Integration Rules for Definite Integrals

1. $\int_a^a f(x) = 0$
2. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ for $c \in (a,b)$ → *This means that *c* is a value of ***x***, lying between *a* and *b*
3. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
4. $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, where f is an **even function.**
5. $\int_{-a}^a f(x)\,dx = 0$, where f is an **odd function.**

Basic Integration Rules

Let *k* be a constant.

$\int 0\,dx = C$	$\int k\,dx = kx + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$	$\int \cos x\,dx = \sin x + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sec^2 x\,dx = \tan x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \csc x \cot x\,dx = -\csc x + C$	

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Integrals

$$\int kf(u)du = k\int f(u)du$$

$$\int du = u + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \tan u du = -\ln|\cos u| + C$$

$$\int \cot u du = \ln|\sin u| + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} arc\sec\left(\frac{|u|}{a}\right) + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$$

Differentiability
No cusps, corners, vertical tangents, or discontinuity

Basic Trig Integrals

1. $\int \sec x \tan x \; dx = \sec x + C$
2. $\int \cos x \; dx = \sin x + C$
3. $\int \sec^2 x \; dx = \tan x + C$
4. $\int \sin x \; dx = -\cos x + C$
5. $\int \csc^2 x \; dx = -\cot x + C$
6. $\int \csc x \cot x \; dx = -\csc x + C$
7. $\int \tan x \; dx = -\ln|\cos x| + C$
8. $\int \cot x \; dx = \ln|\sin x| + C$
9. $\int \sec x \; dx = \ln|\sec x + \tan x| + C$
10. $\int \csc x \; dx = -\ln|\csc x + \cot x| + C$

Integration

Area, Sum, Accumulation → Integrate

Integral of Rate = Total or Net Change

Differentiation

Slope, Instantaneous Rate of Change → Differentiate

Derivative = Slope of Tangent Line

Integration of Trig and Inverse Trig

1. $\int \cos x\,dx = \sin x + c$
2. $\int \sin x\,dx = -\cos x + C$
3. $\int \sec^2 x\,dx = \tan x + C$
4. $\int \csc^2 x\,dx = -\cot x + C$
5. $\int \sec x \tan x\,dx = \sec x + C$
6. $\int \csc x \cot x\,dx = -\csc x + C$
7. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$
8. $\int \frac{-1}{\sqrt{1-x^2}}\,dx = \arccos x + C$

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9. $\int \frac{1}{1+x^2}\,dx = \arctan x + C$
10. $\int \frac{-1}{1+x^2}\,dx = \operatorname{arc}\cot x + C$
11. $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{arc}\sec x + C$
12. $\int \frac{-1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{arc}\csc x + C$

If u is a differentiable function of x, then

1. $\int \frac{u'}{u}\,du = \ln|u| + C$
2. $\int \frac{u'}{\sqrt{1-u^2}}\,du = \arcsin u + C$
3. $\int \frac{u'}{1+u^2}\,du = \arctan u + C$.

Integration of Exponential and Logarithmic Formulas

1. $\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\int \frac{u'}{u}\,du = \ln|u| + C$, where u is a differentiable function of x.
3. $\int \frac{1}{x-a}\,dx = \ln|x-a| + C$, where a = constant.
4. $\int a^x\,dx = \frac{a^x}{\ln a} + C$
5. $\int e^x\,dx = \frac{e^x}{\ln e} + C = e^x + C$
6. $\int a^{u(x)}\,dx = \frac{a^{u(x)}}{(\ln a)u'} + C$
7. $\int e^{u(x)}\,dx = \frac{e^{u(x)}}{u'} + C$

More Integrals

1. $\int du = u + C$

2. $\int u^n du = \frac{u^{n+1}}{n+1} + C \quad n \neq -1$

3. $\int \frac{du}{u} = \ln|u| + C$

4. $\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$

5. $\int e^u du = e^u + C$

4. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

6. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

7. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} arc\sec \frac{|u|}{a} + C$