Fractions

LEAST COMMON MULTIPLE

The LCM of a set of numbers is the smallest number that is a multiple of all the given numbers.

For example, the LCM of 5 and 6 is 30, since 5 and 6 have no factors in common.

GREATEST COMMON FACTOR

The GCF of a set of numbers is the largest number that can be evenly divided into each of the given numbers.

For example, the GCF of 24 and 27 is 3, since both 24 and 27 are divisible by 3, but they are not both divisible by any numbers larger than 3.

FRACTIONS

Fractions are another way to express division. The top number of a fraction is called the **numerator**, and the bottom number is called the **denominator**.

ADDING AND SUBTRACTING FRACTIONS

Fractions must have the same denominator before they can be added or subtracted.

$$\frac{a}{d} + \frac{b}{d} = \frac{a+b}{d}, \text{ with } d \neq 0.$$

$$\frac{a}{d} - \frac{b}{d} = \frac{a-b}{d}, \text{ with } d \neq 0.$$

If the fractions have different denominators, rewrite them as equivalent fractions with a common denominator. Then add or subtract the numerators, keeping the denominators the same. *For example,*

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}.$$

Fractions (continued)

Equivalent fractions are found by multiplying the numerator and denominator of the fraction by the same number. In the previous example,

$$\frac{2}{3} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12} \quad \text{and} \quad \frac{1}{4} = \frac{1 \cdot 3}{4 \cdot 3} = \frac{3}{12}.$$

MULTIPLYING AND DIVIDING FRACTIONS

When multiplying and dividing fractions, a common denominator is not needed. To multiply, take the product of the numerators and the product of the denominators:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{ac}{bd}$$

To divide fractions, invert the second fraction and then multiply the numerators and denominators:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Some examples:

$$\frac{3}{5} \cdot \frac{2}{7} = \frac{6}{35}$$

$$\frac{5}{12} \div \frac{1}{2} = \frac{5}{12} \cdot \frac{2}{1} = \frac{10}{12} = \frac{5}{6}$$

REDUCING FRACTIONS

To **reduce** a fraction, divide both the numerator and denominator by common factors. In the last *example*,

$$\frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}.$$

MIXED NUMBERS

A mixed number has two parts: a whole number part and a fractional part. An example of a mixed number is $5\frac{3}{8}$. This really represents

$$5 + \frac{3}{8},$$

which can be written as

$$\frac{40}{8} + \frac{3}{8} = \frac{43}{8}.$$

Similarly, an improper fraction can be written as a mixed number. *For example,*

$\frac{20}{3}$ can be written as $6\frac{2}{3}$,

since 20 divided by 3 equals 6 with a remainder of 2.