

Factor  $x^2 + 8x + 15$  completely.

Choral Response Questions  
 Expression, equation, or inequality?  
*Expression.*  
 What type of polynomial? *Trinomial.*  
 Is there a GCF other than 1? *No.*

Product	Sum
$\_\_ \bullet \_\_ = 15$	$\_\_ + \_\_ = 8$
$1 \bullet 15 = 15$	$1 + 15 = 16$
$3 \bullet 5 = 15$	$3 + 5 = 8$

**Factoring by Grouping**

Area Model

$$x^2 + 8x + 15$$

Based on our product/sum chart, we will use  $3x$  and  $5x$ .

$$= x^2 + 3x + 5x + 15$$

Fill in the area model.

$x^2$	$3x$
$5x$	$15$

Factor the GCF from each row and column.

$$x \quad +3$$

$x$	$x^2$	$3x$
$+5$	$5x$	$15$

$$= (x + 5)(x + 3)$$

Algebraically

$$x^2 + 8x + 15$$

$$= x^2 + 3x + 5x + 15$$

Group the first two terms and the last two terms.

$$= (x^2 + 3x) + (5x + 15)$$

Factor the GCF from each quantity.

$$= x(x + 3) + 5(x + 3)$$

$$= (x + 5)(x + 3)$$

**Factoring by Guess & Check**

Area Model

$$x^2 + 8x + 15$$

Fill in the first and last terms.

$x^2$	
	$15$

The first factors have to be  $x$  and  $x$ . Strategically guess which factors of 15 to try.

$$x \quad +3$$

$x$	$x^2$	
$+5$		$15$

Fill in the remaining areas and check the diagonal like terms.

$$x \quad +3$$

$x$	$x^2$	$3x$
$+5$	$5x$	$15$

$$= (x + 5)(x + 3)$$

Algebraically

$$x^2 + 8x + 15$$

$$= (x \quad)(x \quad)$$

$$= (x + \quad)(x + \quad)$$

$$= (x + 5)(x + 3)$$

The First and Last terms are the same when we multiply the trial factors. What changes is the middle term. Thus, it is only necessary to check the Outer and Inner terms.

$$(x + 5)(x + 3)$$

$$3x + 5x = 8x \quad \checkmark$$

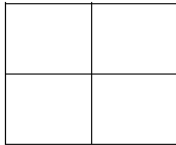
\*Note that when teaching area models, the area represents the trinomial and the dimensions represent the factors.

**Factor  $x^2 + 19x + 60$  completely. Use guess and check with an area model and algebraically.**

If needed students can still use a product/sum chart. The goal is for students to develop their number sense and intuition to factor quadratics without using a product/sum chart.

Product	Sum
$\_ \cdot \_ =$	$\_ + \_ =$

Choral Response Questions  
Expression, equation, or inequality?  
What type of polynomial?  
Is there a GCF other than 1?

<u>Area Model using a Generic Rectangle</u>	<u>Algebraically</u>
$x^2 + 19x + 60$ 	$x^2 + 19x + 60$



## USING THE MIDDLE TERM TO GUESS MORE EFFECTIVELY

**You Try 3:** Factor the following expressions completely. Pay attention to the affect the middle term has on the factors.

$x^2 + 61x + 60$ $= (x \quad)(x \quad)$	$x^2 - 61x + 60$ $= (x \quad)(x \quad)$	$x^2 + 59x - 60$ $= (x \quad)(x \quad)$	$x^2 - 59x - 60$ $= (x \quad)(x \quad)$
$x^2 + 32x + 60$ $= (x \quad)(x \quad)$	$x^2 - 32x + 60$ $= (x \quad)(x \quad)$	$x^2 + 28x - 60$ $= (x \quad)(x \quad)$	$x^2 - 28x - 60$ $= (x \quad)(x \quad)$
$x^2 + 23x + 60$ $= (x \quad)(x \quad)$	$x^2 - 23x + 60$ $= (x \quad)(x \quad)$	$x^2 + 17x - 60$ $= (x \quad)(x \quad)$	$x^2 - 17x - 60$ $= (x \quad)(x \quad)$
$x^2 + 19x + 60$ $= (x \quad)(x \quad)$	$x^2 - 19x + 60$ $= (x \quad)(x \quad)$	$x^2 + 11x - 60$ $= (x \quad)(x \quad)$	$x^2 - 11x - 60$ $= (x \quad)(x \quad)$
$x^2 + 17x + 60$ $= (x \quad)(x \quad)$	$x^2 - 17x + 60$ $= (x \quad)(x \quad)$	$x^2 + 7x - 60$ $= (x \quad)(x \quad)$	$x^2 - 7x - 60$ $= (x \quad)(x \quad)$
$x^2 + 16x + 60$ $= (x \quad)(x \quad)$	$x^2 - 16x + 60$ $= (x \quad)(x \quad)$	$x^2 + 4x - 60$ $= (x \quad)(x \quad)$	$x^2 - 4x - 60$ $= (x \quad)(x \quad)$

What do you notice about how the middle term of the trinomial affects what factors you try first?

- Discuss the closer the middle term is to zero in the trinomial, the closer the factors will be in the binomials.
- Also, discuss the relationship between the sign of larger factor in the binomial and the sign of the middle term from the trinomial for the last two columns. When the last term of the trinomial is negative, the sign of the larger factor in the binomials should match the sign of the sum (note again this is only always true when  $a = 1$ ).
- By choosing a trinomial whose first term is  $x^2$ , and whose last term is  $\pm 60$ , it will help students understand how to make better guesses about which factors to use. Also, by choosing the number 60 which has 12 whole number factors, and varying its sign, students will have the ability to factor 24 similar trinomials.

Factor the following expression using guess and check with an area model and algebraically.

$$x^2 + 5x - 24$$

**Guess & Check**

**Area Model**

**Algebraically**

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Factor the following 24 expressions. Be ready to discuss how we know the signs of the binomials and how the middle term of the trinomial helps you decide which factors of 60 to use.

$x^2 + 61x + 60$ $= (x \quad)(x \quad)$	$x^2 - 61x + 60$ $= (x \quad)(x \quad)$	$x^2 + 59x - 60$ $= (x \quad)(x \quad)$	$x^2 - 59x - 60$ $= (x \quad)(x \quad)$
$x^2 + 32x + 60$ $= (x \quad)(x \quad)$	$x^2 - 32x + 60$ $= (x \quad)(x \quad)$	$x^2 + 28x - 60$ $= (x \quad)(x \quad)$	$x^2 - 28x - 60$ $= (x \quad)(x \quad)$
$x^2 + 23x + 60$ $= (x \quad)(x \quad)$	$x^2 - 23x + 60$ $= (x \quad)(x \quad)$	$x^2 + 17x - 60$ $= (x \quad)(x \quad)$	$x^2 - 17x - 60$ $= (x \quad)(x \quad)$
$x^2 + 19x + 60$ $= (x \quad)(x \quad)$	$x^2 - 19x + 60$ $= (x \quad)(x \quad)$	$x^2 + 11x - 60$ $= (x \quad)(x \quad)$	$x^2 - 11x - 60$ $= (x \quad)(x \quad)$
$x^2 + 17x + 60$ $= (x \quad)(x \quad)$	$x^2 - 17x + 60$ $= (x \quad)(x \quad)$	$x^2 + 7x - 60$ $= (x \quad)(x \quad)$	$x^2 - 7x - 60$ $= (x \quad)(x \quad)$
$x^2 + 16x + 60$ $= (x \quad)(x \quad)$	$x^2 - 16x + 60$ $= (x \quad)(x \quad)$	$x^2 + 4x - 60$ $= (x \quad)(x \quad)$	$x^2 - 4x - 60$ $= (x \quad)(x \quad)$