BC Only: Logistic Growth

A population, P, that experiences a limit factor in the growth of the population based upon the available resources to support the population is said to experience logistic growth.

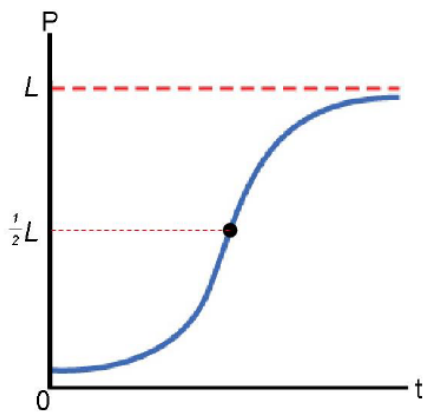
A. Differential Equation: $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$

B. General Solution: $P(t) = \frac{L}{1+be^{-kt}}$

P = population $\quad$ k = constant growth factor $\quad$ L = carrying capacity $\quad$ t = time,

b = constant (found with intital condition)

Graph



Characteristics of Logistics

I. The population is growing the fastest where $P = \frac{L}{2}$

II. The point where $P = \frac{L}{2}$ represents a point of inflection

III. $\lim_{t\to\infty} P(t) = L$

Logistic Growth:

$$\frac{dP}{dt} = kP\cdot\left(1-\frac{P}{L}\right) \qquad P(t) = \frac{L}{1+Ce^{-kt}}$$

$$\text{where}: \begin{cases} k \text{ is the proportionality constant} \\ L \text{ is the Carrying Capacity} \\ C \text{ is the integration constant} \end{cases}$$

Logistics Curves

$$P(t) = \frac{L}{1 + Ce^{-(Lk)t}},$$

where L is carrying capacity
Maximum growth rate occurs when
P = ½ L

$$\frac{dP}{dt} = kP(L - P) \text{ or}$$

$$\frac{dP}{dt} = (Lk)P(1 - \frac{P}{L})$$