Formulas and Theorems

A function $y = f(x)$ is <u>continuous</u> at $x = a$ if

i). $f(a)$ exists

ii). $\lim_{x \to a} f(x)$ exists

iii). $\lim_{x \to a} = f(a)$

Definition of Continuity A function **f is continuous** at c if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $\lvert x - c\rvert < \delta$ and $\lvert f(x) - f(c)\rvert < \varepsilon$. Tip: Rearrange $\lvert f(x) - f(c)\rvert$ to have $\lvert(x - c)\rvert$ as a factor. Since $\lvert x - c\rvert < \delta$ we can find an equation that relates both δ and ε together.	**<u>Prove</u> that $f(x) = x^2 - 1$ is a continuous function.** $\lvert f(x) - f(c)\rvert$ $= \lvert(x^2 - 1) - (c^2 - 1)\rvert$ $= \lvert x^2 - 1 - c^2 + 1\rvert$ $= \lvert x^2 - c^2\rvert$ $= \lvert(x + c)(x - c)\rvert$ $= \lvert(x + c)\rvert\ \lvert(x - c)\rvert$ Since $\lvert(x + c)\rvert \leq \lvert 2c\rvert$ $\lvert f(x) - f(c)\rvert \leq \lvert 2c\rvert\lvert(x - c)\rvert < \varepsilon$ So, given $\varepsilon > 0$, we can **choose** $\boldsymbol{\delta = \left\lvert\frac{1}{2c}\right\rvert \varepsilon > 0}$ in the Definition of Continuity. So, substituting the chosen δ for $\lvert(x - c)\rvert$ we get: $\lvert f(x) - f(c)\rvert \leq \lvert 2c\rvert\left(\left\lvert\frac{1}{2c}\right\rvert \varepsilon\right) = \varepsilon$ Since both conditions are met, $f(x)$ is continuous.

If a function is differentiable at point $x = a$, it is continuous at that point. The converse is false, in other words, continuity does <u>not</u> imply differentiability.