Learning to Work with Fractions … Set 1

A. Greatest Common Factor (GCF):

The GCF of two integers is used to simplify (reduce, rename) a fraction to an equivalent fraction. A <u>factor</u> is an integer <u>multiplier</u>. A <u>prime number</u> is a positive whole number with exactly two positive factors.

example: the prime factorization of 18 is $2 \bullet 3 \bullet 3$, or $2 \bullet 3^2$.

Problems 1-2: Find the prime factorization:

1. 24

2. 42

example: Find the factors of 42.

Factor into primes: $42 = 2 \bullet 3 \bullet 7$

1 is always a factor
2 is a prime factor
3 is a prime factor
4 is a prime factor
7 is a prime factor
$2 \bullet 3 = 6$ is a factor
$2 \bullet 7 = 14$ is a factor
$3 \bullet 7 = 21$ is a factor
$2 \bullet 3 \bullet 7 = 42$ is a factor

Thus 42 has 8 factors.

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Answers

1. $2^3 \bullet 3$
2. $2 \bullet 3 \bullet 7$

Problems 3-4: Find all positive factors:

3. 18	4. 24

To find the GCF:

example: Looking at the factors of 42 and 24, we see that the common factors of both are 1, 2, 3, and 6, of which the greatest it 6; so: the GCF of 42 and 24 is 6. (Notice that "common factor" means "shared factor.")

Problems 5-7: Find the GCF of:

5. 18 and 36	6. 27 and 36	7. 8 and 15

Answers

3. 1, 2, 3, 6. 9, 18
4. 1, 2, 3, 4, 6, 8, 12, 24
5. 18
6. 9
7. 1

B. <u>Simplifying fractions</u>:

> *example:* Reduce $\frac{27}{36}$:
>
> $$\frac{27}{36} = \frac{9 \bullet 3}{9 \bullet 4} = \frac{9}{9} \bullet \frac{3}{4} = 1 \bullet \frac{3}{4} = \frac{3}{4}$$
>
> (Note that you must be able to find a common factor, in this case 9, in both the top and bottom in order to reduce.)

Problems 8-13: Reduce:

8. $\frac{13}{52} =$

9. $\frac{26}{65} =$

10. $\frac{3+6}{3+9} =$

11. $\frac{16}{64} =$

12. $\frac{24}{42} =$

13. $\frac{24}{18} =$

Answers

8. $\frac{1}{4}$
9. $\frac{2}{5}$
10. $\frac{3}{4}$
11. $\frac{1}{4}$
12. $\frac{4}{7}$
13. $\frac{4}{3}$

C. Equivalent Fractions:

example: $\frac{3}{4}$ is equivalent to how many eighths?

$\left(\frac{3}{4} = \frac{}{8}\right)$

$\frac{3}{4} = 1 \bullet \frac{3}{4} = \frac{2}{2} \bullet \frac{3}{4} = \frac{2 \bullet 3}{2 \bullet 4} = \frac{6}{8}$

Problems 14-17: Complete:

14. $\frac{4}{9} = \frac{}{72}$

15. $\frac{3}{5}$ is how many twentieths?

16. $\frac{56}{100} = \frac{}{50}$

17. How many halves are in 3? (Hint: think $3 = \frac{3}{1} = \frac{}{2}$)

Answers

14. 32
15. 12
16. 28
17. 6

D. Ratio:

If the ratio of boys to girls in a class is 2 to 3, it means that for every 2 boys, there are 3 girls. A ratio is like a fraction: think of the ratio 2 to 3 as the fraction $\frac{2}{3}$.

example: If the class had 12 boys, how many girls are there? Write the fraction ratio:

$$\frac{number\ \ of\ \ boys}{number\ \ of\ \ girls} = \frac{2}{3} = \frac{12}{\quad}$$

Complete the equivalent fraction: $\frac{2}{3} = \frac{2 \cdot 6}{3 \cdot 6} = \frac{12}{18}$

So there are 18 girls.

18. If the class had 21 girls and the ratio of boys to girls was 2 to 3, how many boys would be in the class?

19. If the ratio of X to Y is 4 to 3, and there are 462 Y's, how many X's are there?

20. If the ratio of games won to games played is 6 to 7 and 18 games were won, how many games were played?

Answers

18. 14
19. 616
20. 21

E. Least common multiple (LCM):

The LCM of two or more integers is used to find the lowest common denominator of fractions in order to add or subtract them.

> To find the LCM:
> *example:* Find the LCM of 27 and 36.
>
> First factor into primes:
>
> $27 = 3^3$
>
> $36 = 2^2 \bullet 3^2$
>
> Make the LCM by taking each prime factor to its greatest power:
>
> $\text{LCM} = 2^2 \bullet 3^3 = 4 \bullet 27 = 108$

Problems 21-25: Find the LCM:

21. 6 and 15
22. 4 and 8
23. 3 and 5
24. 8 and 12
25. 8, 12, and 15

Answers

21. 30
22. 8
23. 15
24. 24
25. 120

F. Lowest common denominator (LCD):

To find LCD fractions for two or more given fractions:

example: Given $\frac{5}{6}$ and $\frac{8}{15}$

First find LCM of 6 and 15:

$6 = 2 \cdot 3$

$15 = 3 \cdot 5$

$LCM = 2 \cdot 3 \cdot 5 = 30 = LCD$

So $\frac{5}{6} = \frac{25}{30}$ and $\frac{8}{15} = \frac{16}{30}$

Problems 26-32: Find equivalent fractions with the LCD:

26. $\frac{2}{3}$ and $\frac{2}{9}$
27. $\frac{3}{8}$ and $\frac{7}{12}$
28. $\frac{4}{5}$ and $\frac{2}{3}$
29. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$
30. $\frac{7}{8}$ and $\frac{5}{8}$
31. Which is larger, $\frac{5}{7}$ or $\frac{3}{4}$? (Hint: find and compare LCD fractions)
32. Which is larger, $\frac{3}{8}$ or $\frac{1}{3}$?

Answers

26. $\frac{6}{9}$, $\frac{2}{9}$
27. $\frac{9}{24}$, $\frac{14}{24}$
28. $\frac{12}{15}$, $\frac{10}{15}$
29. $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$
30. $\frac{7}{8}$, $\frac{5}{8}$
31. $\frac{3}{4}$ (because $\frac{20}{28} < \frac{21}{28}$)
32. $\frac{3}{8}$ (because $\frac{9}{24} > \frac{8}{24}$)