

Factors of a Monomial

The number(s) and/or variable(s) that are multiplied together to form a monomial

Examples:	Factors	Expanded Form
$5b^2$	$5 \cdot b^2$	$5 \cdot b \cdot b$
$6x^2y$	$6 \cdot x^2 \cdot y$	$2 \cdot 3 \cdot x \cdot x \cdot y$
$\frac{-5p^2q^3}{2}$	$\frac{-5}{2} \cdot p^2 \cdot q^3$	$\frac{1}{2} \cdot (-5) \cdot p \cdot p \cdot q \cdot q \cdot q$

Factoring

(Greatest Common Factor)

Find the greatest common factor (GCF) of all terms of the polynomial and then apply the distributive property.

Example: $20a^4 + 8a$

$$\textcircled{2} \cdot \textcircled{2} \cdot 5 \cdot \textcircled{a} \cdot a \cdot a \cdot a + \textcircled{2} \cdot \textcircled{2} \cdot 2 \cdot \textcircled{a}$$

common factors

$$\text{GCF} = \overbrace{2 \cdot 2 \cdot a} = 4a$$

$$20a^4 + 8a = 4a(5a^3 + 2)$$

Factoring

(Difference of Squares)

$$a^2 - b^2 = (a + b)(a - b)$$

Examples:

$$x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7)$$

$$4 - n^2 = 2^2 - n^2 = (2 - n)(2 + n)$$

$$\begin{aligned} 9x^2 - 25y^2 &= (3x)^2 - (5y)^2 \\ &= (3x + 5y)(3x - 5y) \end{aligned}$$

Factoring

(Perfect Square Trinomials)

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Examples:

$$\begin{aligned}x^2 + 6x + 9 &= x^2 + 2 \cdot 3 \cdot x + 3^2 \\ &= (x + 3)^2\end{aligned}$$

$$\begin{aligned}4x^2 - 20x + 25 &= (2x)^2 - 2 \cdot 2x \cdot 5 + 5^2 \\ &= (2x - 5)^2\end{aligned}$$

Factoring (By Grouping)

For trinomials of the form
 $ax^2 + bx + c$

Example: $3x^2 + 8x + 4$

$$ac = 3 \cdot 4 = 12$$

Find factors of ac that add to equal b

$$12 = 2 \cdot 6 \rightarrow 2 + 6 = 8$$

$$3x^2 + 2x + 6x + 4$$

Rewrite $8x$
as $2x + 6x$

$$(3x^2 + 2x) + (6x + 4)$$

Group factors

$$x(3x + 2) + 2(3x + 2)$$

Factor out a
common
binomial

$$(3x + 2)(x + 2)$$

Factoring

(Sum and Difference of Cubes)

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Examples:

$$\begin{aligned} 27y^3 + 1 &= (3y)^3 + (1)^3 \\ &= (3y + 1)(9y^2 - 3y + 1) \end{aligned}$$

$$x^3 - 64 = x^3 - 4^3 = (x - 4)(x^2 + 4x + 16)$$