Limit of a Continuous Function

If *f(x)* is a continuous function for all real numbers, then $\lim_{x \to c} f(x) = f(c)$

Limits of Rational Functions

A. If *f(x)* is a rational function given by $f(x) = \frac{p(x)}{q(x)}$,such that $p(x)$ and $q(x)$ have no common factors, and *c* is a real number such that $q(c) = 0$, then

I. $\lim_{x \to c} f(x)$ does not exist

II. $\lim_{x \to c} f(x) = \pm\infty$ → x = c is a vertical asymptote

B. If *f(x)* is a rational function given by $f(x) = \frac{p(x)}{q(x)}$, such that reducing a common factor between $p(x)$ and $q(x)$ results in the agreeable function ***k(x)***, then

$$\lim_{x \to c} f(x) = \lim_{x \to c} \frac{p(x)}{q(x)} = \lim_{x \to c} k(x) = k(c)$$ → Hole at the point $(c, k(c))$

Limits of a Function as x Approaches Infinity

If *f(x)* is a rational function given by $(x) = \frac{p(x)}{q(x)}$, such that $p(x)$ and $q(x)$ are both polynomial functions, then

A. If the degree of *p(x)* > *q(x)*, $\lim_{x\to\infty} f(x) = \infty$

B. If the degree of *p(x)* < *q(x)*, $\lim_{x\to\infty} f(x) = 0$ → y = 0 is a horizontal asymptote

C. If the degree of *p(x)* = *q(x)*, $\lim_{x\to\infty} f(x) = c$, where *c* is the ratio of the leading coefficients.
→ y = c is a horizontal asymptote

Special Trig Limits

A. $\lim_{x\to 0} \frac{\sin ax}{ax} = 1$

B. $\lim_{x\to 0} \frac{ax}{\sin ax} = 1$

C. $\lim_{x\to 0} \frac{1-\cos ax}{ax} = 0$