

## TRIGONOMETRY

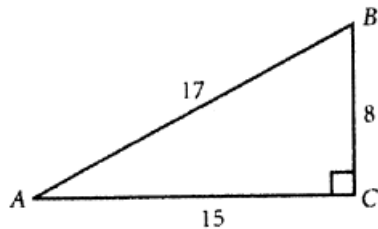
### 96. SINE, COSINE, AND TANGENT OF ACUTE ANGLES

To find the sine, cosine, or tangent of an acute angle, use SOHCAHTOA, which is an abbreviation for the following definitions:

$$\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}$$



In the figure above:

$$\sin A = \frac{8}{17}$$

$$\cos A = \frac{15}{17}$$

$$\tan A = \frac{8}{15}$$

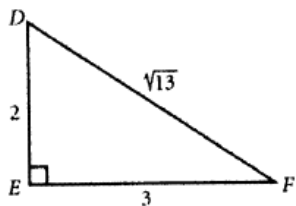
### 97. COTANGENT, SECANT, AND COSECANT OF ACUTE ANGLES

Think of the cotangent, secant, and cosecant as the reciprocals of the SOHCAHTOA functions:

$$\text{Cotangent} = \frac{1}{\text{Tangent}} = \frac{\text{Adjacent}}{\text{Opposite}}$$

$$\text{Secant} = \frac{1}{\text{Cosine}} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$$

$$\text{Cosecant} = \frac{1}{\text{Sine}} = \frac{\text{Hypotenuse}}{\text{Opposite}}$$



In the preceding figure:

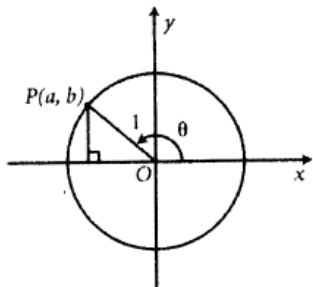
$$\cot D = \frac{2}{3}$$

$$\sec D = \frac{\sqrt{13}}{2}$$

$$\csc D = \frac{\sqrt{13}}{3}$$

### 98. TRIGONOMETRIC FUNCTIONS OF OTHER ANGLES

To find a trigonometric function of an angle greater than  $90^\circ$ , sketch a circle of radius 1 and centered at the origin of the coordinate grid. Start from the point  $(1, 0)$  and rotate the appropriate number of degrees counterclockwise.



In the “unit circle” setup above, the basic trigonometric functions are defined in terms of the coordinates  $a$  and  $b$ :

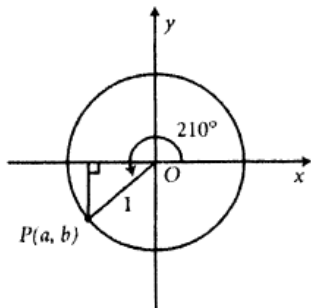
$$\sin \theta = b$$

$$\cos \theta = a$$

$$\tan \theta = \frac{b}{a}$$

**Example:**  $\sin 210^\circ = ?$

**Setup:** Sketch a  $210^\circ$  angle in the coordinate plane:



Because the triangle shown in the figure above is a  $30^\circ$ - $60^\circ$ - $90^\circ$  right triangle, we can determine that the coordinates of point  $P$  are  $-\frac{\sqrt{3}}{2}, -\frac{1}{2}$ .  
The sine is therefore  $-\frac{1}{2}$ .

#### 99. SIMPLIFYING TRIGONOMETRIC EXPRESSIONS

To simplify trigonometric expressions, use the inverse function definitions along with the fundamental trigonometric identity:

$$\sin^2 x + \cos^2 x = 1$$

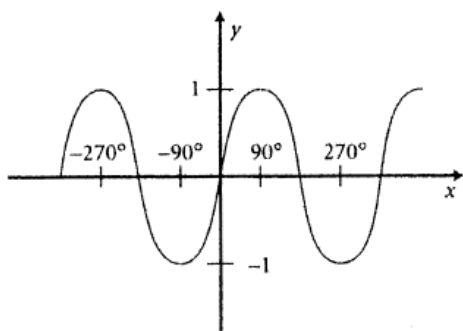
**Example:**  $\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} = ?$

**Setup:** The numerator equals 1, so:

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta$$

### 100. GRAPHING TRIGONOMETRIC FUNCTIONS

To graph trigonometric functions, use the  $x$ -axis for the angle and the  $y$ -axis for the value of the trigonometric function. Use special angles— $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $90^\circ$ ,  $120^\circ$ ,  $135^\circ$ ,  $150^\circ$ ,  $180^\circ$ , etc.—to plot key points.



The figure above shows a portion of the graph of  $y = \sin x$ .