Systems of Equations … All Methods

1. Which linear system has the solution $x = -2$ and $y = 6$?

    a. $x + 3y = 16$
        $4x + 4y = 16$
    b. $x + 3y = 17$
        $2x + y = 15$

    c. $x + 2y = -2$
        $2x + 4y = -4$
    d. $2x + y = -2$
        $x + y = 16$

2. Which linear system has the solution $x = 4$ and $y = -2$?

    a. $x + 4y = 15$
        $4x - 2y = -17$
    b. $2x + 4y = 4$
        $-2x + y = 14$

    c. $4x + y = 14$
        $-2x + 4y = -16$
    d. $x + 4y = 4$
        $2x + 4y = 8$

3. Create a linear system to model this situation:
    A woman is 3 times as old as her son. In thirteen years, she will be 2 times as old as her son will be.

    a. $w = s + 3$
        $w + 13 = 2s$
    b. $w = 3s$
        $w + 13 = 2(s + 13)$

    c. $w = 3s$
        $w = 2s$
    d. $w = 3s$
        $s + 13 = 2(w + 13)$

4. Create a linear system to model this situation:
    A length of outdoor lights is formed from strings that are 5 ft. long and 11 ft. long.
    Fourteen strings of lights are 106 ft. long.

    a. $5x + 11y = 14$
        $x + y = 106$
    b. $x + y = 14$
        $5x + 11y = 106$

    c. $x + y = 14$
        $5x + 11y = 106(14)$
    d. $x + y = 14$
        $x + 2y = 106$

5. Create a linear system to model this situation:
    A rectangular field is 35 m longer than it is wide. The length of the fence around
    the perimeter of the field is 290 m.

    a. $l + 35 = w$
        $2l + 2w = 290$
    b. $l = w + 35$
        $2l + 2w = 290$
    c. $l = w + 35$
        $l + w = 290$
    d. $l = w + 35$
        $lw = 290$

6. Which graph represents the solution of the linear system:
    $y = -2x + 2$
    $y + 6 = 2x$

Systems of Equations … All Methods

Graph A

$(2, -2)$

Graph C

$(2.2, -0.5)$
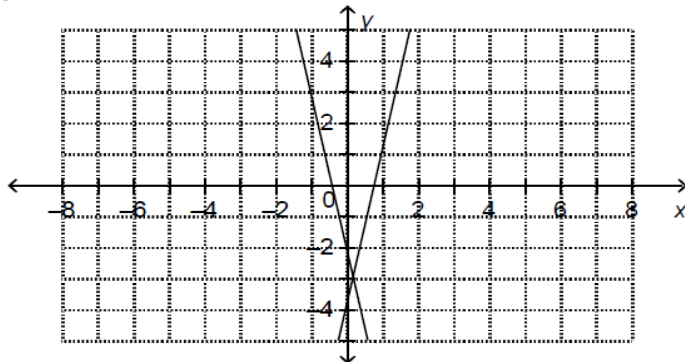
Graph B

$(1, -2)$

Graph D

$(1.4, -0.8)$
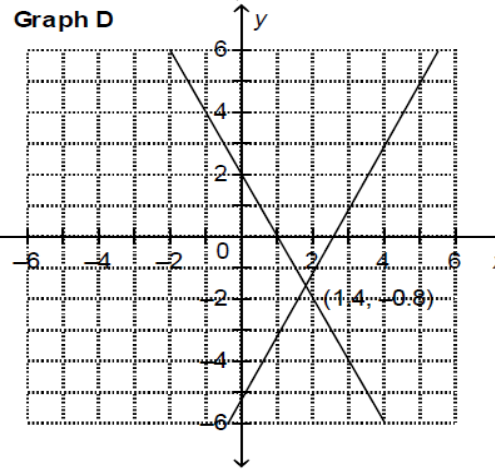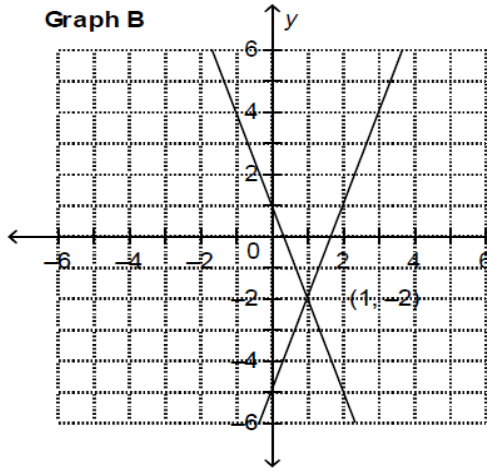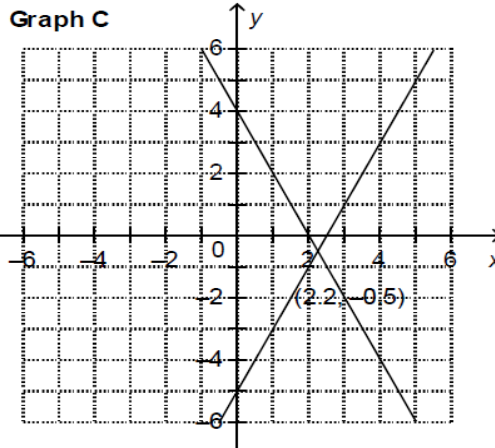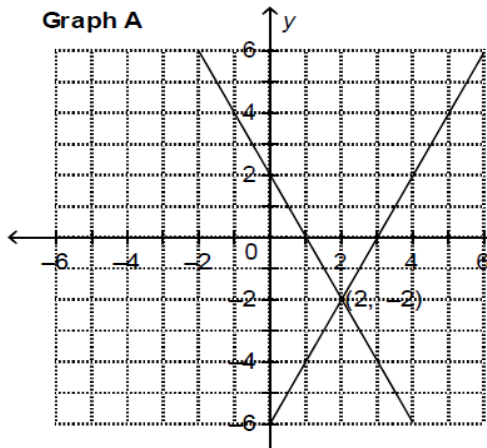
a.　Graph B
b.　Graph A

c.　Graph C
d.　Graph D

Use the graph to approximate the solution of the linear system:
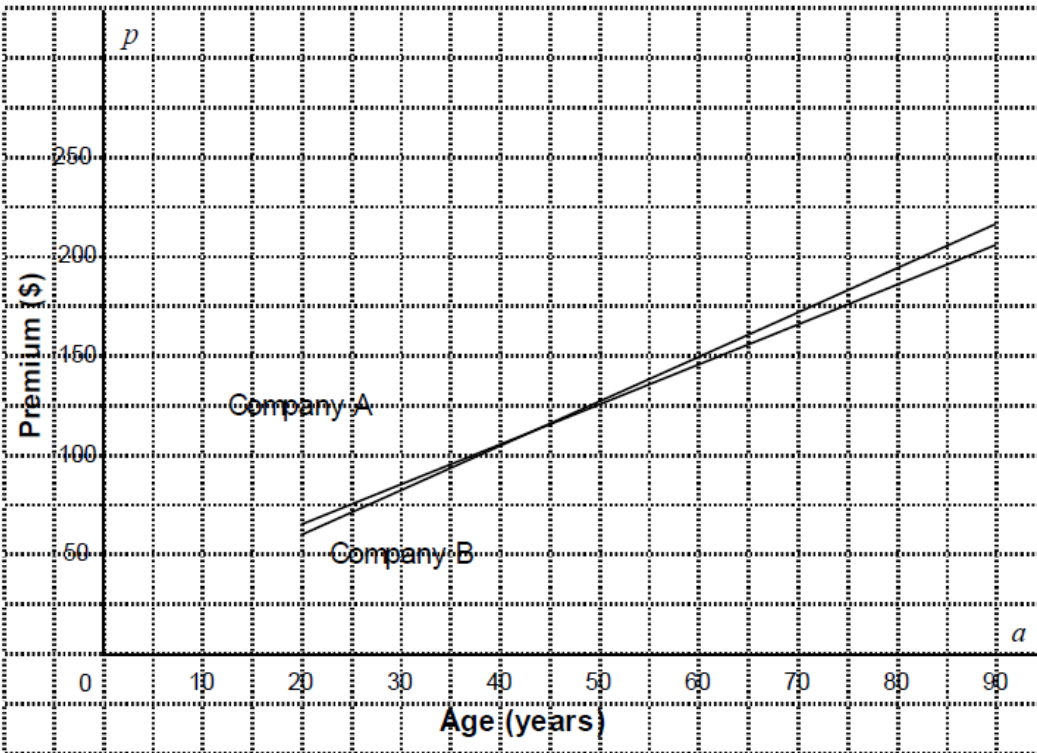$$y = -5x - 2$$
$$y = 5x - 4$$

a.　$(-3, 0.2)$
b.　$(0, -2.8)$

c.　$(0.2, -3)$
d.　$(-2.8, 0)$

8. Two life insurance companies determine their premiums using different formulas:
   Company A: $p = 2a + 24$
   Company B: $p = 2.25a + 13$, where p represents the annual premium,
   and a represents the client's age.

   Use the graph to determine the age at which both companies charge the same premium.

   a.   62 years   b.   24 years   c.   59 years   d.   44 years

9. Use the graph to approximate the solution of this linear system:
   $6x - 7y = -4$
   $-\frac{3}{5}y = 3x + 7$
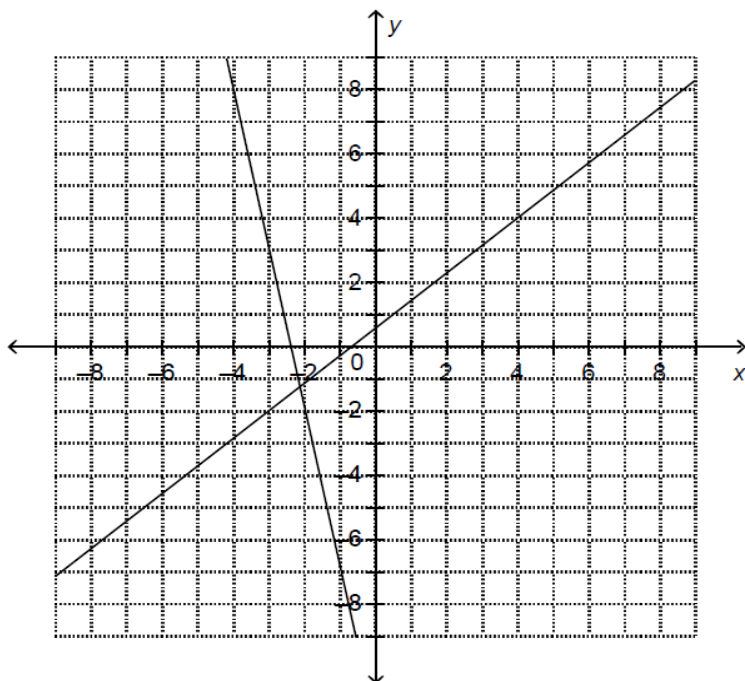
a.   (−0.1, 3.8)   b.   (−2.1, −1.2)   c.   (−1.2, 3.8)   d.   (−2.1, −0.1)

10. Express each equation in slope-intercept form.
$-2x + 4y = 68$
$13x + 4y = 284$

a.
$$y = \frac{1}{2}x - 17$$
$$y = -\frac{13}{4}x - 71$$

b.
$$y = -\frac{284}{13}x + 17$$
$$y = -\frac{13}{4}x + \frac{4}{13}$$

c.
$$y = \frac{1}{2}x + 17$$
$$y = -\frac{13}{4}x + 71$$

d.
$$y = \frac{4}{13}x - \frac{284}{13}$$
$$y = \frac{1}{2}x - \frac{284}{13}$$

11. Use substitution to solve this linear system:
$x - y = 18$
$$\frac{3}{4}x + \frac{3}{4}y = -\frac{15}{2}$$
a. $x = 4; y = 18$ b. $x = -14; y = -14$ c. $x = 4; y = -14$ d. $x = 4; y = 4$

12. Use an elimination strategy to solve this linear system.
$3x - 2y = 5$
$2x + 7y = 20$
a. $x = 3$ and $y = -2$ c. $x = 3$ and $y = 2$

b. $x = \dfrac{1}{5}$ and $y = \dfrac{14}{5}$

d. $x = -3$ and $y = -2$

13. Write an equivalent linear system where both equations have the same x-coefficients.
$2x + 6y = 5$
$8x - 6y = 12$

a. $8x + 6y = 20$ and $8x - 6y = 12$

b. $24x + 8y = 20$ and $6x + 8y = 12$

c. $8x + 24y = 20$ and $8x - 6y = 12$

d. $12x + 24y = 20$ and $12x - 6y = 12$

14. Determine the number of solutions of the linear system:
$14x - 5y = 123$
$14x - 5y = 73$
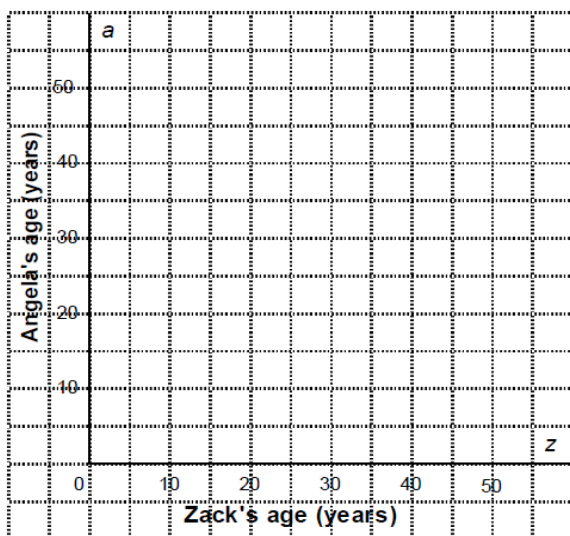
a. no solution

b. infinite solutions

c. two solutions

d. one solution

Answer

15. **a)** Write a linear system to model this situation:
Angela is 24 years older than her cousin Zack. In 13 years, she will be double his age.
b) Use a graph to solve this problem:
How old are Angela and Zack now?

16. Create a linear system to model this situation. Then use substitution to solve the linear system to solve the problem.

At the local fair, the admission fee is $8.00 for an adult and $4.50 for a youth. One Saturday, 209 admissions were purchased, with total receipts of $1304.50. How many adult admissions and how many youth admissions were purchased?

17. In a piggy bank, the number of nickels is 8 more than one-half the number of quarters. The value of the coins is $21.85.
    a) Create a linear system to model the situation.
    b) If the number of quarters is 78, determine the number of nickels.

18. **a)** Write a linear system to model this situation:
    A large tree removes 1.5 kg of pollution from the air each year. A small tree removes 0.04 kg each year. An urban forest has 1650 large and small trees. Together, these trees remove 1818 kg of pollution each year.
    b) Use graphing technology to solve this problem:
    How many of each size of tree are in the forest?
    c) Verify the solution.

19. **a)** Model this situation with a linear system:
    To rent a car, a person is charged a daily rate and a fee for each kilometre driven. When Chena rented a car for 15 days and drove 800 km, the charge was $715.00. When she rented the same car for 25 days and drove 2250 km, the charge was $1512.50.
    b) Determine the daily rate and the fee for each kilometre driven. Verify the solution.

20. Use an elimination strategy to solve this linear system. Verify the solution.
    $$2s - 2c = 10$$
    $$6s + 6c = 50$$

Answers

1. ANS: A

2. ANS: C

3. ANS: B

4. ANS: B

5. ANS: B

6. ANS: B

7. ANS: C

8. ANS: D

9. ANS: B

10. ANS: C

11. ANS: C

12. ANS: C

13. ANS: C

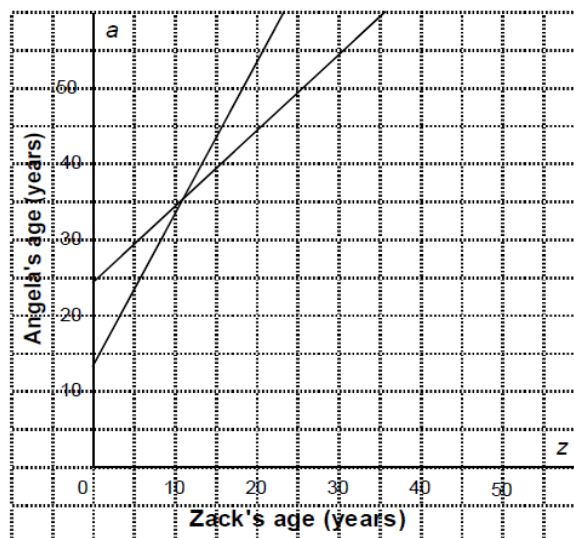14. ANS: A

15. ANS:
    a)

$$a = z + 24$$
$$a + 13 = 2(z + 13)$$

    b)   Zack is approximately 11 years old and Angela is approximately 35 years old.

    PTS: 1   DIF: Moderate   REF: 7.2 Solving a System of Linear Equations Graphically
    LOC: 10.RF9   TOP: Relations and Functions   KEY: Conceptual Understanding

16. ANS:
Let a represent the number of adult admissions, and y represent the number of youth admissions purchased.
$$a + y = 209$$
$$8a + 4.5y = 1304.5$$
104 adult admissions and 105 youth admissions were purchased.

    PTS: 1   DIF: Moderate
    REF: 7.4 Using a Substitution Strategy to Solve a System of Linear Equations
    LOC: 10.RF9   TOP: Relations and Functions   KEY: Conceptual Understanding

LEM

17. ANS:
    a) Let n represent the number of nickels and q represent the number of quarters.

$$n = \frac{1}{2}q + 8$$
$$0.05n + 0.25q = 21.85$$

    b) To determine the number of nickels when the number of quarters is 78:
    Substitute $q = 78$ in one of the equations in part a, then use the other equation to verify.

$$n = \frac{1}{2}q + 8$$

$n = \dfrac{1}{2} (78) + 8$

$n = 39 + 8$

$n = 47$

The number of nickels is 47.

To verify, substitute $q = 78$ and $n = 47$ into $0.05n + 0.25q = 21.85$.

L.S. $= 0.05n + 0.25q$
$= 0.05(47) + 0.25(78)$
$= 2.35 + 19.5$
$= 21.85$

R.S. $= 21.85$

Since the left side equals the right side, the number of nickels in the collection must be 47.

PTS:　1　　　　　　　DIF:　Difficult　　　REF:　7.1 Developing Systems of Linear Equations
LOC:　10.RF9　　　　TOP:　Relations and Functions　　　　　KEY:　Problem-Solving Skills

18.　ANS:
　　a)　Let l represent the number of large trees. Let s represent the number of small trees.
　　　　An urban forest has a total of 1650 trees.
　　　　So, one equation is: $l + s = 1650$
　　　　Together, the trees remove 1818 kg of pollution. So, another equation is:
　　　　$1.5l + 0.04s = 1818$

　　　　Then, a linear system is:
　　　　$l + s = 1650$
　　　　$1.5l + 0.04s = 1818$

　　b)　$l + s = 1650$　　(1)
　　　　$1.5l + 0.04s = 1818$　　(2)

　　　　Write each equation in the form $y = mx + b$.
　　　　Equation (1):
　　　　$l + s = 1650$　　　　　　Subtract l from each side.
　　　　　　$s = -l + 1650$
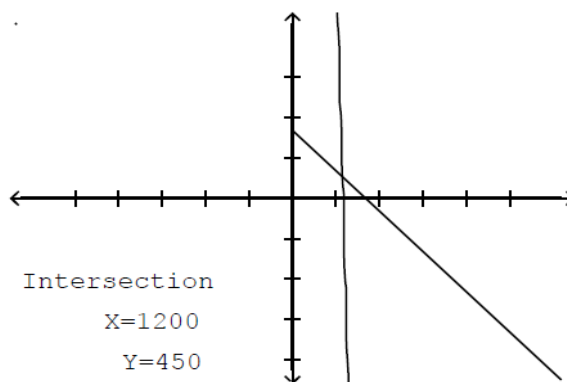　　　　Equation (2):
　　　　$1.5l + 0.04s = 1818$
　　　　　　$0.04s = -1.5l + 1818$　　　　　　　　Divide each side by 0.04.
　　　　　　　　$s = -37.5l + 45\ 450$

On a graphing calculator, input the expressions:

Y1 =(−)X+1650 and

Y2 =(−)37.5*X+45 450

Adjust the window settings so the point of intersection appears in the window, then press graph.

From the calculator screen, the solution is:

$x = 1200$ and $y = 450$

```
Intersection
    X=1200
    Y=450
```

The value of x is the value of l, so the number of large trees is 1200.

The value of y is the value of s, so the number of small trees is 450.

c) The total number of trees is: $1200 + 450 = 1650$; this is the same as the given information.

1200 large trees and 450 small trees remove $1.5(1200) + 0.04(450)$, or 1818 kg of pollution; this is the same as the given information.

The solution is correct.

PTS: 1   DIF: Difficult

REF: 7.3 Using Graphing Technology to Solve a System of Linear Equations

LOC: 10.RF9   TOP: Relations and Functions   KEY: Problem-Solving Skills

19. ANS:

a) Let d dollars represent the daily rate and let k dollars represent the fee for each kilometre driven.

The linear system is:

$15d + 800k = 715$ ①

$25d + 2250k = 1512.5$ ②

b) Multiply equation ① by 25 and equation ② by 15, then subtract to eliminate d.

$25 \times$ equation ①: $25(15d + 800k = 715)$

$375d + 20000k = 17875$ ③

$15 \times$ equation ②: $15(25d + 2250k = 1512.5)$

$375d + 33750k = 22687.5$ ④

Subtract equation ④ from equation ③.

$$375d + 20000k = 17875 \quad ③$$
$$-(375d + 33750k = 22687.5) \quad ④$$
$$-13750k = -4812.5$$
$$k = 0.35$$

Substitute $k = 0.35$ in equation ①.

$$15d + 800k = 715 \qquad \qquad ①$$
$$15d + 800(0.35) = 715$$
$$15d + 280 = 715$$
$$15d = 435$$
$$d = 29$$

Verify the solution.
In each equation, substitute: $k = 0.35$ and $d = 29$

$15d + 800k = 715 \qquad ①$
L.S. $= 15d + 800k$

$\quad = 15(29) + 800(0.35)$

$\quad = 435 + 280$

$\quad = 715$

$\quad =$ R.S.

$25d + 2250k = 1512.5 \quad ②$
L.S. $= 25d + 2250k$

$\quad = 25(29) + 2250(0.35)$

$\quad = 725 + 787.5$

$\quad = 1512.5$

$\quad =$ R.S.

So, the daily rate is \$29 and the fee for each kilometre driven is \$0.35.

PTS: 1   DIF: Difficult
REF: 7.5 Using an Elimination Strategy to Solve a System of Linear Equations
LOC: 10.RF9   TOP: Relations and Functions   KEY: Problem-Solving Skills
20. ANS:
$2s - 2c = 10 \qquad\qquad ①$
$6s + 6c = 50 \qquad\qquad ②$

Multiply equation ① by 3, then add to eliminate c.
$3 \times$ equation ①: $3(2s - 2c = 10)$
$$6s - 6c = 30 \qquad\qquad ③$$

Add:
$$\begin{array}{ll} 6s - 6c = 30 & ③ \\ + \; 6s + 6c = 50 & \\ \hline 12s \qquad = 80 & ② \end{array}$$

$$s = \frac{80}{12}$$

$$s = \frac{20}{3}$$

Substitute $s = \dfrac{20}{3}$ in equation ①.

$$2s - 2c = 10$$

$$2\left(\frac{20}{3}\right) - 2c = 10$$

$$\frac{40}{3} - 2c = 10$$

$$-2c = 10 - \frac{40}{3}$$

$$-2c = \frac{30}{3} - \frac{40}{3}$$

$$-2c = -\frac{10}{3}$$

$$c = \frac{10}{6}$$

$$c = \frac{5}{3}$$

Verify the solution.

In each equation, substitute: $s = \dfrac{20}{3}$ and $c = \dfrac{5}{3}$

$2s - 2c = 10$ $6s + 6c = 50$

L.S. $= 2s - 2c$ L.S. $= 6s + 6c$

$$= 2\left(\frac{20}{3}\right) - 2\left(\frac{5}{3}\right) \qquad\qquad\qquad = 6\left(\frac{20}{3}\right) + 6\left(\frac{5}{3}\right)$$

$$= \frac{40}{3} - \frac{10}{3} \qquad\qquad\qquad\qquad\qquad = 40 + 10$$

$$= \frac{30}{3} \qquad\qquad\qquad\qquad\qquad\qquad = 50$$

$$= 10 \qquad\qquad\qquad\qquad\qquad\qquad = \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$s = \frac{20}{3} \text{ and } c = \frac{5}{3}$$

PTS: 1   DIF: Difficult
REF: 7.5 Using an Elimination Strategy to Solve a System of Linear Equations
LOC: 10.RF9   TOP: Relations and Functions
KEY: Communication | Problem-Solving Skills