

Differential Equations

A differential equation is an equation involving an unknown function and one or more of its derivatives

$$\frac{dy}{dx} = f(x, y) \longrightarrow \text{Usually expressed as a derivative equal to an expression in terms of } x \text{ and/or } y.$$

To solve differential equations, use the technique of separation of variables.

Given the differential equation $\frac{dy}{dx} = \frac{xy}{(x^2+1)}$

Step 1: Separate the variables, putting all y 's on one side, with dy in the numerator, and all x 's on the other side, with dx in the numerator.

$$\frac{1}{y} dy = \frac{x}{(x^2 + 1)} dx$$

Step 2: Integrate both sides of the equation.

$$\ln|y| = \frac{1}{2} \ln \sqrt{x^2 + 1} + C$$

Step 3: Solve the equation for y .

$$y = C\sqrt{x^2 + 1}$$

Given the differential equation $\frac{dy}{dx} = 2x^2$ with the initial condition $y(3) = 10$.

A. The general solution to a differential equation is left with the constant of integration, C , undefined.

$$dy = 2x^2 dx \rightarrow \int dy = \int 2x^2 dx \rightarrow y = \frac{2}{3}x^3 + C$$

B. The particular solution uses the given initial condition to calculate the value of C .

$$10 = \frac{2}{3}(3)^3 + C \rightarrow C = -8 \rightarrow y = \frac{2}{3}x^3 - 8$$