

### BC Only: Improper Integrals

An improper integral is characterized by having a limits of integration that is infinite or the function  $f$  having an infinite discontinuity (asymptote) on the interval  $[a, b]$ .

#### A. Infinite Upper Limit (continuous function)

$$\int_a^{\infty} f(x)dx = \lim_{b \rightarrow \infty} \int_a^b f(x)dx$$

#### B. Infinite Lower Limit (continuous function)

$$\int_{-\infty}^b f(x)dx = \lim_{a \rightarrow -\infty} \int_a^b f(x)dx$$

#### C. Both Infinite Limits (continuous function)

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{a \rightarrow -\infty} \int_a^c f(x)dx + \lim_{b \rightarrow \infty} \int_c^b f(x)dx, \text{ where } c \text{ is an } x \text{ value anywhere on } f.$$

#### D. Infinite Discontinuity (Let $x = k$ represent an infinite discontinuity on $[a, b]$ )

$$\int_a^b f(x)dx = \lim_{x \rightarrow k^-} \int_a^k f(x)dx + \lim_{x \rightarrow k^+} \int_k^b f(x)dx$$