TOPIC 5: EQUATIONS and EXPRESSIONS

A. <u>Operations with literal symbols (letters)</u>:

When letters are used to represent numbers,
addition is shown with a "plus sign" (+), and
subtraction with a "minus sign" (–).

Multiplication is often show by writing letters together: *example: ab* means *a* times *b* So do $a \bullet b,\ \ a \times b,$ and $(a)(b)$ *example:* $7 \bullet 8 = 7 \times 8 = (7)(8)$

example: $a(b+c)$ means a times $(b+c)$

Problems 1-4: What is the meaning of:

1. abc
2. $2a$
3. c^2
4. $3(a-4)$

To show *division*, fractions are often used:
example: 3 divided by 6 may be shown: $3 \div 6$ or $\frac{3}{6}$, or $6\overline{)3}$, and all have value $\frac{1}{2}$, or .5.

5. What does $\frac{4a}{3b}$ mean?

Problems 6-15: Write a fraction form and reduce to find the value (if possible):

6. $36 \div 9 =$
7. $4 \div 36 =$
8. $10\overline{)36} =$
9. $1.2\overline{).06} =$
10. $2x \div a =$
11. $12 \div 5y =$
12. $6b \div 2a =$
13. $8r \div 10s =$
14. $a \div a =$
15. $2x \div x =$

Answers

1. a times b times c
2. 2 times a
3. c times c
4. 3 times $(a - 4)$
5. $4a$ divided by $3b$
6. $\frac{36}{9} = 4$
7. $\frac{4}{36} = \frac{1}{9}$
8. $\frac{36}{10} = \frac{18}{5}$
9. $\frac{.06}{1.2} = \frac{6}{120} = \frac{1}{20}$
10. $\frac{2x}{a}$
11. $\frac{12}{5y}$
12. $\frac{6b}{2a} = \frac{3b}{a}$
13. $\frac{8r}{10s} = \frac{4r}{5s}$
14. $\frac{a}{a} = 1$
15. $\frac{2x}{x} = 2$

In the above exercises, notice that the fraction forms can be reduced if there is a common (shared) factor in the top and bottom:

example: $\frac{36}{9} = \frac{9 \bullet 4}{9 \bullet 1} = \frac{9}{9} \bullet \frac{4}{1} = 1 \bullet 4 = 4$

example: $\frac{4}{36} = \frac{4 \bullet 1}{4 \bullet 9} = \frac{4}{4} \bullet \frac{1}{9} = 1 \bullet \frac{1}{9} = \frac{1}{9}$

example: $\frac{36}{10} = \frac{2 \bullet 18}{2 \bullet 5} = \frac{18}{5}$ $(or\ 3\frac{3}{5})$

example: $\frac{6b}{2a} = \frac{2 \bullet 3 \bullet b}{2 \bullet a} = \frac{3b}{a}$

example: $\frac{a}{a} = 1$

example: $\frac{2x}{x} = \frac{2 \bullet x}{1 \bullet x} = \frac{2}{1} \bullet \frac{x}{x} = \frac{2}{1} \bullet 1 = 2 \bullet 1 = 2$

Problems 16-24: Reduce and simplify:

16. $\frac{3x}{3} =$

17. $\frac{3x}{4x} =$

18. $\frac{x}{2x} =$

19. $\frac{12x}{3x} =$

20. $\frac{12x}{3} =$

21. $\frac{abc}{3ac} =$

22. $\frac{6x}{8xy} =$

23. $\frac{15x^2}{10x} =$

24. $\frac{6x}{5} \bullet 5 =$

(Hint: $\frac{6x}{5} \bullet 5 = \frac{6x}{5} \bullet \frac{5}{1} = \frac{6x \bullet 5}{5 \bullet 1} \ldots$)

The *distributive property* says $a(b+c) = ab + ac$. Since equality $(=)$ goes both ways, the distributive property can also be written $ab + ac = a(b+c)$.

Another form it often takes is
$(a+b)c = ac + bc$, or $ac + bc = (a+b)c$.

example: $3(x-y) = 3x - 3y$. Comparing this with $a(b+c) = ab + ac$, we see $a = 3$, $b = x$, and $c = -y$

Answers

16. x
17. $\frac{3}{4}$
18. $\frac{1}{2}$
19. 4
20. $4x$
21. $\frac{b}{3}$
22. $\frac{3}{4y}$
23. $\frac{3x}{2}$
24. $6x$

example: Compare $4x + 7x = (4+7)x = 11x$ with $ac + bc = (a+b)c$; $a = 4,\ c = x,\ b = 7$

example: $4(2+3) = 4 \bullet 2 + 4 \bullet 3$ (The distributive property says this has value 20, whether you do $4 \bullet 5$ or $8 + 12$.)

example: $4a + 6x - 2 = 2(2a + 3x - 1)$

Problems 25-35: Rewrite, using the distributive property:

25. $6(x-3) =$

26. $4(b+2) =$

27. $(3-x)2 =$

28. $4b - 8c =$

29. $4x - x =$ (Hint: think $4x - 1x$, or 4 cookies minus 1 cookie)

30. $-5(a-1) =$

31. $5a + 7a =$

32. $3a - a =$

33. $3a - a =$

34. $5x - x + 3x =$

35. $x(x+2) =$

Answers

25. $6x - 18$
26. $4b + 8$
27. $6 - 2x$
28. $4(b - 2c)$
29. $(4 - 1)x = 3x$
30. $-5a + 5$
31. $(5 + 7)a = 12a$
32. $(3 - 2)a = 1a = a$
33. $2a$
34. $7x$
35. $x^2 + 2x$