To subtract two integers: add the opposite of the one to be subtracted:

example: 3 subtract -4, or (3) - (-4): The opposite of -4 is 4, so we add 4 (rather than subtract -4). We change the problem from (3) - (-4) to (3)+(4), which we know how to do:

$$(3) + (4) = 3 + 4 = 7$$

example: (-4) – (3): Add the opposite of 3, namely -3: (-4) - (3) = (-4) + (-3) = -7

example:
$$(4) - (3) = (4) + (-3) = 1$$

example:
$$-5 - 8 = (-5) - (8) = (-5) + (-8) = -13$$

Problems 34-43: Calculate:

$$34. (12) - (3) =$$

35.
$$-12 - 3 = (Hint: this means -12 + (-3))$$

$$36. -12 - (-3) = 40. 0 - 3 =$$

$$37. \ \ 3 - 12 =$$
 41. $0 + 4 =$

$$38. -3 - 12 = 42. -12 + 3 =$$

36.
$$-12 - (-3) =$$
 40. $0 - 3 =$ 37. $3 - 12 =$ 41. $0 + 4 =$ 42. $-12 + 3 =$ 43. $(-3) + (-12) =$

- 34. 9
- 35. –15
- 36. –9
- 37. –9
- 38. –15
- 39. 0
- 40. -3
- 41. 4
- 42. –9
- 43. –15

To multiply two integers:		
1 st integer ×	2 nd integer =	Answer
+	+	+
_	+	_
+	_	_
_	_	+

Both positive: multiply as two natural numbers. *example:* $(3) \times (4) = 3 \times 4 = 12$

Both negative: multiply as if positive; and make the answer positive, and remember, two negatives make a positive. When multiplying two negative numbers, you always get a positive answer.

example: (-3)(-4) so $3 \times 4 = 12$; make it positive, and the answer is 12.

One positive, one negative: When multiplying a negative number and a positive number, the answer is always negative.

example: (3)(-4) so $3 \times 4 = 12$; make the answer negative; answer -12.

Problems 44-55: Multiply:

$$44.3 \times (-4) =$$

44.
$$3 \times (-4) =$$

45. $(3) \cdot (-4) =$

46. $(3)(-4) =$

47. $3(-4) =$

48. $(-3)(-4) =$

49. $-3(-4) =$

50. $(-4) \cdot 0 =$

51. $0^2 =$

52. $(-3)^2 =$

53. $(4)^2 =$

54. $(-3) \cdot 4 =$

55. $3 \cdot 4 =$

46.
$$(3)(-4)=$$

$$47. \ 3(-4) =$$

48.
$$(-3)(-4) =$$

49.
$$-3(-4) =$$

50.
$$(-4) \cdot 0 =$$

51.
$$0^2 =$$

52.
$$(-3)^2 =$$

53.
$$(4)^2 =$$

- 44. -12
- 45. -12
- 46. -12
- 47. -12
- 48. 12
- 49. 12
- 50. 0
- 51. 0
- 52. 9
- 53. 16
- 54. -12
- 55. 12

Reciprocals are used for dividing. Every integer except zero has a reciprocal. The reciprocal is the number that multiplies the integer to give 1.

example:
$$6 \cdot \frac{1}{6} = 1$$
, so the reciprocal of 6 is $\frac{1}{6}$.
(And the reciprocal of $\frac{1}{6}$ is 6.)

example:
$$(-4)(-\frac{1}{4}) = 1$$
, so the reciprocal of -4 is $-\frac{1}{4}$.

Problems 56-59: Find the reciprocal:

- 60. What number is its own reciprocal? (Can you find "more than one"?)
- Using the reciprocal definition, explain why there is no reciprocal of zero.

Answers

56.
$$-\frac{1}{5}$$

58.
$$\frac{1}{10}$$

61. no number times 0 = 1

To divide two integers: multiply by the reciprocal of the one to be divided by:

example: 20 divided by
$$-5 = 20 \div (-5)$$
.

The reciprocal of -5 is $-\frac{1}{5}$ so we multiply by

$$-\frac{1}{5}$$
: 20 ÷ (-5) = 20 × ($-\frac{1}{5}$) =

$$\frac{20}{1} \times \left(-\frac{1}{5}\right) = -\frac{20 \cdot 1}{1 \cdot 5} = -\frac{20}{5} = -4$$

example:
$$\frac{-5}{20} = -5 \div 20 = -5 \cdot \frac{1}{20} = -\frac{1}{4}$$

example:
$$\frac{-3}{-6} = -3 \div (-6) = -3 \cdot (-\frac{1}{6}) = \frac{3}{6} = \frac{1}{2}$$

(Note negative times negative is positive.)

example:
$$\frac{0}{3} = 0 \div 3 = 0 \cdot \frac{1}{3} = 0$$

Problems 62-67: Calculate:

62.
$$(-14) \div (-2) = 65. \frac{-15}{3} =$$

62.
$$(-14) \div (-2) = 65$$
. $\frac{-15}{3} = 66$. $\frac{-5}{0} = (careful)^*$
64. $3 \div 2 = 67$. $\frac{0}{7} = 67$

64.
$$3 \div 2 =$$
 67. $\frac{0}{7} =$

- * Problem 61 says $\frac{1}{0}$ has no value (you cannot divide by zero).
- 68. From the rule for division, why is it impossible to divide by zero?

- 62. 7
- 63. $\frac{2}{3}$
- 64. $\frac{3}{2}$
- 65. –5
- 66. no value (not defined)
- 67. 0
- 68. zero has no reciprocal

To "sum" it all up:

Positive+positive = larger positive Negative+negative = more negative Positive+negative = in between both

Positive × positive = positive Negative × negative = positive Positive × negative = negative

To subtract add the opposite.

To divide, multiply by the reciprocal.

- 69. Given the statement "Two negatives make a positive." Provide an example of a situation where the statement would be true and another when it would be false.
- 70. Write "18 divided by 30" in three ways: using ÷, , and using a fraction bar —.

Problems 71-80: Calculate:

71.
$$4 - 10 + 3 - 2 = [76. -2[(-6)(8) + 9] =$$

72.
$$4 - (10 + 3 - 2) = |77.5 + (3 - 7) =$$

73.
$$4 + 3 - (10 - 2) = 78.5 - (3 - 7) =$$

74.
$$6(8-3) =$$
 79. $5-3+7 =$

75.
$$(-6)(8) + 9 =$$
 80. $-1 + 2 - 3 + 4 =$

81. What is the meaning of "sum", "product", quotient", and "difference"?

- 69. true if \times , false if +.
- 70. $18 \div 30$, 30)18, 18/30
- 71. -5
- 72. -7
- 73. –1
- 74. 30
- 75. -39
- 76. 78
- 77. 1
- 78. 9
- 79. 9
- 80. 2
- 81. +, x, ÷, -