

Summer Packet ... for Algebra 2 Students

Order of Operations

Hints/Guide:

The rules for multiplying integers are:

positive \times positive = positive

positive \times negative = negative

negative \times negative = positive

negative \times positive = negative

The rules for dividing integers are the same as for multiplying integers.

REMEMBER: Order of Operations

(PEMDAS)

P - parenthesis

E - exponents

M/D - multiply/divide which comes first

A/S - add/subtract which comes first

Exercises: Solve the following problems. Show all work.

1. $\frac{100 - 15}{9 + 8}$

2. $3 + 4[13 - 2(6 - 3)]$

3. $5[2(8 + 5) - 15]$

4. $14 + 6 \cdot 2 - 8 \div 4$

5. $\frac{7(14) - 3(6)}{2}$

6. $14 \div [3(8 - 2) - 11]$

7. $32 \div (-7 + 5)^3$

8. $4^3 - (2 - 5)^3$

Use grouping symbols to make each equation true.

9. $6 + 8 \div 4 \cdot 2 = 7$

10. $5 + 4 \cdot 3 - 1 = 18$

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Solving Equations

Hint/Guide:

Equation-Solving Procedure

1. Multiply on both sides to clear the equation of fractions or decimals.
2. Distribute.
3. Collect like terms on each side, if necessary.
4. Get all terms with variables on one side and all constant terms on the other side, using the addition principle.
5. Collect like terms again, if necessary.
6. Multiply or divide to solve for the variable, using the multiplication principle.
7. Check all possible solutions in the original equation.

Example: $\frac{2x}{3} - \frac{1}{6} + \frac{1x}{2} = \frac{7}{6} + 2x$

$$6 \frac{2x}{3} - \frac{1}{6} + \frac{1x}{2} = 6 \frac{7}{6} + 2x$$

Multiply by LCM of 6 both sides.

$$4x - 1 + 3x = 7 + 12x$$

Simplify.

$$7x - 1 = 7 + 12x$$

Combine like terms.

$$7x - 12x = 7 + 1$$

Collect variables on one side and constants on other.

$$-5x = 8$$

Combine like terms.

$$x = \frac{-8}{5}$$

Divide by -5.

Exercises: Solve each equation. Show all work.

1. $3(r - 6) + 2 = 4(r + 2) - 21$

2. $5(t + 3) + 9 = 3(t - 2) + 6$

3. $\frac{1}{3}(6x + 24) - 20 = -\frac{1}{4}(12x - 72)$

4. $\frac{4}{5}(3x + 4) = 20$

5. $0.7(3x + 6) = 1.1 - (x + 2)$

6. $a + (a - 3) = (a + 2) - (a + 1)$

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Exponents

Hint/Guide:

Rules for Exponents	$a^1 = a$	$a^0 = 1$	when $a \neq 0$
<i>Negative Exponents:</i>	$a^{-n} = \frac{1}{a^n}$		when $a \neq 0$
<i>Product Rule:</i>	$a^m \cdot a^n = a^{m+n}$	<i>Quotient Rule:</i>	$\frac{a^m}{a^n} = a^{m-n}$
<i>Power Rule:</i>	$(a^m)^n = a^{mn}$	<i>Product to Power:</i>	$(ab)^n = a^n b^n$
<i>Quotient to a Power:</i>	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$		

Exercises: Simplify using the Rules for Exponents.

1. $6^{-2} \cdot 6^{-3}$

2. $x^6 \cdot x^2 \cdot x$

3. $(4a)^3 \cdot (4a)^8$

4. $\frac{3^5}{3^2}$

5. $\frac{x^3}{x^8}$

6. $\frac{(2x)^5}{(2x)^8}$

7. $(x^3)^2$

8. $(-3y^2)^3$

9. $(2a^3b)^4$

10. $(3x^2)^3(-2x^5)^3$

11. $3(x^2)^3(-2x^5)^3$

12. $(2x)^2(-3x^2)^4$

13. Express using a positive exponent:

$$5^{-3}$$

14. Express using a negative exponent:

$$\frac{1}{y^8}$$

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Addition and Subtraction Polynomials

Hint/Guide:

- Only like terms can be added or subtracted.
- Like terms have the same variables with the same exponents.
- Only the coefficients (numbers) are added or subtracted.
- A subtraction sign in front of the parenthesis changes each term in the parenthesis to the opposite.

Examples:

1) Add the polynomial. $(3x^2 - 2x + 2) + (5x^3 - 2x^2 + 3x - 4)$
 $= 5x^3 + (3 - 2)x^2 + (-2 + 3)x + (2 - 4)$
 $= 5x^3 + x^2 + x - 2$

2) Subtract the polynomial. $(9x^5 + x^3 - 2x^2 + 4) - (2x^5 + x^4 - 4x^3 - 3x^2)$
 $= 9x^5 + x^3 - 2x^2 + 4 - 2x^5 - x^4 + 4x^3 + 3x^2$
 $= 7x^5 - x^4 + 5x^3 + x^2 + 4$

Exercises: Add or subtract the polynomials. Show all work.

1. $(3x + 2) + (-4x + 3)$

2. $(-6x + 2) + (x^2 + x - 3)$

3. $(1.2x^3 + 4.5x^2 - 3.8x) +$
 $(-3.4x^3 - 4.7x^2 + 23)$

4. $(1/4 x^4 + 2/3 x^3 + 5/8 x^2 + 7) +$
 $(-3/4 x^4 + 3/8 x^2 - 7)$

5. $(6x + 1) - (-7x + 2)$

6. $(x^2 - 5x + 4) - (8x - 9)$

7. $(0.5x^4 - 0.6x^2 + 0.7) -$
 $(2.3x^4 + 1.8x - 3.9)$

8. $(1/5x^3 + 2x^2 - 0.1) -$
 $(-2/5x^3 + 2x^2 + 0.01)$

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Multiplying Polynomials

Hint/Guide:

- Multiply the coefficients and use the rule of exponents for the variables.
- Remember: FOIL F - first O - outers I - inners L - last **OR** Box Method

Examples:

$$\begin{aligned} 1. & (x^2 + 2x - 3)(x^2 + 4) \\ &= x^2 \cdot x^2 + 2x \cdot x^2 - 3 \cdot x^2 + x^2 \cdot 4 + 2x \cdot 4 - 3 \cdot 4 \\ &= x^4 + 2x^3 - 3x^2 + 4x^2 + 8x - 12 \\ &= x^4 + 2x^3 + x^2 + 8x - 12 \end{aligned}$$

$$2. (x + 5)(x + 4)$$

$$x \quad 5$$

x	x^2	$5x$
4	$4x$	20

$$= x^2 + 9x + 20$$

Exercises: Multiply the polynomials. Show all work.

1. $-3x(x - 1)$

2. $-4x(2x^3 - 6x^2 - 5x + 1)$

3. $(x + 5)(x - 2)$

4. $(5 - x)(5 - 2x)$

5. $(3a^2 - 5a + 2)(2a^2 - 3a + 4)$

6. $(x^3 + x^2 + x + 1)(x - 1)$

7. $(3 - 2x^3)^2$

8. $(x - 4x^3)^2$

9. $(3x + 2)(4x^2 + 5)$

10. $(1/5x^2 + 9)(3/5x^2 - 7)$

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Division of Polynomials

Hint/Guide:

- Divide the coefficients.
- Use the rules of exponents to divide the variables.

Example:

$$\begin{aligned}\frac{x^3 + 10x^2 + 8x}{2x} &= \frac{x^3}{2x} + \frac{10x^2}{2x} + \frac{8x}{2x} \\ &= \frac{1}{2}x^{3-1} + \frac{10}{2}x^{2-1} + \frac{8}{2}x^{1-1} \\ &= \frac{1}{2}x^2 + 5x + 4\end{aligned}$$

Exercises: Divide the polynomials. Show all work.

1. $\frac{24x^4 - 4x^3 + x^2 - 16}{8}$

2. $\frac{u - 2u^2 - u^5}{u}$

3. $\frac{50x^5 - 7x^4 + x^2}{x}$

4. $(25t^3 + 15t^2 - 30t) \div (5t)$

5. $(24x^6 + 32x^5 - 8x^2) \div (-8x^2)$

6. $(18x^6 - 27x^5 - 3x^3) \div (9x^3)$

7. $\frac{9r^2s^2 + 3r^2s - 6rs^2}{3rs}$

8. $(4x^4y - 8x^6y^2 + 12x^8y^6) \div (4x^4y)$

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Factor Polynomials I

Hint/Guide:

- Always look for a common factor first. Don't forget to include the variable in the common factor.
- Check your answer by multiplying.

Example: Factor $15x^5 - 12x^4 + 27x^3 - 3x^2$

Question: What number is common to the coefficients of 15, 12, 27, and 3?

Answer: 3

Question: What exponent is common to variables of x^5 , x^4 , x^3 , and x^2 ?

Answer: x^2

$$= (3x^2)(5x^3) - (3x^2)(4x^2) + (3x^2)(9x) - (3x^2)(1)$$

$$= 3x^2(5x^3 - 4x^2 + 9x - 1)$$

Exercises: Factor the polynomials. Show all work.

1. $x^2 - 4x$

2. $x^3 + 6x^2$

3. $8x^4 - 24x^2$

4. $6x^2 + 3x - 15$

5. $17x^5y^3 + 34x^3y^2 + 51xy$

6. $16x^6y^4 - 32x^5y^3 - 48xy^2$

7. $x^9y^6 - x^7y^5 + x^4y^4 + x^3y^3$

8. $1.6x^4 - 2.4x^3 + 3.2x^2 + 6.4x$

9. $5/3x^6 + 4/3x^5 + 1/3x^4 + 1/3x^3$

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Factor Polynomials II

Hints/Guide:

- Write the terms in descending order.
- List the factors for the constant term.
- Add those factors to find the match for the coefficient of the middle term.

Example: Factor $t^2 - 24 + 5t$.

Write in descending order: $t^2 + 5t - 24$

List the factors:

Pairs of Factors	Sums of Factors
-1, 24	23
-2, 12	10
-3, 8	5
-4, 6	2

The factors are: $(t - 3)(t + 8)$

Exercises: Factor the polynomials. Show all work.

1. $x^2 + 5x + 6$

2. $y^2 + 11y + 28$

3. $x^2 - 8x + 15$

4. $x^2 + 2x - 15$

5. $-2x - 99 + x^2$

6. $x^2 - 72 + 6x$

7. $a^4 + 2a^2 - 35$

8. $2 - x^2 - x^4$

9. $x^2 + 20x + 100$

10. $x^2 - 25x + 144$

11. $a^2 - 2ab - 3b^2$

12. $m^2 + 5mn + 4n^2$

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Factor Polynomials III

Hint/Guide:

To Factor Polynomials of the type $ax^2 + bx + c$, when $a \neq 1$:

- Write the terms in descending order.
- Factor all common factors.
- List the factors of the coefficient of the first term.
- List the factors for the constant term.
- Multiply a factor of 1st term with 3rd term. Multiply the other factor of 1st term with 3rd term. Add the two together, to find middle term. Continue this process until the correct factor combination is found.

Example: Factor $24x^2 - 76x + 40$.

Factor the common factor: $4(6x^2 - 19x + 10)$

Find the factors of the first and third terms:

Factors of 1 st Term	Factors of 3 rd Term	Middle Term
1, 6 OR -1, -6	1, 10 OR -1, -10	
2, 3 OR -2, -3	2, 5 OR -2, -5	

Try combinations of factors:

- a) $3 \cdot -1 + 10 \cdot 2 = -23$ wrong
- b) $3 \cdot -10 + -1 \cdot -2 = -32$ wrong
- c) $3 \cdot -2 + -5 \cdot 2 = -16$ wrong
- d) $3 \cdot -5 + -2 \cdot 2 = -19$ **correct**

The factors are: $4(3x - 2)(2x - 5)$

Exercises: Factor the polynomials. Show all work.

1. $2x^2 - 7x - 4$

2. $3x^2 - 4x - 15$

3. $3x^2 + 4x + 1$

4. $9x^2 + 6x - 8$

5. $9x^2 + 18x - 16$

6. $18x^2 - 3x - 10$

7. $14x^2 + 19x - 3$

8. $6 - 13x + 6x^2$

9. $24x^2 + 47x - 2$

10. $18x^3 - 21x^2 - 9x$

11. $18x^2 - 6xy - 24y^2$

12. $15a^2 - 5ab - 20b^2$

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Solving Systems of Equations by Substitution

Hint/Guide:

- Solve one equation for a variable with a coefficient of 1.
- Substitute what the variable equals into the other equation of the original pair. (The new equation should have only one variable.)
- Solve for the variable.
- Use that answer to solve for the other variable.
- Answers are ordered pairs: (x, y)

Example: Solve $x - 2y = 6$
 $3x + 2y = 4$.

Solve the first equation for x : $x = 6 + 2y$
Substitute your answer above into the second equation: $3(6 + 2y) + 2y = 4$
Distribute: $18 + 6y + 2y = 4$
Combine like terms: $18 + 8y = 4$
Collect like terms to one side (subtract 18 both sides): $8y = -14$
Isolate the variable (divide by 8 both sides): $y = \frac{-14}{8}$ OR $\frac{-7}{4}$

Substitute the y value into an original equation to solve for x :
 $x - 2(-7/4) = 6$
 $x - (-14/4) = 6$
 $x = 10/4$ or $5/2$

The solution to the system of equations: $(5/2, -7/4)$

Exercises: Solve the system of equations using the substitution method. Show all work.

1. $s + t = -4$
 $s - t = 2$

2. $x - y = 6$
 $x + y = -2$

3. $y - 2x = -6$
 $2y - x = 5$

4. $x - y = 5$
 $x + 2y = 7$

5. $2x + 3y = -2$
 $2x - y = 9$

6. $x + 2y = 10$
 $3x + 4y = 8$

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Solving Systems of Equations by Elimination

Hint/Guide:

- Answers are ordered pairs (x, y) .
- Eliminate one variable by adding the two equations together.
- Sometimes, one equation must be multiplied by a number to have a variable with the same coefficient.

Examples:

1. Solve $2x + 3y = 8$
 $x + 3y = 7$

Multiply the equation by -1 to make the y coefficients opposite:
Add the equations together:

$$\begin{array}{r} 2x + 3y = 8 \\ -x - 3y = -7 \\ \hline x + 0y = 1 \\ x = 1 \end{array}$$

Solve for y by substituting the value of x into the original equation:
Solve the equation for y :

$$\begin{array}{r} 2(1) + 3y = 8 \\ 3y = 6 \\ y = 2. \end{array}$$

The solution for this system: $(1, 2)$

2. Solve $3x + 6y = -6$
 $5x - 2y = 14$

Multiply the second equation by 3 to get the y coefficients the same:
Add the equations together:
Solve for x :

$$\begin{array}{r} 3x + 6y = -6 \\ 15x - 6y = 42 \\ \hline 18x + 0y = 36 \\ x = 2 \end{array}$$

Solve for y by substituting the value of x into the original equation:
Solve the equation for y :

$$\begin{array}{r} 3(2) + 6y = -6 \\ 6y = -12 \\ y = -2 \end{array}$$

The solution for this system: $(2, -2)$

Exercises: Solve the systems of equations by elimination. Show all work.

1. $x + y = 10$ 2. $x - y = 7$ 3. $3x - y = 9$ 4. $4x - y = 1$
 $x - y = 8$ $x + y = 3$ $2x + y = 6$ $3x + y = 13$

5. $-x - y = 8$ 6. $3x - y = 8$ 7. $2w - 3z = -1$ 8. $3x - 4y = 16$
 $2x - y = -1$ $x + 2y = 5$ $3w + 4z = 24$ $5x + 6y = 14$

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Simplifying Radicals

Hint/Guide:

- For any real number that is not negative, $\sqrt{x^2} = x$
- *Assume that the radical sign ($\sqrt{\quad}$) extends over the entire expression in parenthesis.*

Examples:

1. Simplify $\sqrt{(3x)^2} = 3x$
2. Simplify $\sqrt{a^2 b^2} = ab$
3. Simplify $\sqrt{x^2 + 2x + 1} = \sqrt{(x+1)(x+1)} = x + 1$
4. Simplify by factoring: $\sqrt{32x^{15}} = \sqrt{[(16 \cdot 2)(x^{14} \cdot x)]} = 4x^7\sqrt{2x}$

Exercises: Simplify the radical expressions.

1. $\sqrt{t^2}$
2. $\sqrt{9x^2}$
3. $\sqrt{(ab)^2}$
4. $\sqrt{(6y)^2}$
5. $\sqrt{(34d)^2}$
6. $\sqrt{(53b)^2}$
7. $\sqrt{(x-7)^2}$
8. $\sqrt{d^2 - 10a + 25}$
9. $\sqrt{4x^2 - 20x + 25}$
10. $\sqrt{9p^2 + 12p + 4}$
11. $\sqrt{75}$
12. $\sqrt{20}$
13. $\sqrt{48x}$
14. $\sqrt{64y^2}$
15. $\sqrt{20x^2}$
16. $\sqrt{8x^2 + 8x + 2}$
17. $\sqrt{27x^2 - 36x + 12}$
18. $\sqrt{x - 2x^2 + x^3}$

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Rationalizing Radicals

Hint/Guide:

- *Assume that the radical extends over the entire expression in parenthesis.*
- To rationalize a radical expression, first simplify through division if possible.
- Multiply the numerator and the denominator by the denominator.
- Simplify the expression.

Examples:

1. Rationalize the radical expression: $\frac{\sqrt{30a^5}}{\sqrt{6a^2}} = \sqrt{5a^3} = \sqrt{a^2 \cdot 5a} = \sqrt{a^2} \cdot \sqrt{5a} = a\sqrt{5a}$

2. Rationalize the radical expression: $\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{6}}{\sqrt{4}} = \frac{\sqrt{6}}{2}$

3. Rationalize the radical expression: $\frac{\sqrt{5}}{\sqrt{x}} = \frac{\sqrt{5} \cdot \sqrt{x}}{\sqrt{x} \cdot \sqrt{x}} = \frac{\sqrt{5x}}{x}$

Exercises: Rationalize the radical expressions. Show all work.

1. $\frac{\sqrt{18}}{\sqrt{2}}$

2. $\frac{\sqrt{60}}{\sqrt{15}}$

3. $\frac{\sqrt{3}}{\sqrt{75}}$

4. $\frac{\sqrt{18}}{\sqrt{32}}$

5. $\frac{\sqrt{18b}}{\sqrt{2b}}$

6. $\frac{\sqrt{48x^3}}{\sqrt{3x}}$

7. $\frac{\sqrt{2}}{\sqrt{5}}$

8. $\frac{2}{\sqrt{2}}$

9. $\frac{\sqrt{3}}{\sqrt{x}}$

10. $\frac{\sqrt{27c}}{\sqrt{32c^3}}$

11. $\frac{\sqrt{y^5}}{\sqrt{xy^2}}$

12. $\frac{\sqrt{16a^4b^6}}{\sqrt{128a^6b^6}}$

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Solving Radical Equations

Hint/Guide:

- *Assume that the radical extends over the entire expression in parenthesis.*
- Isolate the radical.
- Square both sides to remove the radicals.
- Combine like terms.
- Isolate the variable to one side.

Examples:

1. Solve $2\sqrt{x+2} = \sqrt{x+10}$

Square both sides: $(2\sqrt{x+2})^2 = (\sqrt{x+10})^2$
 $4(x+2) = x+10$

Distribute: $4x+8 = x+10$

Collect like terms: $4x-x = 10-8$
 $3x = 2$

Divide to isolate the variable: $x = 2/3$

2. Solve $x-5 = \sqrt{x+7}$

Square both sides: $(x-5)^2 = (\sqrt{x+7})^2$
 $x^2 - 10x + 25 = x+7$

Combine like terms: $x^2 - 11x + 18 = 0$

Factor: $(x-9)(x-2) = 0$

Solve $x-9 = 0$ $x-2 = 0$
 $x = 9$ OR $x = 2$

Exercises: Solve the radical equations. Show all work.

1. $\sqrt{x} = 5$

2. $\sqrt{x} = 7$

3. $\sqrt{x+3} = 20$

4. $\sqrt{x+4} = 11$

5. $3 + \sqrt{x-1} = 5$

6. $4 + \sqrt{y-3} = 11$

7. $\sqrt{5x-7} = \sqrt{x+10}$

8. $\sqrt{4x-5} = \sqrt{x+9}$

9. $\sqrt{2y+6} = \sqrt{2y-5}$

10. $x-7 = \sqrt{x-5}$

11. $x-9 = \sqrt{x-3}$

12. $2\sqrt{x-1} = x-1$

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Quadratic Formula

Hint/Guide:

- Assume that the radical extends over the expression in parenthesis.
- Equation must be in the form $ax^2 + bx + c = 0$ (standard form).
- Try to factor first.
- If you cannot find factors, then use the quadratic equation.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

Solve $x^2 = 4x + 7$

Write the equation in standard form:

$$x^2 - 4x - 7 = 0$$

Identify a, b, and c for the formula:

$$a = 1, \quad b = -4, \quad c = -7$$

Substitute into the formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)}$$

Simplify:

$$x = \frac{4 \pm \sqrt{16 + 28}}{2}$$

$$x = \frac{4 \pm \sqrt{44}}{2}$$

Simplify the radical:

$$x = \frac{4 \pm 2\sqrt{11}}{2}$$

$$x = 2 \pm \sqrt{11}$$

Solutions: $x = 2 + \sqrt{11}$ or $x = 2 - \sqrt{11}$

Exercises: Solve using the quadratic formula. Show all work.

1. $x^2 - 4x = 21$

2. $x^2 = 6x - 9$

3. $3y^2 - 7y + 4 = 0$

4. $x^2 - 9 = 0$

5. $x^2 - 2x - 2 = 0$

6. $x^2 - 4x - 7 = 0$

7. $y^2 - 10y + 22 = 0$

8. $y^2 + 6y = 1$

9. $2y^2 - 6y = 10$