

### Arc Length

For a function,  $f(x)$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

For a polar graph,  $r(\theta)$

$$L = \int_{\theta_1}^{\theta_2} \sqrt{[r(\theta)]^2 + [r'(\theta)]^2} d\theta$$

#### BC Only: Arc Length (Length of a Curve)

A. If the function  $y = f(x)$  is a differentiable function, then the length of the arc on  $[a, b]$  is

$$\int_a^b \sqrt{1 + [f'(x)]^2} dx$$

B. If the function  $x = f(y)$  is a differentiable function, then the length of the arc on  $[a, b]$  is

$$\int_a^b \sqrt{1 + [f'(y)]^2} dy$$

C. Parametric Arc Length: If a smooth curve is given by  $x(t)$  and  $y(t)$ , then the arc length over the interval  $a \leq t \leq b$  is

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Arc Length of a Function:

For a function  $f(x)$  with a continuous derivative on  $[a, b]$ :

*Arc Length is:* 
$$s = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$