(x) = \dfrac{2 - \frac{1}{x}}{x - 3}$

**34.** $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x + 1}\right)$

**35.** $f(x) = (2x^3 + 5x)(x - 3)(x + 2)$

**36.** $f(x) = (x^3 - x)(x^2 + 2)(x^2 + x - 1)$

**37.** $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$, $c$ is a constant

**38.** $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$, $c$ is a constant

**In Exercises 39–54, find the derivative of the trigonometric function.**

**39.** $f(t) = t^2 \sin t$

**40.** $f(\theta) = (\theta + 1) \cos \theta$

**41.** $f(t) = \dfrac{\cos t}{t}$

**42.** $f(x) = \dfrac{\sin x}{x^3}$

**43.** $f(x) = -x + \tan x$

**44.** $y = x + \cot x$

**45.** $g(t) = \sqrt[4]{t} + 6 \csc t$

**46.** $h(x) = \dfrac{1}{x} - 12 \sec x$

**47.** $y = \dfrac{3(1 - \sin x)}{2 \cos x}$

**48.** $y = \dfrac{\sec x}{x}$

**49.** $y = -\csc x - \sin x$

**50.** $y = x \sin x + \cos x$

**51.** $f(x) = x^2 \tan x$

**52.** $f(x) = \sin x \cos x$

**53.** $y = 2x \sin x + x^2 \cos x$

**54.** $h(\theta) = 5\theta \sec \theta + \theta \tan \theta$

**CAS** **In Exercises 55–58, use a computer algebra system to differentiate the function.**

**55.** $g(x) = \left(\dfrac{x + 1}{x + 2}\right)(2x - 5)$

**56.** $f(x) = \left(\dfrac{x^2 - x - 3}{x^2 + 1}\right)(x^2 + x + 1)$

**57.** $g(\theta) = \dfrac{\theta}{1 - \sin \theta}$

**58.** $f(\theta) = \dfrac{\sin \theta}{1 - \cos \theta}$

*The symbol* **CAS** *indicates an exercise in which you are instructed to specifically use a computer algebra system.*

**In Exercises 59–62, evaluate the derivative of the function at the given point. Use a graphing utility to verify your result.**

| *Function* | *Point* |
|---|---|
| 59. $y = \dfrac{1 + \csc x}{1 - \csc x}$ | $\left(\dfrac{\pi}{6}, -3\right)$ |
| 60. $f(x) = \tan x \cot x$ | $(1, 1)$ |
| 61. $h(t) = \dfrac{\sec t}{t}$ | $\left(\pi, -\dfrac{1}{\pi}\right)$ |
| 62. $f(x) = \sin x(\sin x + \cos x)$ | $\left(\dfrac{\pi}{4}, 1\right)$ |

**In Exercises 63–68, (a) find an equation of the tangent line to the graph of $f$ at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.**

63. $f(x) = (x^3 + 4x - 1)(x - 2), \quad (1, -4)$

64. $f(x) = (x + 3)(x^2 - 2), \quad (-2, 2)$

65. $f(x) = \dfrac{x}{x + 4}, \quad (-5, 5)$

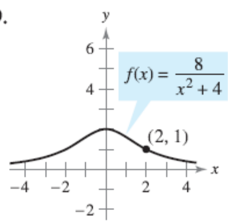
66. $f(x) = \dfrac{(x - 1)}{(x + 1)}, \quad \left(2, \dfrac{1}{3}\right)$

67. $f(x) = \tan x, \quad \left(\dfrac{\pi}{4}, 1\right)$

68. $f(x) = \sec x, \quad \left(\dfrac{\pi}{3}, 2\right)$

***Famous Curves*** **In Exercises 69–72, find an equation of the tangent line to the graph at the given point. (The graphs in Exercises 69 and 70 are called *Witches of Agnesi*. The graphs in Exercises 71 and 72 are called *serpentines*.)**

69.

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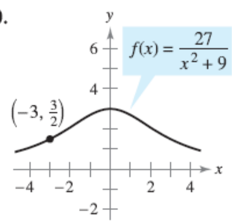


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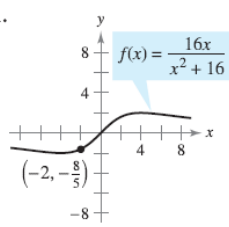


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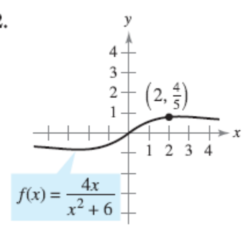



**In Exercises 73–76, determine the point(s) at which the graph of the function has a horizontal tangent line.**

73. $f(x) = \dfrac{2x - 1}{x^2}$

74. $f(x) = \dfrac{x^2}{x^2 + 1}$

75. $f(x) = \dfrac{x^2}{x - 1}$

76. $f(x) = \dfrac{x - 4}{x^2 - 7}$

77. ***Tangent Lines*** Find equations of the tangent lines to the graph of $f(x) = (x + 1)/(x - 1)$ that are parallel to the line $2y + x = 6$. Then graph the function and the tangent lines.

78. ***Tangent Lines*** Find equations of the tangent lines to the graph of $f(x) = x/(x - 1)$ that pass through the point $(-1, 5)$. Then graph the function and the tangent lines.

**In Exercises 79 and 80, verify that $f'(x) = g'(x)$, and explain the relationship between $f$ and $g$.**

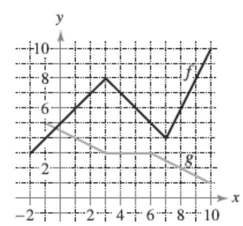
79. $f(x) = \dfrac{3x}{x + 2}, \quad g(x) = \dfrac{5x + 4}{x + 2}$

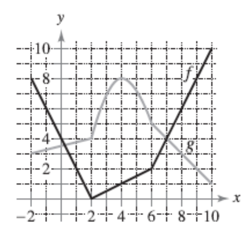
80. $f(x) = \dfrac{\sin x - 3x}{x}, \quad g(x) = \dfrac{\sin x + 2x}{x}$

**In Exercises 81 and 82, use the graphs of $f$ and $g$. Let $p(x) = f(x)g(x)$ and $q(x) = f(x)/g(x)$.**

81. (a) Find $p'(1)$.
(b) Find $q'(4)$.

82. (a) Find $p'(4)$.
(b) Find $q'(7)$.




83. ***Area*** The length of a rectangle is given by $6t + 5$ and its height is $\sqrt{t}$, where $t$ is time in seconds and the dimensions are in centimeters. Find the rate of change of the area with respect to time.

84. ***Volume*** The radius of a right circular cylinder is given by $\sqrt{t + 2}$ and its height is $\frac{1}{2}\sqrt{t}$, where $t$ is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time.

85. ***Inventory Replenishment*** The ordering and transportation cost $C$ for the components used in manufacturing a product is

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \geq 1$$

where $C$ is measured in thousands of dollars and $x$ is the order size in hundreds. Find the rate of change of $C$ with respect to $x$ when (a) $x = 10$, (b) $x = 15$, and (c) $x = 20$. What do these rates of change imply about increasing order size?

86. ***Boyle's Law*** This law states that if the temperature of a gas remains constant, its pressure is inversely proportional to its volume. Use the derivative to show that the rate of change of the pressure is inversely proportional to the square of the volume.

87. ***Population Growth*** A population of 500 bacteria is introduced into a culture and grows in number according to the equation

$$P(t) = 500\left(1 + \frac{4t}{50 + t^2}\right)$$

where $t$ is measured in hours. Find the rate at which the population is growing when $t = 2$.

**88.** ***Gravitational Force*** Newton's Law of Universal Gravitation states that the force $F$ between two masses, $m_1$ and $m_2$, is

$$F = \frac{Gm_1m_2}{d^2}$$

where $G$ is a constant and $d$ is the distance between the masses. Find an equation that gives an instantaneous rate of change of $F$ with respect to $d$. (Assume that $m_1$ and $m_2$ represent moving points.)

**89.** Prove the following differentiation rules.

(a) $\frac{d}{dx}[\sec x] = \sec x \tan x$ (b) $\frac{d}{dx}[\csc x] = -\csc x \cot x$

(c) $\frac{d}{dx}[\cot x] = -\csc^2 x$

**90.** ***Rate of Change*** Determine whether there exist any values of $x$ in the interval $[0, 2\pi)$ such that the rate of change of $f(x) = \sec x$ and the rate of change of $g(x) = \csc x$ are equal.

**91.** ***Modeling Data*** The table shows the quantities $q$ (in millions) of personal computers shipped in the United States and the values $v$ (in billions of dollars) of these shipments for the years 1999 through 2004. The year is represented by $t$, with $t = 9$ corresponding to 1999. *(Source: U.S. Census Bureau)*

| Year, $t$ | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|
| $q$ | 19.6 | 15.9 | 14.6 | 12.9 | 15.0 | 15.8 |
| $v$ | 26.8 | 22.6 | 18.9 | 16.2 | 14.7 | 15.3 |

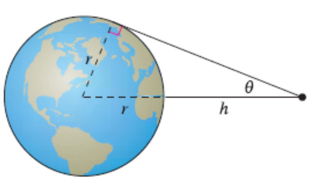
(a) Use a graphing utility to find cubic models for the quantity of personal computers shipped $q(t)$ and the value $v(t)$ of the personal computers.

(b) Graph each model found in part (a).

(c) Find $A = v(t)/q(t)$, then graph $A$. What does this function represent?

(d) Interpret $A'(t)$ in the context of these data.

**92.** ***Satellites*** When satellites observe Earth, they can scan only part of Earth's surface. Some satellites have sensors that can measure the angle $\theta$ shown in the figure. Let $h$ represent the satellite's distance from Earth's surface and let $r$ represent Earth's radius.




(a) Show that $h = r(\csc \theta - 1)$.

(b) Find the rate at which $h$ is changing with respect to $\theta$ when $\theta = 30°$. (Assume $r = 3960$ miles.)

**In Exercises 93–100, find the second derivative of the function.**

**93.** $f(x) = x^4 + 2x^3 - 3x^2 - x$ **94.** $f(x) = 8x^6 - 10x^5 + 5x^3$

**95.** $f(x) = 4x^{3/2}$ **96.** $f(x) = x + 32x^{-2}$

**97.** $f(x) = \frac{x}{x - 1}$ **98.** $f(x) = \frac{x^2 + 2x - 1}{x}$

**99.** $f(x) = x \sin x$ **100.** $f(x) = \sec x$

**In Exercises 101–104, find the given higher-order derivative.**

**101.** $f'(x) = x^2, \quad f''(x)$ **102.** $f''(x) = 2 - \frac{2}{x}, \quad f'''(x)$

**103.** $f'''(x) = 2\sqrt{x}, \quad f^{(4)}(x)$ **104.** $f^{(4)}(x) = 2x + 1, \quad f^{(6)}(x)$

**In Exercises 105–108, use the given information to find $f'(2)$.**

$g(2) = 3$ and $g'(2) = -2$

$h(2) = -1$ and $h'(2) = 4$

**105.** $f(x) = 2g(x) + h(x)$ **106.** $f(x) = 4 - h(x)$

**107.** $f(x) = \frac{g(x)}{h(x)}$ **108.** $f(x) = g(x)h(x)$

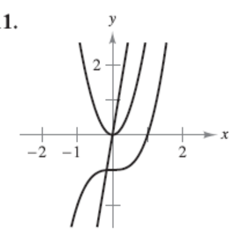
**WRITING ABOUT CONCEPTS**

**109.** Sketch the graph of a differentiable function $f$ such that $f(2) = 0$, $f' < 0$ for $-\infty < x < 2$, and $f' > 0$ for $2 < x < \infty$. Explain how you found your answer.

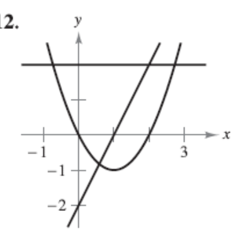
**110.** Sketch the graph of a differentiable function $f$ such that $f > 0$ and $f' < 0$ for all real numbers $x$. Explain how you found your answer.

**In Exercises 111 and 112, the graphs of $f$, $f'$, and $f''$ are shown on the same set of coordinate axes. Identify each graph. Explain your reasoning. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

**111.**

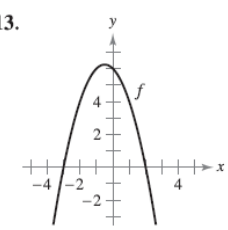



**112.**

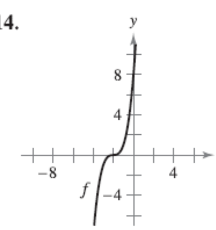



**In Exercises 113–116, the graph of $f$ is shown. Sketch the graphs of $f'$ and $f''$. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

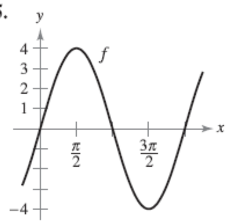
**113.**



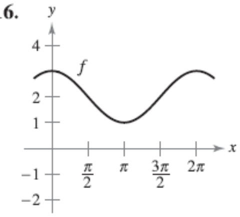

**114.**



**115.**




**116.**




**117.** ***Acceleration*** The velocity of an object in meters per second is $v(t) = 36 - t^2, 0 \le t \le 6$. Find the velocity and acceleration of the object when $t = 3$. What can be said about the speed of the object when the velocity and acceleration have opposite signs?

**118.** ***Acceleration*** An automobile's velocity starting from rest is

$$v(t) = \frac{100t}{2t + 15}$$

where $v$ is measured in feet per second. Find the acceleration at (a) 5 seconds, (b) 10 seconds, and (c) 20 seconds.

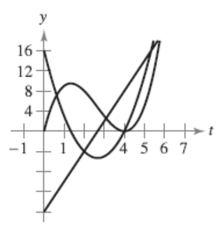
**119.** ***Stopping Distance*** A car is traveling at a rate of 66 feet per second (45 miles per hour) when the brakes are applied. The position function for the car is $s(t) = -8.25t^2 + 66t$, where $s$ is measured in feet and $t$ is measured in seconds. Use this function to complete the table, and find the average velocity during each time interval.

| $t$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $s(t)$ | | | | | |
| $v(t)$ | | | | | |
| $a(t)$ | | | | | |

**CAPSTONE**

**120.** ***Particle Motion*** The figure shows the graphs of the position, velocity, and acceleration functions of a particle.

(a) Copy the graphs of the functions shown. Identify each graph. Explain your reasoning. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.




(b) On your sketch, identify when the particle speeds up and when it slows down. Explain your reasoning.

***Finding a Pattern*** **In Exercises 121 and 122, develop a general rule for $f^{(n)}(x)$ given $f(x)$.**

**121.** $f(x) = x^n$ **122.** $f(x) = \frac{1}{x}$

**123.** ***Finding a Pattern*** Consider the function $f(x) = g(x)h(x)$.

(a) Use the Product Rule to generate rules for finding $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$.

(b) Use the results of part (a) to write a general rule for $f^{(n)}(x)$.

**124.** ***Finding a Pattern*** Develop a general rule for $[xf(x)]^{(n)}$ where $f$ is a differentiable function of $x$.

**In Exercises 125 and 126, find the derivatives of the function $f$ for $n = 1, 2, 3$, and 4. Use the results to write a general rule for $f'(x)$ in terms of $n$.**

**125.** $f(x) = x^n \sin x$ **126.** $f(x) = \frac{\cos x}{x^n}$

***Differential Equations*** **In Exercises 127–130, verify that the function satisfies the differential equation.**

| | *Function* | *Differential Equation* |
|---|---|---|
| **127.** | $y = \frac{1}{x}, x > 0$ | $x^3 y'' + 2x^2 y' = 0$ |
| **128.** | $y = 2x^3 - 6x + 10$ | $-y''' - xy'' - 2y' = -24x^2$ |
| **129.** | $y = 2 \sin x + 3$ | $y'' + y = 3$ |
| **130.** | $y = 3 \cos x + \sin x$ | $y'' + y = 0$ |

***True or False?*** **In Exercises 131–136, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

**131.** If $y = f(x)g(x)$, then $dy/dx = f'(x)g'(x)$.

**132.** If $y = (x + 1)(x + 2)(x + 3)(x + 4)$, then $d^5y/dx^5 = 0$.

**133.** If $f'(c)$ and $g'(c)$ are zero and $h(x) = f(x)g(x)$, then $h'(c) = 0$.

**134.** If $f(x)$ is an $n$th-degree polynomial, then $f^{(n+1)}(x) = 0$.

**135.** The second derivative represents the rate of change of the first derivative.

**136.** If the velocity of an object is constant, then its acceleration is zero.

**137.** Find a second-degree polynomial $f(x) = ax^2 + bx + c$ such that its graph has a tangent line with slope 10 at the point $(2, 7)$ and an $x$-intercept at $(1, 0)$.

**138.** Consider the third-degree polynomial

$$f(x) = ax^3 + bx^2 + cx + d, \quad a \ne 0.$$

Determine conditions for $a$, $b$, $c$, and $d$ if the graph of $f$ has (a) no horizontal tangents, (b) exactly one horizontal tangent, and (c) exactly two horizontal tangents. Give an example for each case.

**139.** Find the derivative of $f(x) = x|x|$. Does $f''(0)$ exist?

**140.** ***Think About It*** Let $f$ and $g$ be functions whose first and second derivatives exist on an interval $I$. Which of the following formulas is (are) true?

(a) $fg'' - f''g = (fg' - f'g)'$ (b) $fg'' + f''g = (fg)''$

**141.** Use the Product Rule twice to prove that if $f$, $g$, and $h$ are differentiable functions of $x$, then

$$\frac{d}{dx}[f(x)g(x)h(x)] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x).$$