CHAPTER 1

Tools of Geometry

Then

You graphed points on the coordinate plane and evaluated mathematical expressions.

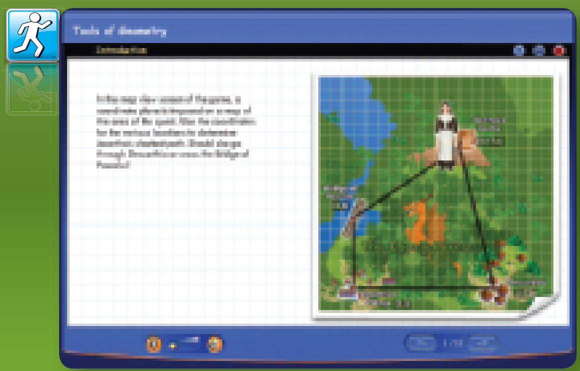
Now

In this chapter, you will:

- Find distances between points and midpoints of line segments.
- Identify angle relationships.
- Find perimeters, areas, surface areas, and volumes.

Why?

MAPS Geometric figures and terms can be used to represent and describe real-world situations. On a map, locations of cities can be represented by points, highways or streets by lines, and national parks by polygons that have both perimeter and area. The map itself is representative of a plane.



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LatinStock Collection/Alamy

Get Ready for the Chapter

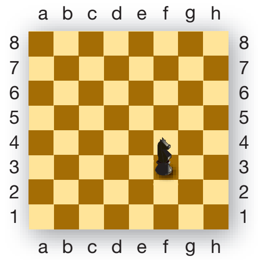
Diagnose Readiness | You have two options for checking Prerequisite Skills.

1 Textbook Option
Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Graph and label each point in the coordinate plane.

1. $W(5, 2)$
2. $X(0, 6)$
3. $Y(-3, -1)$
4. $Z(4, -2)$
5. **GAMES** Carolina is using the diagram to record her chess moves. She moves her knight 2 spaces up and 1 space to the left from f3. What is the location of the knight after Carolina completes her turn?

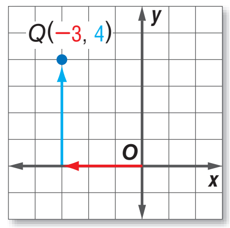


QuickReview

Example 1

Graph and label the point $Q(-3, 4)$ in the coordinate plane.

Start at the origin. Since the x-coordinate is negative, move 3 units to the left. Then move 4 units up since the y-coordinate is positive. Draw a dot and label it Q.



QuickCheck

Find each sum or difference.

6. $\frac{2}{3} + \frac{5}{6}$
7. $2\frac{1}{18} + 4\frac{3}{4}$
8. $\frac{13}{18} - \frac{5}{9}$
9. $14\frac{3}{5} - 9\frac{7}{15}$
10. **FOOD** Alvin ate $\frac{1}{3}$ of a pizza for dinner and took $\frac{1}{6}$ of it for lunch the next day. How much of the pizza does he have left?

QuickReview

Example 2

Find $3\frac{1}{6} + 2\frac{3}{4}$.

$3\frac{1}{6} + 2\frac{3}{4} = \frac{19}{6} + \frac{11}{4}$ — Write as improper fractions.

$= \frac{19}{6}\left(\frac{2}{2}\right) + \frac{11}{4}\left(\frac{3}{3}\right)$ — The LCD is 12.

$= \frac{38}{12} + \frac{33}{12}$ — Multiply.

$= \frac{71}{12}$ or $5\frac{11}{12}$ — Simplify.

QuickCheck

Evaluate each expression.

11. $(-4 - 5)^2$
12. $(6 - 10)^2$
13. $(8 - 5)^2 + [9 - (-3)]^2$

Solve each equation.

14. $6x + 5 + 2x - 11 = 90$
15. $8x - 7 = 53 - 2x$

QuickReview

Example 3

Evaluate the expression $[-2 - (-7)]^2 + (1 - 8)^2$.

Follow the order of operations.

$[-2 - (-7)]^2 + (1 - 8)^2$

$= 5^2 + (-7)^2$ — Subtract.

$= 25 + 49$ — $5^2 = 25, (-7)^2 = 49$

$= 74$ — Add.

2 Online Option
Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

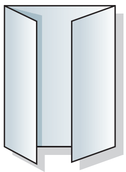
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 1. To get ready, identify important terms and organize your resources. You may refer to Chapter 0 to review prerequisite skills.

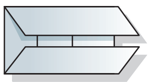
FOLDABLES® **Study**Organizer

Tools of Geometry Make this Foldable to help you organize your Chapter 1 notes about points, lines, and planes; angles and angle relationships; and formulas and notes for distance, midpoint, perimeter, area, and volume. Begin with a sheet of 11″ × 17″ paper.

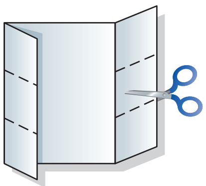
1. **Fold** the short sides to meet in the middle.



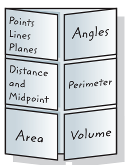
2. **Fold** the booklet in thirds lengthwise.



3. **Open and cut** the booklet in thirds lengthwise.



4. **Label** the tabs as shown.



NewVocabulary

English		Español
collinear	p. 5	colineal
coplanar	p. 5	coplanar
congruent	p. 16	congruente
midpoint	p. 27	punto medio
segment bisector	p. 29	bisectriz de segmento
angle	p. 36	angulo
vertex	p. 36	vertice
angle bisector	p. 39	bisectriz de un angulo
perpendicular	p. 48	perpendicular
polygon	p. 56	poligono
perimeter	p. 58	perimetro
volume	p. 69	volumen

ReviewVocabulary

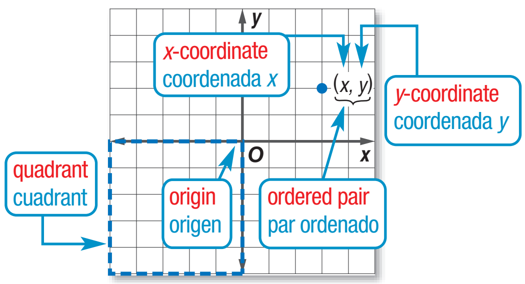
ordered pair par ordenado a set of numbers or coordinates used to locate any point on a coordinate plane, written in the form (x, y)

origin origen the point where the two axes intersect at their zero points

quadrants cuadrantes the four regions into which the x-axis and y-axis separate the coordinate plane

x-coordinate coordenada x the first number in an ordered pair

y-coordinate coordenada y the second number in an ordered pair



LESSON 1-1 Points, Lines, and Planes

Then

- You used basic geometric concepts and properties to solve problems.

Now

1. Identify and model points, lines, and planes.
2. Identify intersecting lines and planes.

Why?

- On a subway map, the locations of stops are represented by *points*. The route the train can take is modeled by a series of connected paths that look like *lines*. The flat surface of the map on which these points and lines lie is representative of a *plane*.

Brand X Pictures/Jupiterimages

NewVocabulary

undefined term
point
line
plane
collinear
coplanar
intersection
definition
defined term
space

Common Core State Standards

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

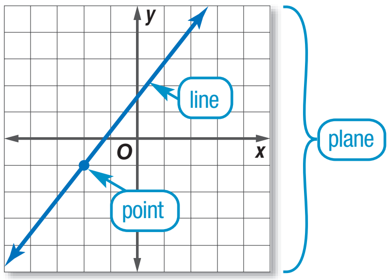
Mathematical Practices

4 Model with mathematics.
6 Attend to precision.

1 Points, Lines, and Planes

Unlike the real-world objects that they model, shapes, points, lines, and planes do not have any actual size. In geometry, *point*, *line*, and *plane* are considered **undefined terms** because they are only explained using examples and descriptions.

You are already familiar with the terms point, line, and plane from algebra. You graphed on a coordinate *plane* and found ordered pairs that represented *points* on *lines*. In geometry, these terms have a similar meaning.



The phrase *exactly one* in a statement such as, "There is exactly one line through any two points," means that there is *one and only one*.

KeyConcept Undefined Terms

A **point** is a location. It has neither shape nor size.

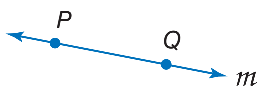
Named by a capital letter

Example point *A*

A **line** is made up of points and has no thickness or width. There is exactly one line through any two points.

Named by the letters representing two points on the line or a lowercase script letter

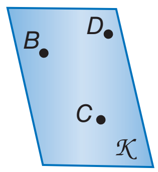
Example line *m*, line *PQ* or $\overleftrightarrow{PQ}$, line *QP* or $\overleftrightarrow{QP}$



A **plane** is a flat surface made up of points that extends infinitely in all directions. There is exactly one plane through any three points not on the same line.

Named by a capital script letter or by the letters naming three points that are not all on the same line

Example plane $\mathcal{K}$, plane *BCD*, plane *CDB*, plane *DCB*, plane *DBC*, plane *CBD*, plane *BDC*

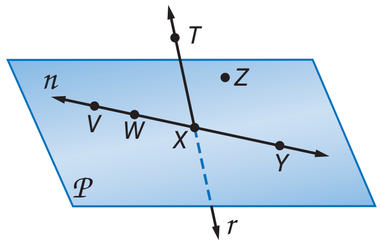


Collinear points are points that lie on the same line. *Noncollinear* points do not lie on the same line. **Coplanar** points are points that lie in the same plane. *Noncoplanar* points do not lie in the same plane.

Example 1 Name Lines and Planes

Use the figure to name each of the following.

a. a line containing point *W*



The line can be named as line *n*, or any two of the four points on the line can be used to name the line.

$\overleftrightarrow{VW}$ $\overleftrightarrow{WV}$ $\overleftrightarrow{VX}$ $\overleftrightarrow{XV}$ $\overleftrightarrow{VY}$ $\overleftrightarrow{YV}$
$\overleftrightarrow{WX}$ $\overleftrightarrow{XW}$ $\overleftrightarrow{WY}$ $\overleftrightarrow{YW}$ $\overleftrightarrow{XY}$ $\overleftrightarrow{YX}$

b. a plane containing point *X*

One plane that can be named is plane 𝒫. You can also use the letters of any three *noncollinear* points to name this plane.

plane *XZY*	plane *VZW*	plane *VZX*
plane *VZY*	plane *WZX*	plane *WZY*

The letters of each of these names can be reordered to create other acceptable names for this plane. For example, *XZY* can also be written as *XYZ*, *ZXY*, *ZYX*, *YXZ*, and *YZX*. In all, there are 36 different three-letter names for this plane.

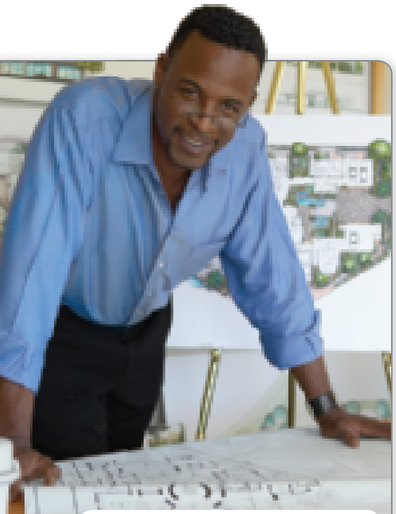
Study Tip

Additional Planes Although not drawn in Example 1b, there is another plane that contains point *X*. Since points *W*, *T*, and *X* are noncollinear, point *X* is also in plane *WTX*.

Guided Practice

1A. a plane containing points *T* and *Z*

1B. a line containing point *T*

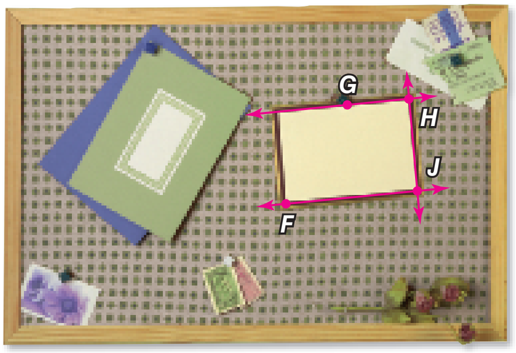


Real-World Career

Drafter Drafters use perspective to create drawings to build everything from toys to school buildings. Drafters need skills in math and computers. They get their education at trade schools, community colleges, and some 4-year colleges. Refer to Exercises 50 and 51.

Real-World Example 2 Model Points, Lines, and Planes

MESSAGE BOARD **Name the geometric terms modeled by the objects in the picture.**



The push pin models point *G*.

The maroon border on the card models line *GH*.

The edge of the card models line *HJ*.

The card itself models plane *FGJ*.

Guided Practice

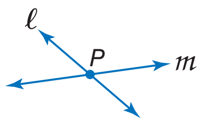
Name the geometric term modeled by each object.

2A. stripes on a sweater

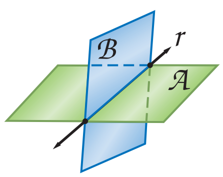
2B. the corner of a box

2 Intersections of Lines and Planes

The **intersection** of two or more geometric figures is the set of points they have in common. Two lines intersect in a point. Lines can intersect planes, and planes can intersect each other.



P represents the intersection of lines ℓ and *m*.



Line *r* represents the intersection of planes 𝒜 and ℬ.

Digital Vision/Getty Images

Example 3 Draw Geometric Figures

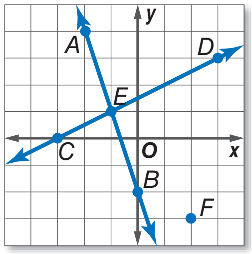
Draw and label a figure for each relationship.

a. ALGEBRA Lines *AB* and *CD* intersect at *E* for *A*(−2, 4), *B*(0, −2), *C*(−3, 0), and *D*(3, 3) on a coordinate plane. Point *F* is coplanar with these points, but not collinear with $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

Graph each point and draw $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

Label the intersection point as *E*.

An infinite number of points are coplanar with *A*, *B*, *C*, *D* and *E* but not collinear with $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. In the graph, one such point is *F*(2, −3).

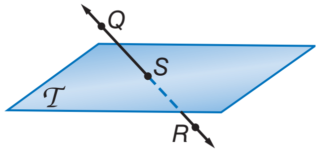


b. *QR* intersects plane $\mathcal{T}$ at point *S*.

Draw a surface to represent plane $\mathcal{T}$ and label it.

Draw a dot for point *S* anywhere on the plane and a dot that is not on plane $\mathcal{T}$ for point *Q*.

Draw a line through points *Q* and *S*. Dash the line to indicate the portion hidden by the plane. Then draw another dot on the line and label it *R*.



StudyTip

Three-Dimensional Drawings Because it is impossible to show an entire plane in a figure, edged shapes with different shades of color are used to represent planes.

GuidedPractice

3A. Points *J*(−4, 2), *K*(3, 2), and *L* are collinear.

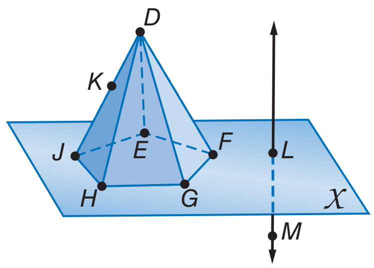
3B. Line *p* lies in plane $\mathcal{N}$ and contains point *L*.

Definitions or **defined terms** are explained using undefined terms and/or other defined terms. **Space** is defined as a boundless, three-dimensional set of all points. Space can contain lines and planes.

Example 4 Interpret Drawings

a. How many planes appear in this figure?

Six: plane $\mathcal{X}$, plane *JDH*, plane *JDE*, plane *EDF*, plane *FDG*, and plane *HDG*.



b. Name three points that are collinear.

Points *J*, *K*, and *D* are collinear.

c. Name the intersection of plane *HDG* with plane $\mathcal{X}$.

Plane *HDG* intersects plane $\mathcal{X}$ in $\overleftrightarrow{HG}$.

d. At what point do $\overleftrightarrow{LM}$ and $\overleftrightarrow{EF}$ intersect? Explain.

It does not appear that these lines intersect. $\overleftrightarrow{EF}$ lies in plane $\mathcal{X}$, but only point *L* of $\overleftrightarrow{LM}$ lies in $\mathcal{X}$.

StudyTip

CCSS Precision A point has no dimension. A line exists in one dimension. However, a circle is two-dimensional, and a pyramid is three-dimensional.

GuidedPractice

Explain your reasoning.

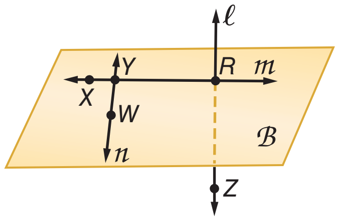
4A. Are points *E*, *D*, *F*, and *G* coplanar?

4B. At what point or in what line do planes *JDH*, *JDE*, and *EDF* intersect?

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Use the figure to name each of the following.**



1. a line containing point X
2. a line containing point Z
3. a plane containing points W and R

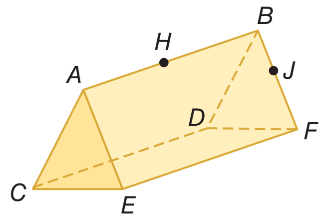
Example 2 **Name the geometric term modeled by each object.**

4. a beam from a laser
5. a floor

Example 3 **Draw and label a figure for each relationship.**

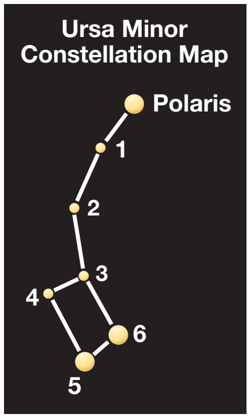
6. A line in a coordinate plane contains $A(0, -5)$ and $B(3, 1)$ and a point C that is not collinear with $\overleftrightarrow{AB}$.
7. Plane Z contains lines x, y, and w. Lines x and y intersect at point V and lines x and w intersect at point P.

Example 4 **Refer to the figure.**



8. How many planes are shown in the figure?
9. Name three points that are collinear.
10. Are points A, H, J, and D coplanar? Explain.
11. Are points B, D, and F coplanar? Explain.

12. **ASTRONOMY** Ursa Minor, or the Little Dipper, is a constellation made up of seven stars in the northern sky including the star Polaris.

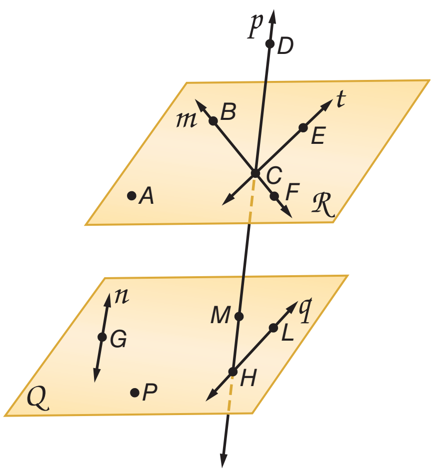


   a. What geometric figures are modeled by the stars?
   b. Are Star 1, Star 2, and Star 3 collinear on the constellation map? Explain.
   c. Are Polaris, Star 2, and Star 6 coplanar on the map?

Practice and Problem Solving

Extra Practice is on page R1.

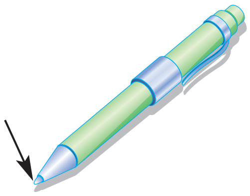
Example 1 **Refer to the figure.**



13. Name the lines that are only in plane Q.
14. How many planes are labeled in the figure?
15. Name the plane containing the lines m and t.
16. Name the intersection of lines m and t.
17. Name a point that is not coplanar with points A, B, and C.
18. Are points F, M, G, and P coplanar? Explain.
19. Name the points not contained in a line shown.
20. What is another name for line t?
21. Does line n intersect line q? Explain.

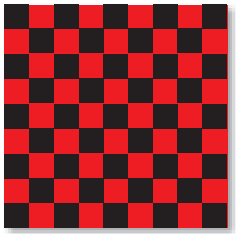
Example 2 **Name the geometric term(s) modeled by each object.**

22.

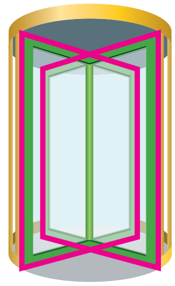


23.

24.



25.



26. a blanket

27. a knot in a rope

28. a telephone pole

29. the edge of a desk

30. two connected walls

31. a partially opened folder

Example 3 **Draw and label a figure for each relationship.**

32. Line m intersects plane $\mathcal{R}$ at a single point.

33. Two planes do not intersect.

34. Points X and Y lie on $\overleftrightarrow{CD}$.

35. Three lines intersect at point J but do not all lie in the same plane.

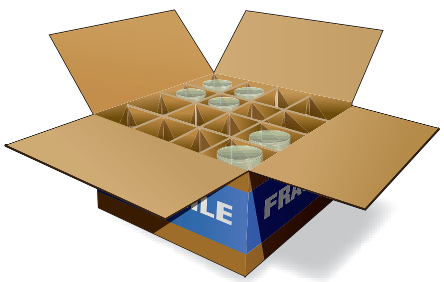
36. Points $A(2, 3)$, $B(2, -3)$, C and D are collinear, but A, B, C, D, and F are not.

37. Lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{NP}$ are coplanar but do not intersect.

38. $\overleftrightarrow{FG}$ and $\overleftrightarrow{JK}$ intersect at $P(4, 3)$, where point F is at $(-2, 5)$ and point J is at $(7, 9)$.

39. Lines s and t intersect, and line v does not intersect either one.

Example 4 CCSS **MODELING** **When packing breakable objects such as glasses, movers frequently use boxes with inserted dividers like the one shown.**

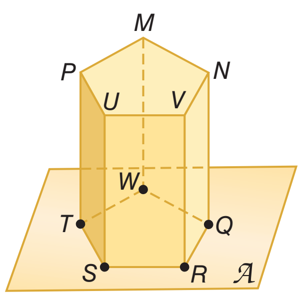


40. How many planes are modeled in the picture?

41. What parts of the box model lines?

42. What parts of the box model points?

Refer to the figure at the right.



43. Name two collinear points.

44. How many planes appear in the figure?

45. Do plane $\mathcal{A}$ and plane MNP intersect? Explain.

46. In what line do planes $\mathcal{A}$ and QRV intersect?

47. Are points T, S, R, Q, and V coplanar? Explain.

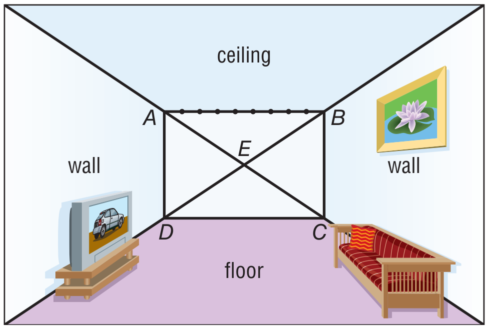
48. Are points T, S, R, Q, and W coplanar? Explain.

49 **FINITE PLANES** A *finite plane* is a plane that has boundaries, or does not extend indefinitely. The street signs shown are finite planes.

a. If the pole models a line, name the geometric term that describes the intersection between the signs and the pole.

b. What geometric term(s) describes the intersection between the two finite planes? Explain your answer with a diagram if necessary.

50. **ONE-POINT PERSPECTIVE** One-point perspective drawings use lines to convey depth. Lines representing horizontal lines in the real object can be extended to meet at a single point called the *vanishing point*. Suppose you want to draw a tiled ceiling in the room below with nine tiles across.

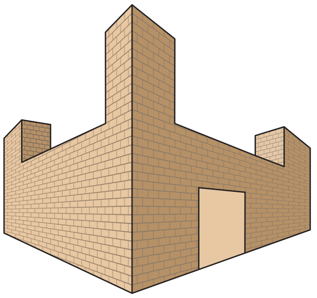


a. What point represents the vanishing point in the drawing?

b. Trace the figure. Then draw lines from the vanishing point through each of the eight points between *A* and *B*. Extend these lines to the top edge of the drawing.

c. How could you change the drawing to make the back wall of the room appear farther away?

51. **TWO-POINT PERSPECTIVE** Two-point perspective drawings use two vanishing points to convey depth.



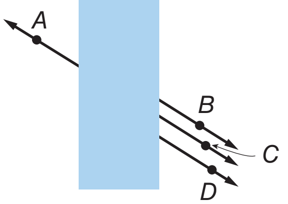
a. Trace the drawing of the castle shown. Draw five of the vertical lines used to create the drawing.

b. Draw and extend the horizontal lines to locate the vanishing points and label them.

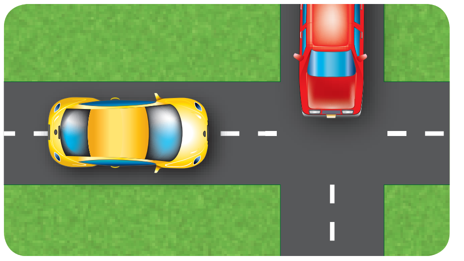
c. What do you notice about the vertical lines as they get closer to the vanishing point?

d. Draw a two-point perspective of a home or a room in a home.

52. CCSS **ARGUMENTS** Name two points on the same line in the figure. How can you support your assertion?



53. **TRANSPORTATION** When two cars enter an intersection at the same time on opposing paths, one of the cars must adjust its speed or direction to avoid a collision. Two airplanes, however, can cross paths while traveling in different directions without colliding. Explain how this is possible.



54. **MULTIPLE REPRESENTATIONS** Another way to describe a group of points is called a locus. A **locus** is a set of points that satisfy a particular condition. In this problem, you will explore the locus of points that satisfy an equation.

    a. **Tabular** Represent the locus of points satisfying the equation $2 + x = y$ using a table of at least five values.

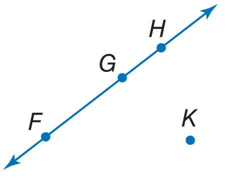
    b. **Graphical** Represent this same locus of points using a graph.

    c. **Verbal** Describe the geometric figure that the points suggest.

55. **PROBABILITY** Three of the labeled points are chosen at random.

    a. What is the probability that the points chosen are collinear?

    b. What is the probability that the points chosen are coplanar?



56. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the locus of points that satisfy an inequality.

    a. **Tabular** Represent the locus of points satisfying the inequality $y < -3x - 1$ using a table of at least ten values.

    b. **Graphical** Represent this same locus of points using a graph.

    c. **Verbal** Describe the geometric figure that the points suggest.

H.O.T. Problems Use Higher-Order Thinking Skills

57. **OPEN ENDED** Sketch three planes that intersect in a line.

58. **ERROR ANALYSIS** Camille and Hiroshi are trying to determine the most number of lines that can be drawn using any two of four random points. Is either correct? Explain.

Camille	Hiroshi
Since there are four points, $4 \cdot 3$ or 12 lines can be drawn between the points.	You can draw $3 \cdot 2 \cdot 1$ or 6 lines between the points.

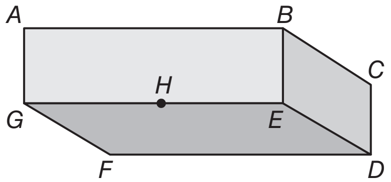
59. **CCSS ARGUMENTS** What is the greatest number of planes determined using any three of the points A, B, C, and D if no three points are collinear?

60. **REASONING** Is it possible for two points on the surface of a prism to be neither collinear nor coplanar? Justify your answer.

61. **WRITING IN MATH** Refer to Exercise 49. Give a real-life example of a finite plane. Is it possible to have a real-life object that is an infinite plane? Explain your reasoning.

Standardized Test Practice

62. Which statement about the figure below is *not* true?



A Point H lies in planes AGE and GED.

B Planes GAB, GFD and BED intersect at point E.

C Points F, E, and B are coplanar.

D Points A, H, and D are collinear.

63. ALGEBRA What is the value of x if $3x + 2 = 8$?

F -2 **G** 0 **H** 2 **J** 6

64. GRIDDED RESPONSE An ice chest contains 3 types of drinks: 10 apple juices, 15 grape juices, and 15 bottles of water. What is the probability that a drink selected randomly from the ice chest does *not* contain fruit juice?

65. SAT/ACT A certain school's enrollment increased 6% this year over last year's enrollment. If the school now has 1378 students enrolled, how many students were enrolled last year?

A 1295 **C** 1350 **E** 1500

B 1300 **D** 1460

Spiral Review

Simplify. (Lesson 0-9)

66. $\sqrt{72}$

67. $\sqrt{18} \cdot \sqrt{14}$

68. $\sqrt{44x^4y^3}$

69. $\dfrac{3}{\sqrt{18}}$

70. $\sqrt{\dfrac{28}{75}}$

71. $\dfrac{\sqrt{8a^6}}{\sqrt{108}}$

72. $\dfrac{5}{4 - \sqrt{2}}$

73. $\dfrac{4\sqrt{3}}{2 + \sqrt{5}}$

74. FINANCIAL LITERACY Suppose you buy 3 shirts and 2 pairs of slacks on sale at a clothing store for $72. The next day, a friend buys 2 shirts and 4 pairs of slacks for $96. If the shirts you each bought were all the same price and the slacks were also all the same price, then what was the cost of each shirt and each pair of slacks? (Lesson 0-8)

Graph the following points, and connect them in order to form a figure. (Lesson 0-7)

75. $A(-5, 3)$, $B(3, -4)$, and $C(-2, -3)$

76. $P(-2, 1)$, $Q(3, 4)$, $R(5, 1)$, and $S(0, -2)$

GROCERIES **Find an approximate metric weight for each item.** (Lesson 0-2)

77.

Net Wt: 15 oz

78.

Net Wt: 8.2 oz

79.

Net Wt: 2.5 lb

Skills Review

Replace each ● with >, <, or = to make a true statement.

80. $\frac{1}{4}$ in. ● $\frac{1}{2}$ in.

81. $\frac{3}{4}$ in. ● $\frac{5}{8}$ in.

82. $\frac{3}{8}$ in. ● $\frac{6}{16}$ in.

83. 18 mm ● 2 cm

84. 32 mm ● 3.2 cm

85. 0.8 m ● 8 cm

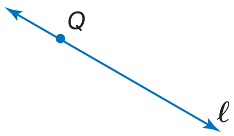
EXTEND 1-1

Geometry Lab
Describing What You See

When you are learning geometric concepts, it is critical to have accurate drawings to represent the information. It is helpful to know what words and phrases can be used to describe figures. Likewise, it is important to know how to read a geometric description and be able to draw the figure it describes.

Common Core State Standards
Content Standards
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★
Mathematical Practices 6

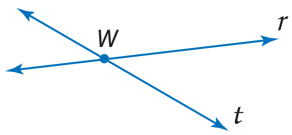
The figures and descriptions below help you visualize and write about points, lines, and planes.



Point Q is on ℓ.

Line ℓ contains Q.

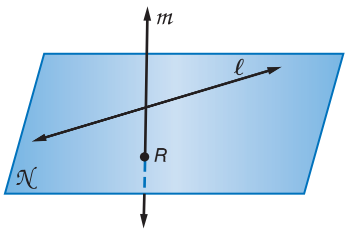
Line ℓ passes through Q.



Lines r and t intersect at W.

Point W is the intersection of r and t.

Point W is on r. Point W is on t.



Line ℓ and point R are in $\mathcal{N}$.

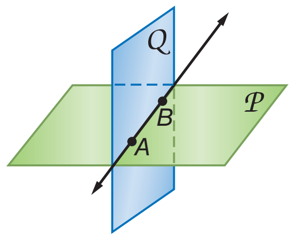
Point R lies in $\mathcal{N}$.

Plane $\mathcal{N}$ contains R and ℓ.

Line m intersects $\mathcal{N}$ at R.

Point R is the intersection of m with $\mathcal{N}$.

Lines ℓ and m do not intersect.



$\overleftrightarrow{AB}$ is in $\mathcal{P}$ and $\mathcal{Q}$.

Points A and B lie in both $\mathcal{P}$ and $\mathcal{Q}$.

Planes $\mathcal{P}$ and $\mathcal{Q}$ both contain $\overleftrightarrow{AB}$.

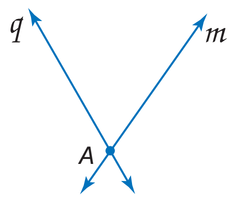
Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in $\overleftrightarrow{AB}$.

$\overleftrightarrow{AB}$ is the intersection of $\mathcal{P}$ and $\mathcal{Q}$.

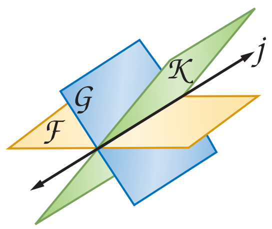
Exercises

Write a description for each figure.

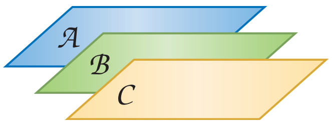
1.



2.



3.



4. Draw and label a figure for the statement Planes $\mathcal{N}$ and $\mathcal{P}$ contain line a.

LESSON 1-2 Linear Measure

Then

- You identified and modeled points, lines, and planes.

Now

1. Measure segments.
2. Calculate with measures.

Why?

- When the ancient Egyptians found a need for a measurement system, they used the human body as a guide. The cubit was the length of an arm from the elbow to the fingertips. Eventually the Egyptians standardized the length of a cubit, with ten *royal cubits* equivalent to one *rod*.

NewVocabulary
line segment
betweenness of points
between
congruent segments
construction

Common Core State Standards

Content Standards
G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices
6 Attend to precision.

1 Measure Line Segments

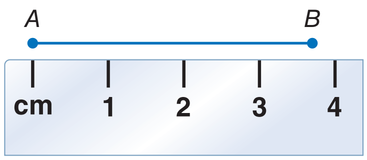
Unlike a line, a **line segment**, or *segment*, can be measured because it has two endpoints. A segment with endpoints A and B can be named as $\overline{AB}$ or $\overline{BA}$. The *measure* of $\overline{AB}$ is written as AB. The length or measure of a segment always includes a unit of measure, such as meter or inch.

All measurements are approximations dependent upon the smallest unit of measure available on the measuring instrument.

Example 1 Length in Metric Units

Find the length of $\overline{AB}$ using each ruler.

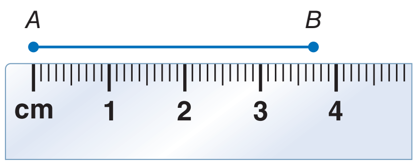
a.



The ruler is marked in centimeters. Point B is closer to the 4-centimeter mark than to 3 centimeters.

Thus, $\overline{AB}$ is about 4 centimeters long.

b.



The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter.

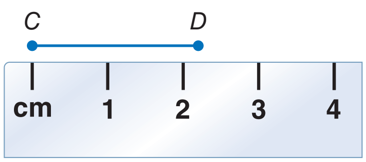
Thus, $\overline{AB}$ is about 3.7 centimeters long.

GuidedPractice

1A. Measure the length of a dollar bill in centimeters.

1B. Measure the length of a pencil in millimeters.

1C. Find the length of $\overline{CD}$.



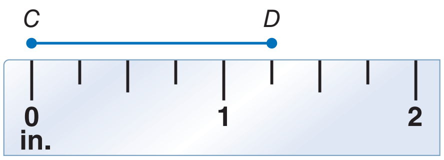
StudyTip

Using a Ruler The zero point on a ruler may not be clearly marked. For some rulers, zero is the left edge of the ruler. On others, it may be a fine line farther in on the scale. If it is not clear where the zero is, align one endpoint on 1 and subtract 1 from the measurement at the other endpoint.

Example 2 Length in Standard Units

Find the length of $\overline{CD}$ using each ruler.

a.

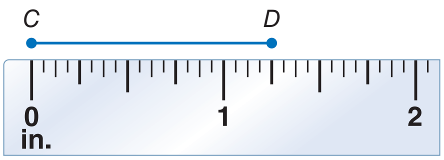


Each inch is divided into fourths.

Point D is closer to the $1\frac{1}{4}$-inch mark.

$\overline{CD}$ is about $1\frac{1}{4}$ inches long.

b.



Each inch is divided into sixteenths.

Point D is closer to the $1\frac{4}{16}$-inch mark.

$\overline{CD}$ is about $1\frac{4}{16}$ or $1\frac{1}{4}$ inches long.

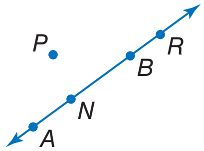
GuidedPractice

2A. Measure the length of a dollar bill in inches.

2B. Measure the length of a pencil in inches.

2 Calculate Measures

Recall that for any two real numbers a and b, there is a real number n that is *between* a and b such that $a < n < b$. This relationship also applies to points on a line and is called **betweenness of points**. In the figure, point N is between points A and B, but points R and P are not.



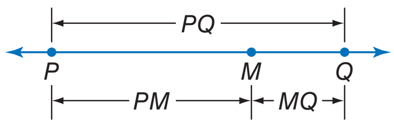
Measures are real numbers, so all arithmetic operations can be used with them. You know that the whole usually equals the sum of its parts. That is also true of line segments in geometry.

KeyConcept Betweenness of Points

Words

Point M is **between** points P and Q if and only if P, Q, and M are collinear and $PM + MQ = PQ$.

Model



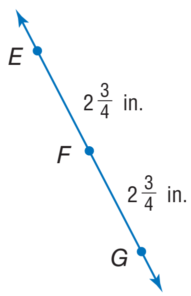
StudyTip

Comparing Measures Because measures are real numbers, you can compare them. If points X, Y, and Z are collinear in that order, then one of these statements is true: $XY = YZ$, $XY > YZ$, or $XY < YZ$.

Example 3 Find Measurements by Adding

Find EG. Assume that the figure is not drawn to scale.

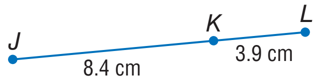
EG is the measure of $\overline{EG}$. Point F is between E and G. Find EG by adding EF and FG.



$EF + FG = EG$	Betweenness of points
$2\frac{3}{4} + 2\frac{3}{4} = EG$	Substitution
$5\frac{1}{2}$ in. $= EG$	Add.

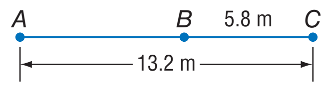
GuidedPractice

3. Find JL. Assume that the figure is not drawn to scale.



Example 4 Find Measurements by Subtracting

Find AB. Assume that the figure is not drawn to scale.

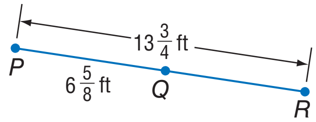


Point B is between A and C.

$AB + BC = AC$	Betweenness of points
$AB + 5.8 = 13.2$	Substitution
$AB + 5.8 - 5.8 = 13.2 - 5.8$	Subtract 5.8 from each side.
$AB = 7.4$ m	Simplify.

GuidedPractice

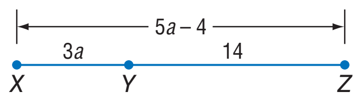
4. Find QR. Assume that the figure is not drawn to scale.



Example 5 Write and Solve Equations to Find Measurements

ALGEBRA Find the value of a and XY if Y is between X and Z, $XY = 3a$, $XZ = 5a - 4$, and $YZ = 14$.

Draw a figure to represent this information.



$XZ = XY + YZ$	Betweenness of points
$5a - 4 = 3a + 14$	Substitution
$5a - 4 - 3a = 3a + 14 - 3a$	Subtract 3a from each side.
$2a - 4 = 14$	Simplify.
$2a - 4 + 4 = 14 + 4$	Add 4 to each side.
$2a = 18$	Simplify.
$\frac{2a}{2} = \frac{18}{2}$	Divide each side by 2.
$a = 9$	Simplify.

Now find XY.

$XY = 3a$	Given
$= 3(9)$ or 27	$a = 9$

GuidedPractice

5. Find x and BC if B is between A and C, $AC = 4x - 12$, $AB = x$, and $BC = 2x + 3$.

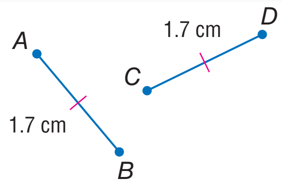
Segments that have the same measure are called **congruent segments**.

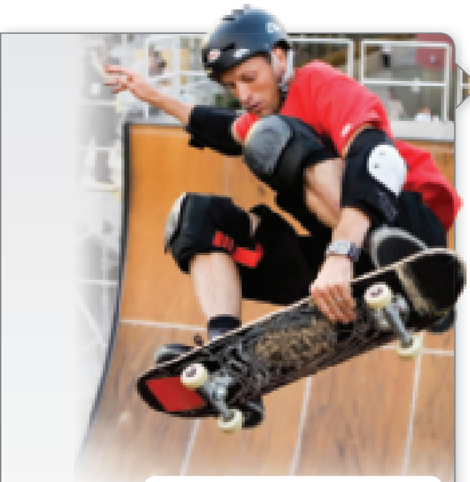
WatchOut!

Equal vs. Congruent Lengths are equal and segments are congruent. It is correct to say that $AB = CD$ and $\overline{AB} \cong \overline{CD}$. However, it is *not* correct to say that $\overline{AB} = \overline{CD}$ or that $AB \cong CD$.

KeyConcept Congruent Segments

Words	Congruent segments have the same measure.
Symbols	$\cong$ is read *is congruent to*. Red slashes on the figure also indicate congruence.
Example	$\overline{AB} \cong \overline{CD}$





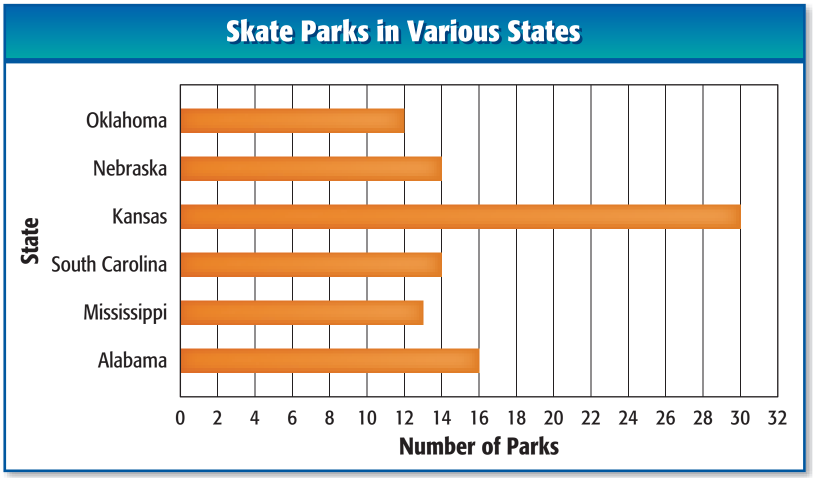
Real-WorldLink

The first commercial skateboard was introduced in 1959. Now there are more than 500 skate parks in the United States.

Source: *Encyclopaedia Britannica*

Real-World Example 6 Congruent Segments

SKATE PARKS **In the graph, suppose a segment was drawn along the top of each bar. Which states would have segments that are congruent? Explain.**



Source: SITE Design Group, Inc.

The segments on the bars for Nebraska and South Carolina would be congruent because they both represent the same number of skate parks.

GuidedPractice

6A. Suppose Oklahoma added another skate park. The segment drawn along the bar representing Oklahoma would be congruent to which other segment?

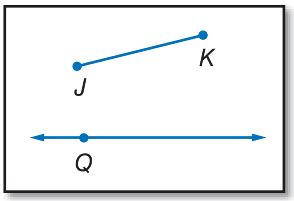
6B. Name the congruent segments in the sign shown.

Drawings of geometric figures are created using measuring tools such as a ruler and protractor. **Constructions** are methods of creating these figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used in constructions. *Sketches* are created without the use of any of these tools.

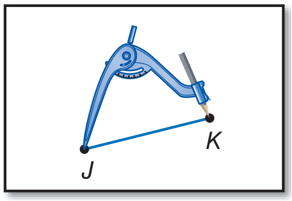
You can construct a segment that is congruent to a given segment.

Construction Copy a Segment

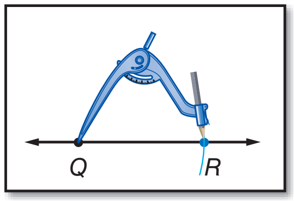
Step 1 Draw a segment $\overline{JK}$. Elsewhere on your paper, draw a line and a point on the line. Label the point Q.



Step 2 Place the compass at point J and adjust the compass setting so that the pencil is at point K.



Step 3 Using that setting, place the compass point at Q and draw an arc that intersects the line. Label the point of intersection R. $\overline{JK} \cong \overline{QR}$

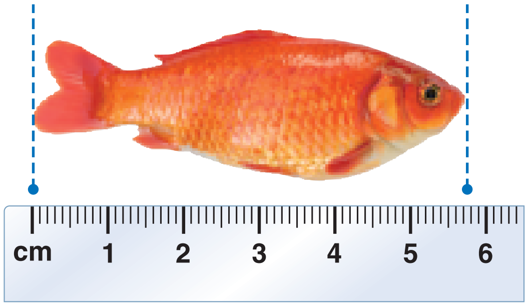


Check Your Understanding

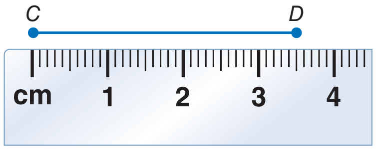
= Step-by-Step Solutions begin on page R14.

Example 1 Find the length of each line segment or object.

1.

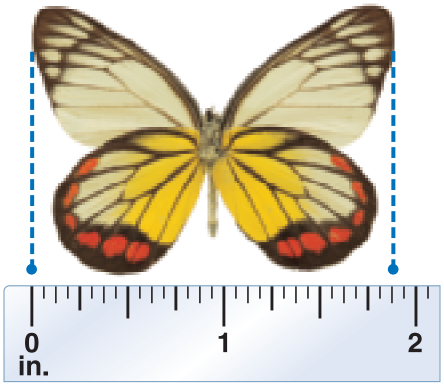


2.

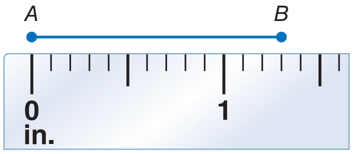


Example 2

3.

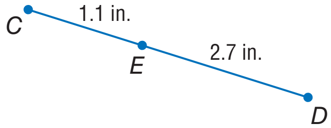


4.

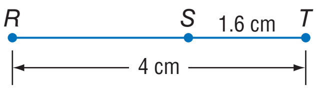


Examples 3–4 Find the measurement of each segment. Assume that each figure is not drawn to scale.

5. $\overline{CD}$



6. $\overline{RS}$



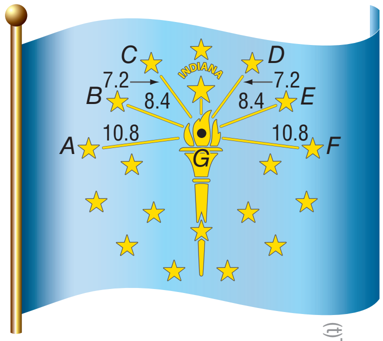
Example 5 **ALGEBRA** Find the value of x and BC if B is between C and D.

7. $CB = 2x$, $BD = 4x$, and $BD = 12$

8. $CB = 4x - 9$, $BD = 3x + 5$, and $CD = 17$

Example 6

9. **CCSS STRUCTURE** The Indiana State Flag was adopted in 1917. The measures of the segments between the stars and the flame are shown on the diagram in inches. List all of the congruent segments in the figure.

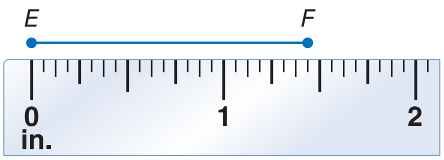


Practice and Problem Solving

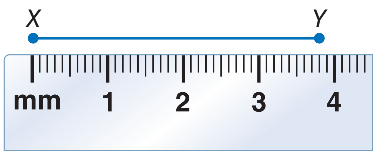
Extra Practice is on page R1.

Examples 1–2 Find the length of each line segment.

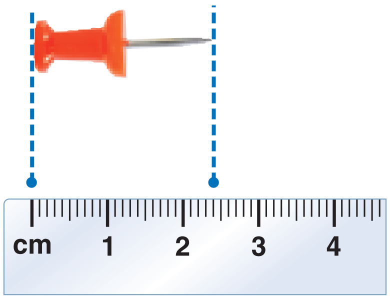
10.



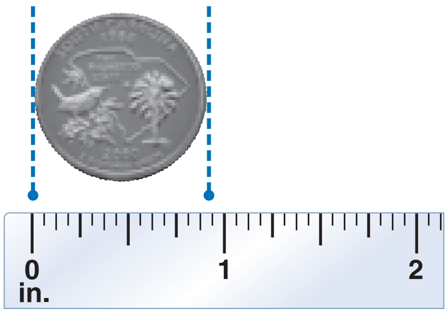
11.



12.

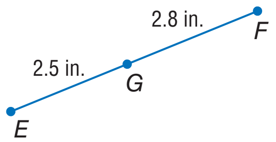


13.



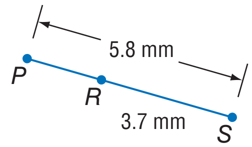
Examples 3–4 **Find the measurement of each segment. Assume that each figure is not drawn to scale.**

14. $\overline{EF}$

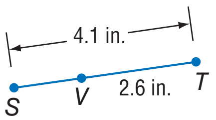


15. $\overline{JL}$

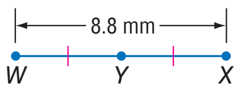
16. $\overline{PR}$



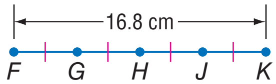
17. $\overline{SV}$



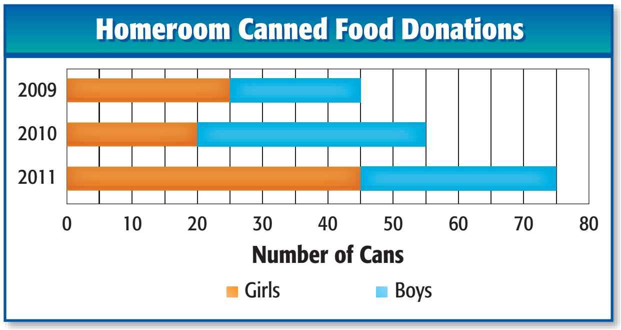
18. $\overline{WY}$



19. $\overline{FG}$



20. CCSS **SENSE-MAKING** The stacked bar graph shows the number of canned food items donated by the girls and the boys in a homeroom class over three years. Use the concept of betweenness of points to find the number of cans donated by the boys for each year. Explain your method.



Example 5 **ALGEBRA** **Find the value of the variable and *YZ* if *Y* is between *X* and *Z*.**

21. $XY = 11, YZ = 4c, XZ = 83$

22. $XY = 6b, YZ = 8b, XZ = 175$

23. $XY = 7a, YZ = 5a, XZ = 6a + 24$

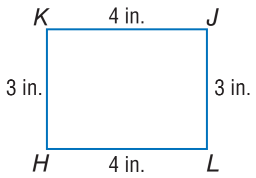
24. $XY = 11d, YZ = 9d - 2, XZ = 5d + 28$

25. $XY = 4n + 3, YZ = 2n - 7, XZ = 22$

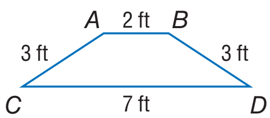
26. $XY = 3a - 4, YZ = 6a + 2, XZ = 5a + 22$

Example 6 **Determine whether each pair of segments is congruent.**

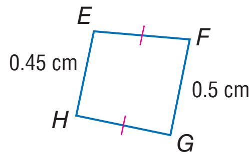
27 $\overline{KJ}, \overline{HL}$



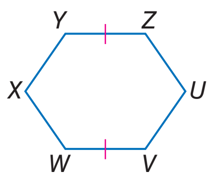
28. $\overline{AC}, \overline{BD}$



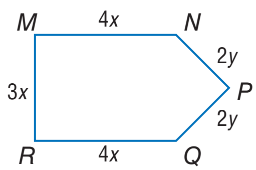
29. $\overline{EH}, \overline{FG}$



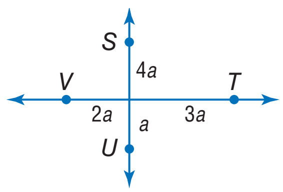
30. $\overline{VW}, \overline{UZ}$



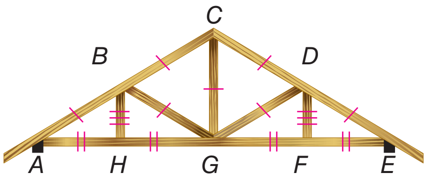
31. $\overline{MN}, \overline{RQ}$



32. $\overline{SU}, \overline{VT}$



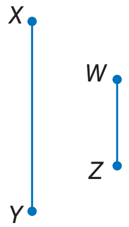
33 **TRUSSES** A truss is a structure used to support a load over a span, such as a bridge or the roof of a house. List all of the congruent segments in the figure.



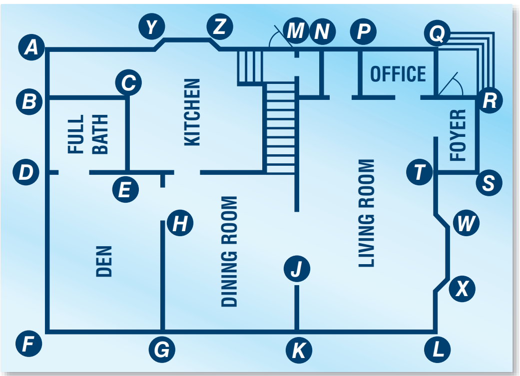
34. **CONSTRUCTION** For each expression:

- construct a segment with the given measure,
- explain the process you used to construct the segment, and
- verify that the segment you constructed has the given measure.

a. $2(XY)$ **b.** $6(WZ) - XY$



35. **BLUEPRINTS** Use a ruler to determine at least five pairs of congruent segments with labeled endpoints in the blueprint at the right.



36. **MULTIPLE REPRESENTATIONS** Betweenness of points ensures that a line segment may be divided into an infinite number of line segments.

a. **Geometric** Use a ruler to draw a line segment 3 centimeters long. Label the endpoints *A* and *D*. Draw two more points along the segment and label them *B* and *C*. Draw a second line segment 6 centimeters long. Label the endpoints *K* and *P*. Add four more points along the line and label them *L*, *M*, *N*, and *O*.

b. **Tabular** Use a ruler to measure the length of the line segment between each of the points you have drawn. Organize the lengths of the segments in $\overline{AD}$ and $\overline{KP}$ into a table. Include a column in your table to record the sum of these measures.

c. **Algebraic** Give an equation that could be used to find the lengths of $\overline{AD}$ and $\overline{KP}$. Compare the lengths determined by your equation to the actual lengths.

H.O.T. Problems Use Higher-Order Thinking Skills

37. **WRITING IN MATH** If point *B* is between points *A* and *C*, explain how you can find *AC* if you know *AB* and *BC*. Explain how you can find *BC* if you know *AB* and *AC*.

38. **OPEN ENDED** Draw a segment $\overline{AB}$ that measures between 2 and 3 inches long. Then sketch a segment $\overline{CD}$ congruent to $\overline{AB}$, draw a segment $\overline{EF}$ congruent to $\overline{AB}$, and construct a segment $\overline{GH}$ congruent to $\overline{AB}$. Compare your methods.

39. **CHALLENGE** Point *K* is between points *J* and *L*. If $JK = x^2 - 4x$, $KL = 3x - 2$, and $JL = 28$, write and solve an equation to find the lengths of *JK* and *KL*.

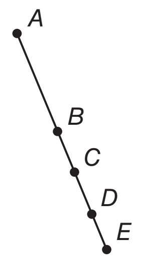
40. CCSS **REASONING** Determine whether the statement *If point M is between points C and D, then CD is greater than either CM or MD* is *sometimes*, *never*, or *always* true. Explain.

41. **WRITING IN MATH** Why is it important to have a standard of measure?

Standardized Test Practice

42. SHORT RESPONSE A 36-foot-long ribbon is cut into three pieces. The first piece of ribbon is half as long as the second piece of ribbon. The third piece is 1 foot longer than twice the length of the second piece of ribbon. How long is the longest piece of ribbon?

43. In the figure, points A, B, C, D, and E are collinear. If $AE = 38$, $BD = 15$, and $\overline{BC} \cong \overline{CD} \cong \overline{DE}$, what is the length of $\overline{AD}$?



A 7.5 **C** 22.5

B 15 **D** 30.5

44. SAT/ACT If $f(x) = 7x^2 - 4x$, what is the value of $f(2)$?

F -8 **J** 17

G 2 **K** 20

H 6

45. ALGEBRA
Simplify $(3x^2 - 2)(2x + 4) - 2x^2 + 6x + 7$.

A $4x^2 + 14x - 1$

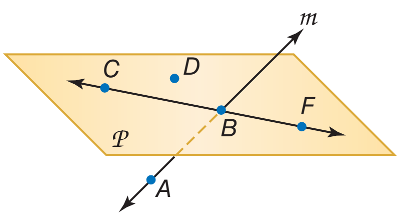
B $4x^2 - 14x + 15$

C $6x^3 + 12x^2 + 2x - 1$

D $6x^3 + 10x^2 + 2x - 1$

Spiral Review

Refer to the figure. (Lesson 1-1)



46. What are two other names for $\overleftrightarrow{AB}$?

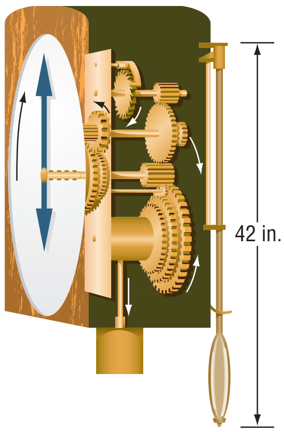
47. Give another name for plane $\mathcal{P}$.

48. Name the intersection of plane $\mathcal{P}$ and $\overleftrightarrow{AB}$.

49. Name three collinear points.

50. Name two points that are not coplanar.

51. CLOCKS The period of a pendulum is the time required for it to make one complete swing back and forth. The formula of the period P in seconds of a pendulum is $P = 2\pi\sqrt{\frac{\ell}{32}}$, where ℓ is the length of the pendulum in feet. (Lesson 0-9)



a. What is the period of the pendulum in the clock shown to the nearest tenth of a second?

b. About how many inches long should the pendulum be in order for it to have a period of 1 second?

Solve each inequality. (Lesson 0-6)

52. $-14n \geq 42$

53. $p + 6 > 15$

54. $-2a - 5 < 20$

55. $5x \leq 3x - 26$

Skills Review

Evaluate each expression if $a = -7$, $b = 4$, $c = -3$, and $d = 5$.

56. $b - c$

57. $|a - d|$

58. $|d - c|$

59. $\frac{b - a}{2}$

60. $(a - c)^2$

61. $\sqrt{(a - b)^2 + (c - d)^2}$

EXTEND 1-2

Extension Lesson Precision and Accuracy

Objective

1 Determine precision of measurements.

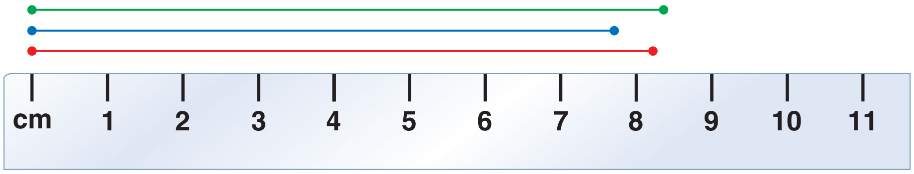
2 Determine accuracy of measurements.

As stated in Lesson 1-2, all measurements are approximations. Two main factors are considered when determining the quality of such an approximation.

- How *precise* is the measure?
- How *accurate* is the measure?

1 Precision

Precision refers to the clustering of a group of measurements. It depends only on the smallest unit of measure available on a measuring tool. Suppose you are told that a segment measures 8 centimeters. The length, to the nearest centimeter, of each segment shown below is 8 centimeters.



Notice that the exact length of each segment above is between 7.5 and 8.5 centimeters, or within 0.5 centimeter of 8 centimeters. The **absolute error** of a measurement is equal to one half the unit of measure. The smaller the unit of measure, the more precise the measurement.

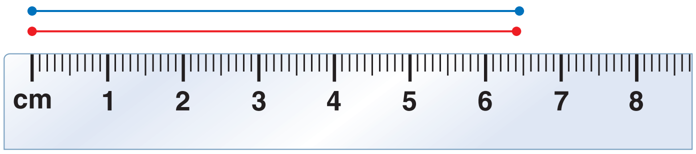
> **StudyTip**
>
> **Precision** The absolute error of a measurement in customary units is determined before reducing the fraction. For example, if you measure the length of an object to be $1\frac{4}{16}$ inches, then the absolute error measurement is precise to within $\frac{1}{32}$ inch.

Example 1 Find Absolute Error

Find the absolute error of each measurement. Then explain its meaning.

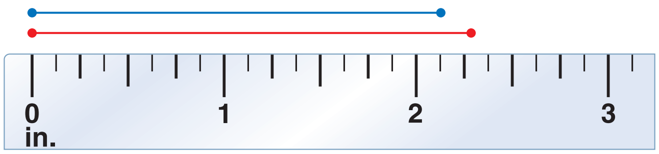
a. 6.4 centimeters

The measure is given to the nearest 0.1 centimeter, so the absolute error of this measurement is $\frac{1}{2}(0.1)$ or 0.05 centimeter. Therefore, the exact measurement could be between 6.35 and 6.45 centimeters. The two segments below measure 6.4 ± 0.05 centimeters.



b. $2\frac{1}{4}$ inches

The measure is given to the nearest $\frac{1}{4}$ inch, so the absolute error of this measurement is $\frac{1}{2}\left(\frac{1}{4}\right)$ or $\frac{1}{8}$ inch. Therefore, the exact measurement could be between $2\frac{1}{8}$ and $2\frac{3}{8}$ inches. The two segments below measure $2\frac{1}{4} \pm \frac{1}{8}$ inches.

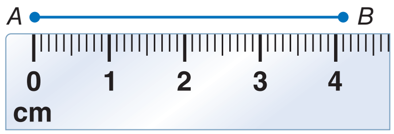


GuidedPractice

1A. $1\frac{1}{2}$ inches

1B. 4 centimeters

Precision in a measurement is usually expressed by the number of **significant digits** reported. Reporting that the measure of $\overline{AB}$ is 4 centimeters is *less precise* than reporting that the measure of $\overline{AB}$ is 4.1 centimeters.



To determine whether digits are considered significant, use the following rules.

- Nonzero digits are always significant.
- In whole numbers, zeros are significant if they fall between nonzero digits.
- In decimal numbers greater than or equal to 1, every digit is significant.
- In decimal numbers less than 1, the first nonzero digit and every digit to the right are significant.

Example 2 Significant Digits

Determine the number of significant digits in each measurement.

a. 430.008 meters

Since this is a decimal number greater than 1, every digit is significant. So, this measurement has 6 significant digits.

b. 0.00750 centimeter

This is a decimal number less than 1. The first nonzero digit is 7, and there are two digits to the right of 7, 5 and 0. So, this measurement has 3 significant digits.

GuidedPractice

2A. 779,000 mi **2B.** 50,008 ft **2C.** 230.004500 m

Real-World Link

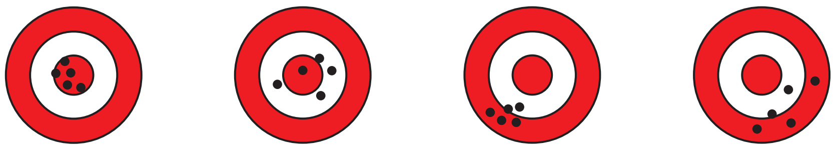
Precision in measurement in the real world usually comes at a price.

- Precision in a process to 3 significant digits, commercial quality, can cost $100.
- Precision in a process to 4 significant digits, industrial quality, can cost $500.
- Precision in a process to 5 significant digits, scientific quality, can cost $2500.

Source: Southwest Texas Junior College

2 Accuracy

Accuracy refers to how close a measured value comes to the actual or desired value. Consider the target practice results shown below.



accurate and precise | accurate but not precise | precise but not accurate | not accurate and not precise

The relative error of a measure is the ratio of the absolute error to the expected measure. A measurement with a smaller relative error is said to be more accurate.

Example 3 Find Relative Error

MANUFACTURING A manufacturer measures each part for a piece of equipment to be 23 centimeters in length. Find the relative error of this measurement.

$$\text{relative error} = \frac{\text{absolute error}}{\text{expected measure}} = \frac{0.5 \text{ cm}}{23 \text{ cm}} \approx 0.022 \text{ or } 2.2\%$$

GuidedPractice

Find the relative error of each measurement.

3A. 3.2 mi **3B.** 1 ft **3C.** 26 ft

StudyTip

Accuracy The accuracy or relative error of a measurement depends on both the absolute error and the size of the object being measured.

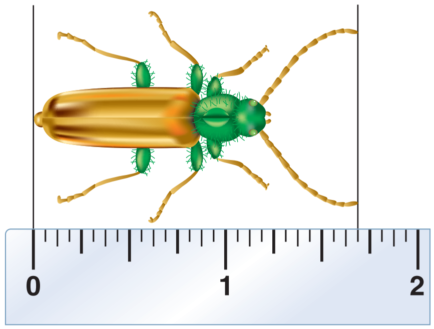
Precision and Accuracy *Continued*

Practice and Problem Solving

Find the absolute error of each measurement. Then explain its meaning.

1. 12 yd **2.** $50\frac{4}{16}$ in. **3.** 3.28 ft **4.** 2.759 cm

5. **ERROR ANALYSIS** In biology class, Manuel and Jocelyn measure a beetle as shown. Manuel says that the beetle measures between $1\frac{5}{8}$ and $1\frac{3}{4}$ inches. Jocelyn says that it measures between $1\frac{9}{16}$ and $1\frac{5}{8}$ inches. Is either of their statements about the beetle's measure correct? Explain your reasoning.

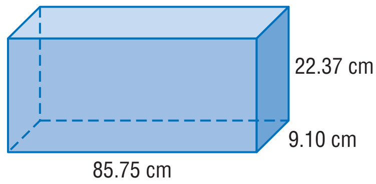


6. **PYRAMIDS** Research suggests that the design dimensions of the Great Pyramid of Giza in Egypt were 440 by 440 royal cubits. The sides of the pyramid are precise within 0.05%. What are the greatest and least possible lengths of the sides?

Determine the number of significant digits in each measurement.

7. 4.05 in. **8.** 53,000 mi **9.** 0.0005 mm **10.** 750,001 ft

11. **VOLUME** When multiplying or dividing measures, the product or quotient should have only as many significant digits as the multiplied or divided measurement showing the least number of significant digits. To how many significant digits should the volume of the rectangle prism shown be reported? Report the volume to this number of significant digits.



Find the relative error of each measurement.

12. 48 in. **13.** 2.0 mi **14.** 11.14 cm **15.** 0.6 m

Determine which measurement is more precise and which is more accurate. Explain your reasoning.

16. 22.4 ft; 5.82 ft **17.** 25 mi; 8 mi **18.** 9.2 cm; 42 mm **19.** $18\frac{1}{4}$ in.; 125 yd

For each situation, determine the level of accuracy needed. Explain.

20. You are estimating the height of a person. Which unit of measure should you use: 1 foot, 1 inch, or $\frac{1}{16}$ inch?

21. You are estimating the height of a mountain. Which unit of measure should you use: 1 foot, 1 inch, or $\frac{1}{16}$ inch?

22. **PERIMETER** The *perimeter* of a geometric figure is the sum of the lengths of its sides. Jermaine uses a ruler divided into inches and measures the sides of a rectangle to be $2\frac{1}{4}$ inches and $4\frac{3}{4}$ inches. What are the least and greatest possible perimeters of the rectangle? Explain.

23. **WRITING IN MATH** How precise is precise enough?

LESSON 1-3 Distance and Midpoints

Then
- You graphed points on the coordinate plane.

Now
1. Find the distance between two points.
2. Find the midpoint of a segment.

Why?
- The location of a city on a map is given in degrees of latitude and longitude. For short distances, the Pythagorean Theorem can be used to approximate the distance between two locations.

NewVocabulary
distance
irrational number
midpoint
segment bisector

Common Core State Standards

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices
2 Reason abstractly and quantitatively.
7 Look for and make use of structure.

1 Distance Between Two Points

The **distance** between two points is the length of the segment with those points as its endpoints. The coordinates of the points can be used to find this length. Because $\overline{PQ}$ is the same as $\overline{QP}$, the order in which you name the endpoints is not important when calculating distance.

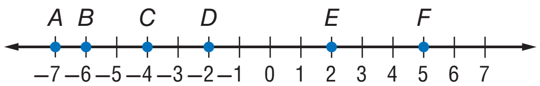
KeyConcept Distance Formula (on Number Line)

Words The distance between two points is the absolute value of the difference between their coordinates.

Symbols If P has coordinate x_1 and Q has coordinate x_2, $PQ = |x_2 - x_1|$ or $|x_1 - x_2|$.

Example 1 Find Distance on a Number Line

Use the number line to find *BE*.



The coordinates of B and E are −6 and 2.

$BE = |x_2 - x_1|$ Distance Formula

$= |2 - (-6)|$ $x_1 = -6$ and $x_2 = 2$

$= 8$ Simplify.

GuidedPractice

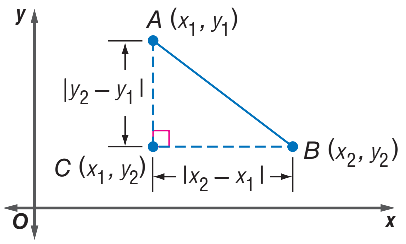
Use the number line above to find each measure.

1A. AC **1B.** CF **1C.** FB

StudyTip

Pythagorean Theorem Recall that the Pythagorean Theorem is often expressed as $a^2 + b^2 = c^2$, where a and b are the measures of the shorter sides (legs) of a right triangle, and c is the measure of the longest side (hypotenuse). You will prove and learn about other applications of the Pythagorean Theorem in Lesson 8-2.

To find the distance between two points A and B in the coordinate plane, you can form a right triangle with $\overline{AB}$ as its hypotenuse and point C as its vertex as shown. Then use the Pythagorean Theorem to find AB.



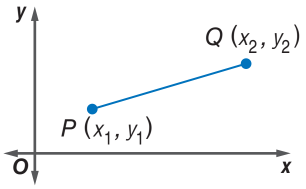
$(CB)^2 + (AC)^2 = (AB)^2$	Pythagorean Theorem
$(\lvert x_2 - x_1 \rvert)^2 + (\lvert y_2 - y_1 \rvert)^2 = (AB)^2$	$CB = \lvert x_2 - x_1 \rvert$, $AC = \lvert y_2 - y_1 \rvert$
$(x_2 - x_1)^2 + (y_2 - y_1)^2 = (AB)^2$	The square of a number is always positive.
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = AB$	Take the positive square root of each side.

This gives us a Distance Formula for points in the coordinate plane. Because this formula involves taking the square root of a real number, distances can be irrational. Recall that an **irrational number** is a number that cannot be expressed as a terminating or repeating decimal.

KeyConcept Distance Formula (in Coordinate Plane)

If P has coordinates (x_1, y_1) and Q has coordinates (x_2, y_2), then

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The order of the x- and y-coordinates in each set of parentheses is not important.

Example 2 Find Distance on a Coordinate Plane

Find the distance between $C(-4, -6)$ and $D(5, -1)$.

$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{[5 - (-4)]^2 + [-1 - (-6)]^2}$	$(x_1, y_1) = (-4, -6)$ and $(x_2, y_2) = (5, -1)$
$= \sqrt{9^2 + 5^2}$ or $\sqrt{106}$	Subtract.

The distance between C and D is $\sqrt{106}$ units. Use a calculator to find that $\sqrt{106}$ units is approximately 10.3 units.

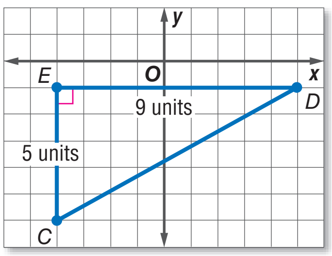
CHECK Graph the ordered pairs and check by using the Pythagorean Theorem.

$(CD)^2 \stackrel{?}{=} (EC)^2 + (ED)^2$

$(CD)^2 \stackrel{?}{=} 5^2 + 9^2$

$(CD)^2 \stackrel{?}{=} 106$

$CD = \sqrt{106}$ ✔



GuidedPractice

Find the distance between each pair of points.

2A. $E(-5, 6)$ and $F(8, -4)$

2B. $J(4, 3)$ and $K(-3, -7)$

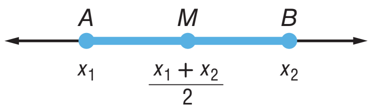
2 Midpoint of a Segment

The **midpoint** of a segment is the point halfway between the endpoints of the segment. If X is the midpoint of $\overline{AB}$, then $AX = XB$ and $\overline{AX} \cong \overline{XB}$. You can find the midpoint of a segment on a number line by finding the *mean*, or the average, of the coordinates of its endpoints.

KeyConcept Midpoint Formula (on Number Line)

If $\overline{AB}$ has endpoints at x_1 and x_2 on a number line, then the midpoint M of $\overline{AB}$ has coordinate

$$\frac{x_1 + x_2}{2}.$$

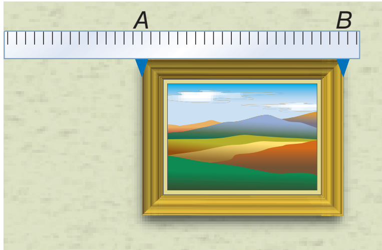


StudyTip

Alternative Method In Example 3, the coordinate of the midpoint could also have been located by first finding the length of AB, which is $37.5 - 15$ or 22.5 inches. Half of this measure is the distance from one endpoint to the point midway between A and B, $\frac{22.5}{2}$ or 11.25. Add this distance to point A's distance from the left wall. So the midpoint between A and B is $15 + 11.25$ or 26.25 inches from the left wall.

Real-World Example 3 Find Midpoint on a Number Line

DECORATING Jacinta hangs a picture 15 inches from the left side of a wall. How far from the edge of the wall should she mark the location for the nail the picture will hang on if the right edge is 37.5 inches from the wall's left side?



The coordinates of the endpoints of the top of the picture frame are 15 inches and 37.5 inches. Let M be the midpoint of $\overline{AB}$.

$M = \frac{x_1 + x_2}{2}$ — Midpoint Formula

$= \frac{15 + 37.5}{2}$ — $x_1 = 15$, $x_2 = 37.5$

$= \frac{52.5}{2}$ or 26.25 — Simplify.

The midpoint is located at 26.25 or $26\frac{1}{4}$ inches from the left edge of the wall.

GuidedPractice

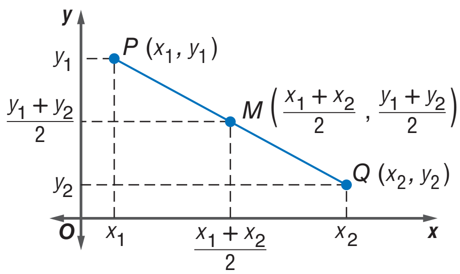
3. **TEMPERATURE** The temperature on a thermometer dropped from a reading of 25° to −8°. Find the midpoint of these temperatures.

You can find the midpoint of a segment on the coordinate plane by finding the average of the x-coordinates and of the y-coordinates of the endpoints.

KeyConcept Midpoint Formula (in Coordinate Plane)

If $\overline{PQ}$ has endpoints at $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane, then the midpoint M of $\overline{PQ}$ has coordinates

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



When finding the midpoint of a segment, the order of the coordinates of the endpoints is not important.

Example 4 Find Midpoint in Coordinate Plane

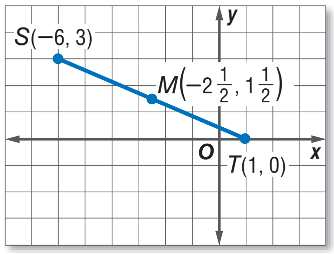
Find the coordinates of M, the midpoint of $\overline{ST}$, for $S(-6, 3)$ and $T(1, 0)$.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint Formula

$= \left(\frac{-6 + 1}{2}, \frac{3 + 0}{2}\right)$ $(x_1, y_1) = S(-6, 3), (x_2, y_2) = T(1, 0)$

$= \left(\frac{-5}{2}, \frac{3}{2}\right)$ or $M\left(-2\frac{1}{2}, 1\frac{1}{2}\right)$ Simplify.

CHECK Graph S, T, and M. The distance from S to M does appear to be the same as the distance from M to T, so our answer is reasonable.



Guided Practice

Find the coordinates of the midpoint of a segment with the given coordinates.

4A. $A(5, 12)$, $B(-4, 8)$

4B. $C(-8, -2)$, $D(5, 1)$

You can also find the coordinates of the endpoint of a segment if you know the coordinates of its other endpoint and its midpoint.

Example 5 Find the Coordinates of an Endpoint

Find the coordinates of J if $K(-1, 2)$ is the midpoint of $\overline{JL}$ and L has coordinates $(3, -5)$.

Step 1 Let J be (x_1, y_1) and L be (x_2, y_2) in the Midpoint Formula.

$K\left(\frac{x_1 + 3}{2}, \frac{y_1 + (-5)}{2}\right) = K(-1, 2)$ $(x_2, y_2) = (3, -5)$

Step 2 Write two equations to find the coordinates of J.

$\frac{x_1 + 3}{2} = -1$ Midpoint Formula

$x_1 + 3 = -2$ Multiply each side by 2.

$x_1 = -5$ Subtract 3 from each side.

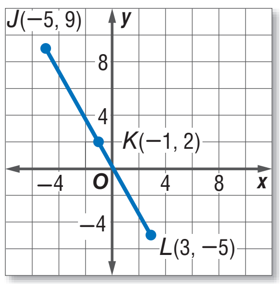
$\frac{y_1 + (-5)}{2} = 2$ Midpoint Formula

$y_1 - 5 = 4$ Multiply each side by 2.

$y_1 = 9$ Add 5 to each side.

The coordinates of J are $(-5, 9)$.

CHECK Graph J, K, and L. The distance from J to K does appear to be the same as the distance from K to L, so our answer is reasonable.



Guided Practice

Find the coordinates of the missing endpoint if P is the midpoint of $\overline{EG}$.

5A. $E(-8, 6)$, $P(-5, 10)$

5B. $P(-1, 3)$, $G(5, 6)$

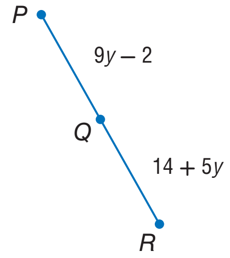
Study Tip

Check for Reasonableness Always graph the given information and the calculated coordinates of the third point to check the reasonableness of your answer.

You can use algebra to find a missing measure or value in a figure that involves the midpoint of a segment.

Example 6 Use Algebra to Find Measures

ALGEBRA **Find the measure of $\overline{PQ}$ if Q is the midpoint of $\overline{PR}$.**



Understand You know that Q is the midpoint of $\overline{PR}$.
You are asked to find the measure of $\overline{PQ}$.

Plan Because Q is the midpoint, you know that $PQ = QR$. Use this equation to find a value for y.

Solve

$PQ = QR$	Definition of midpoint
$9y - 2 = 14 + 5y$	$PQ = 9y - 2$, $QR = 14 + 5y$
$4y - 2 = 14$	Subtract $5y$ from each side.
$4y = 16$	Add 2 to each side.
$y = 4$	Divide each side by 4.

Now substitute 4 for y in the expression for PQ.

$PQ = 9y - 2$	Original measure
$= 9(4) - 2$	$y = 4$
$= 36 - 2$ or 34	Simplify.

The measure of $\overline{PQ}$ is 34.

Check Since $PQ = QR$, when the expression for QR is evaluated for 4, it should also be 34.

$QR = 14 + 5y$	Original measure
$\stackrel{?}{=} 14 + 5(4)$	$y = 4$
$= 34$ ✓	Simplify.

Guided Practice

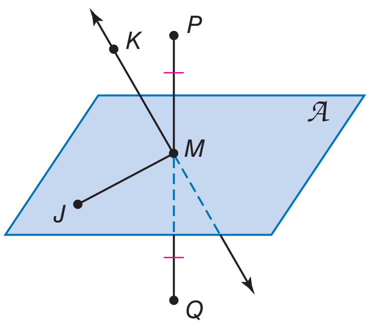
6A. Find the measure of $\overline{YZ}$ if Y is the midpoint of $\overline{XZ}$ and $XY = 2x - 3$ and $YZ = 27 - 4x$.

6B. Find the value of x if C is the midpoint of $\overline{AB}$, $AC = 4x + 5$, and $AB = 78$.

Study Tip

CCSS Sense-Making and Perseverance The four-step problem solving plan is a tool for making sense of any problem. When making and executing your plan, continually ask yourself, "Does this make sense?" Monitor and evaluate your progress and change course if necessary.

Any segment, line, or plane that intersects a segment at its midpoint is called a **segment bisector**. In the figure at the right, M is the midpoint of $\overline{PQ}$. Plane $\mathcal{A}$, $\overline{MJ}$, $\overleftrightarrow{KM}$, and point M are all bisectors of $\overline{PQ}$. We say that they *bisect* $\overline{PQ}$.



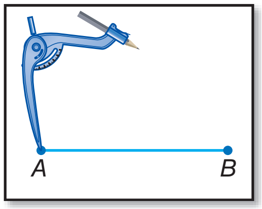
Study Tip

Segment Bisectors There can be an infinite number of bisectors and each must contain the midpoint of the segment.

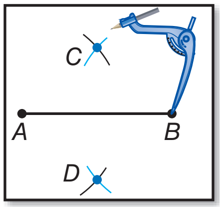
The construction on the following page shows how to construct a line that bisects a segment to find the midpoint of a given segment.

Construction Bisect a Segment

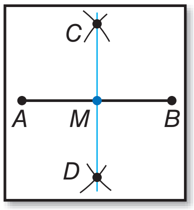
Step 1 Draw a segment and name it $\overline{AB}$. Place the compass at point A. Adjust the compass so that its width is greater than $\frac{1}{2}AB$. Draw arcs above and below $\overline{AB}$.



Step 2 Using the same compass setting, place the compass at point B and draw arcs above and below $\overline{AB}$ so that they intersect the two arcs previously drawn. Label the points of the intersection of the arcs as C and D.



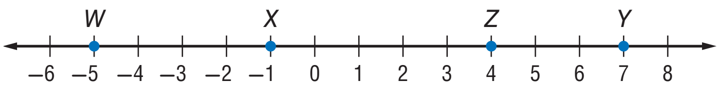
Step 3 Use a straightedge to draw $\overline{CD}$. Label the point where it intersects $\overline{AB}$ as M. Point M is the midpoint of $\overline{AB}$, and $\overline{CD}$ is a bisector of $\overline{AB}$.



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

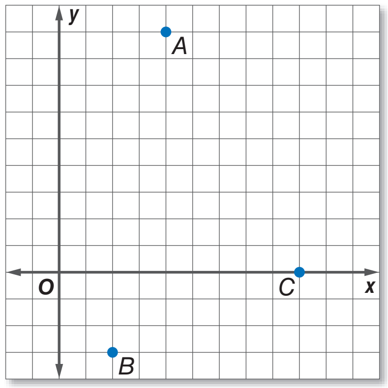
Example 1 **Use the number line to find each measure.**



1. XY

2. WZ

Example 2 **TIME CAPSULE** Graduating classes have buried time capsules on the campus of East Side High School for over twenty years. The points on the diagram show the position of three time capsules. Find the distance between each pair of time capsules.



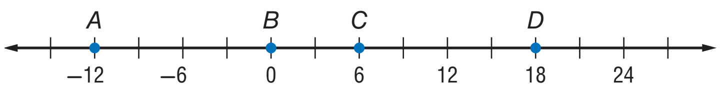
3. $A(4, 9)$, $B(2, -3)$

4. $A(4, 9)$, $C(9, 0)$

5. $B(2, -3)$, $C(9, 0)$

6. CCSS **REASONING** Which two time capsules are the closest to each other? Which are farthest apart?

Example 3 **Use the number line to find the coordinate of the midpoint of each segment.**



7. $\overline{AC}$

8. $\overline{BD}$

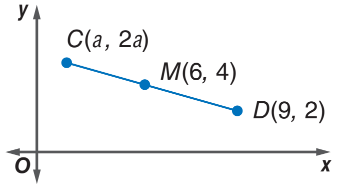
Example 4 **Find the coordinates of the midpoint of a segment with the given endpoints.**

9. $J(5, -3)$, $K(3, -8)$

10. $M(7, 1)$, $N(4, -1)$

Example 5 **11** Find the coordinates of G if $F(1, 3.5)$ is the midpoint of $\overline{GJ}$ and J has coordinates $(6, -2)$.

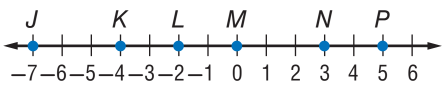
Example 6 **12.** **ALGEBRA** Point M is the midpoint of $\overline{CD}$. What is the value of a in the figure?



Practice and Problem Solving

Extra Practice is on page R1.

Example 1 **Use the number line to find each measure.**

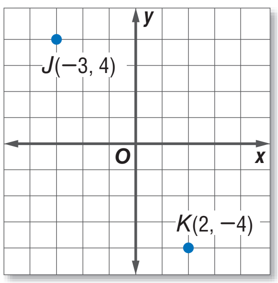


13. JL **14.** JK **15.** KP

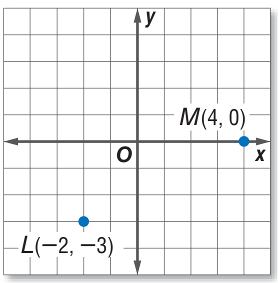
16. NP **17.** JP **18.** LN

Example 2 **Find the distance between each pair of points.**

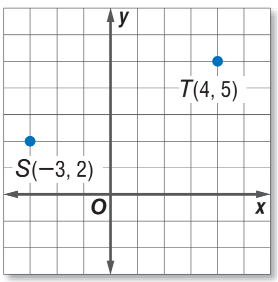
19.



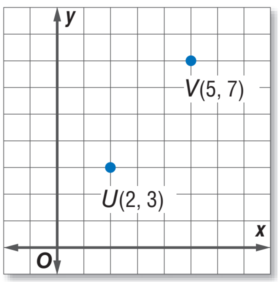
20.



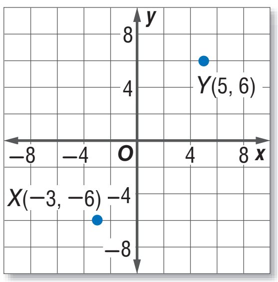
21.



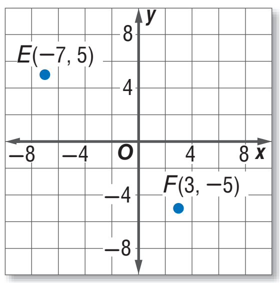
22.



23.



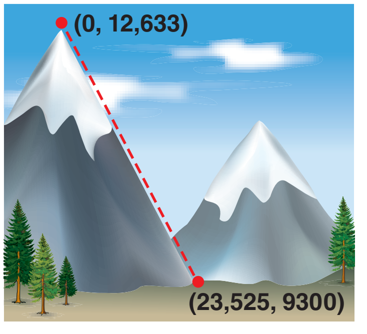
24.



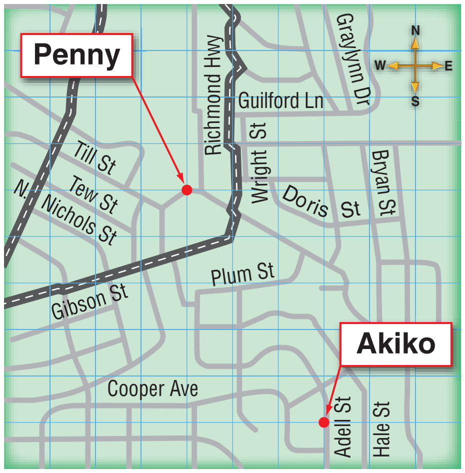
25. $X(1, 2), Y(5, 9)$ **26.** $P(3, 4), Q(7, 2)$ **27.** $M(-3, 8), N(-5, 1)$

28. $Y(-4, 9), Z(-5, 3)$ **29.** $A(2, 4), B(5, 7)$ **30.** $C(5, 1), D(3, 6)$

31. CCSS **REASONING** Vivian is planning to hike to the top of Humphreys Peak on her family vacation. The coordinates of the peak of the mountain and of the base of the trail are shown. If the trail can be approximated by a straight line, estimate the length of the trail. (*Hint:* 1 mi = 5280 ft)



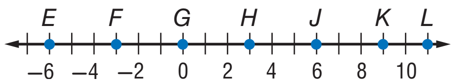
32. CCSS **MODELING** Penny and Akiko live in the locations shown on the map below.



a. If each square on the grid represents one block and the bottom left corner of the grid is the location of the origin, what is the straight-line distance from Penny's house to Akiko's?

b. If Penny moves three blocks to the north and Akiko moves 5 blocks to the west, how far apart will they be?

Example 3 **Use the number line to find the coordinate of the midpoint of each segment.**

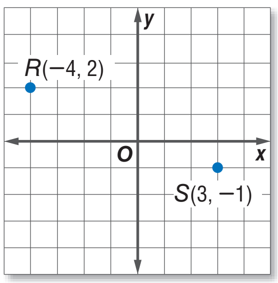


33. $\overline{HK}$
34. $\overline{JL}$
35. $\overline{EF}$
36. $\overline{FG}$
37. $\overline{FK}$
38. $\overline{EL}$

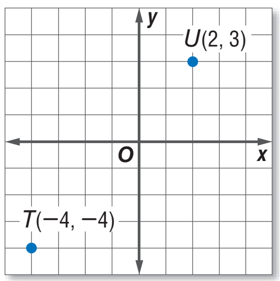
Example 4 **Find the coordinates of the midpoint of a segment with the given endpoints.**

39. $C(22, 4)$, $B(15, 7)$
40. $W(12, 2)$, $X(7, 9)$
41. $D(-15, 4)$, $E(2, -10)$
42. $V(-2, 5)$, $Z(3, -17)$
43. $X(-2.4, -14)$, $Y(-6, -6.8)$
44. $J(-11.2, -3.4)$, $K(-5.6, -7.8)$

45.



46.



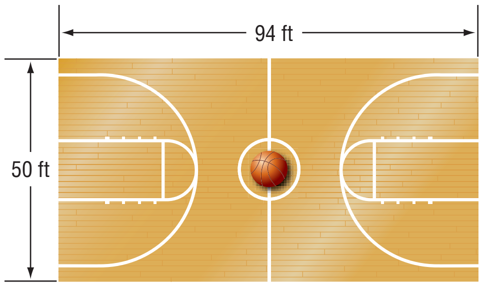
Example 5 **Find the coordinates of the missing endpoint if B is the midpoint of $\overline{AC}$.**

47. $C(-5, 4)$, $B(-2, 5)$
48. $A(1, 7)$, $B(-3, 1)$
49. $A(-4, 2)$, $B(6, -1)$
50. $C(-6, -2)$, $B(-3, -5)$
51. $A(4, -0.25)$, $B(-4, 6.5)$
52. $C\left(\frac{5}{3}, -6\right)$, $B\left(\frac{8}{3}, 4\right)$

Example 6 **ALGEBRA** **Suppose M is the midpoint of $\overline{FG}$. Use the given information to find the missing measure or value.**

53. $FM = 3x - 4$, $MG = 5x - 26$, $FG = ?$
54. $FM = 5y + 13$, $MG = 5 - 3y$, $FG = ?$
55. $MG = 7x - 15$, $FG = 33$, $x = ?$
56. $FM = 8a + 1$, $FG = 42$, $a = ?$

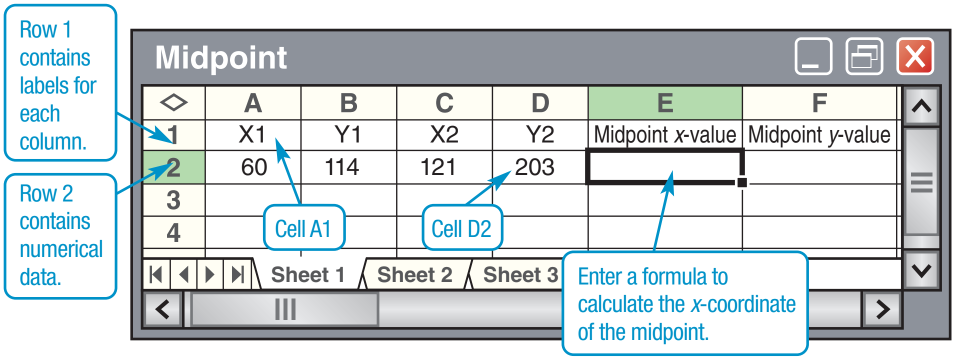
57 **BASKETBALL** The dimensions of a basketball court are shown below. Suppose a player throws the ball from a corner to a teammate standing at the center of the court.



a. If center court is located at the origin, find the ordered pair that represents the location of the player in the bottom right corner.

b. Find the distance that the ball travels.

CCSS TOOLS **Spreadsheets can be used to perform calculations quickly. The spreadsheet below can be used to calculate the distance between two points. Values are used in formulas by using a specific cell name. The value of x_1 is used in a formula using its cell name, A2.**



Write a formula for the indicated cell that could be used to calculate the indicated value using the coordinates (x_1, y_1) and (x_2, y_2) as the endpoint of a segment.

58. E2; the x-value of the midpoint of the segment

59. F2; the y-value of the midpoint of the segment

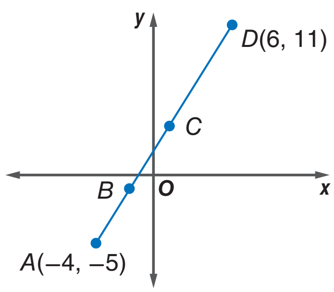
60. G2; the length of the segment

Name the point(s) that satisfy the given condition.

61. two points on the x-axis that are 10 units from (1, 8)

62. two points on the y-axis that are 25 units from (−24, 3)

63. **COORDINATE GEOMETRY** Find the coordinates of B if B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{AD}$.

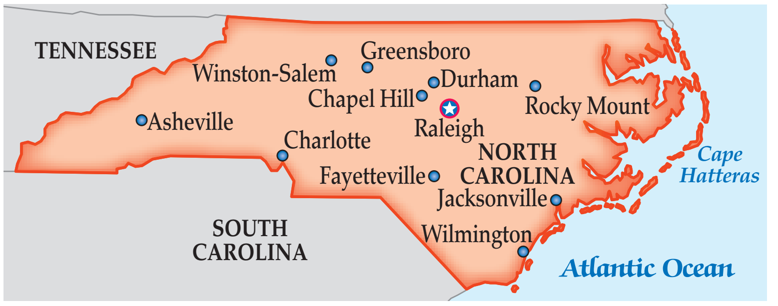


ALGEBRA **Determine the value(s) of n.**

64. $J(n, n + 2), K(3n, n - 1), JK = 5$

65. $P(3n, n - 7), Q(4n, n + 5), PQ = 13$

66. CCSS **PERSEVERANCE** Wilmington, North Carolina, is located at (34.3°, 77.9°), which represents north latitude and west longitude. Winston-Salem is in the northern part of the state at (36.1°, 80.2°).



a. Find the latitude and longitude of the midpoint of the segment between Wilmington and Winston-Salem.

b. Use an atlas or the Internet to find a city near the location of the midpoint.

c. If Winston-Salem is the midpoint of the segment with one endpoint at Wilmington, find the latitude and longitude of the other endpoint.

d. Use an atlas or the Internet to find a city near the location of the other endpoint.

67. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between a midpoint of a segment and the midpoint between the endpoint and the midpoint.

a. Geometric Use a straightedge to draw three different line segments. Label the endpoints *A* and *B*.

b. Geometric On each line segment, find the midpoint of $\overline{AB}$ and label it *C*. Then find the midpoint of $\overline{AC}$ and label it *D*.

c. Tabular Measure and record *AB*, *AC*, and *AD* for each line segment. Organize your results into a table.

d. Algebraic If $AB = x$, write an expression for the measures *AC* and *AD*.

e. Verbal Make a conjecture about the relationship between *AB* and each segment if you were to continue to find the midpoints of a segment and a midpoint you previously found.

H.O.T. Problems Use Higher-Order Thinking Skills

68. **WRITING IN MATH** Explain how the Pythagorean Theorem and the Distance Formula are related.

69. **REASONING** Is the point one third of the way from (x_1, y_1) to (x_2, y_2) *sometimes*, *always*, or *never* the point $\left(\frac{x_1 + x_2}{3}, \frac{y_1 + y_2}{3}\right)$? Explain.

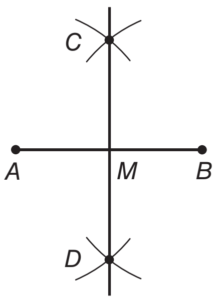
70. **CHALLENGE** Point *P* is located on the segment between point *A*(1, 4) and point *D*(7, 13). The distance from *A* to *P* is twice the distance from *P* to *D*. What are the coordinates of point *P*?

71. **OPEN ENDED** Draw a segment and name it $\overline{AB}$. Using only a compass and a straightedge, construct a segment $\overline{CD}$ such that $CD = 5\frac{1}{4}AB$. Explain and then justify your construction.

72. **WRITING IN MATH** Describe a method of finding the midpoint of a segment that has one endpoint at (0, 0). Give an example using your method, and explain why your method works.

Standardized Test Practice

73. Which of the following best describes the first step in bisecting $\overline{AB}$?



A From point A, draw equal arcs on $\overline{CD}$ using the same compass width.

B From point A, draw equal arcs above and below $\overline{AB}$ using a compass width of $\frac{1}{3}\overline{AB}$.

C From point A, draw equal arcs above and below $\overline{AB}$ using a compass width greater than $\frac{1}{2}\overline{AB}$.

D From point A, draw equal arcs above and below $\overline{AB}$ using a compass width less than $\frac{1}{2}\overline{AB}$.

74. ALGEBRA Beth paid \$74.88 for 3 pairs of jeans. All 3 pairs of jeans were the same price. How much did each pair of jeans cost?

F \$24.96 **H** \$74.88
G \$37.44 **J** \$224.64

75. SAT/ACT If $5^{2x-3} = 1$, then $x =$

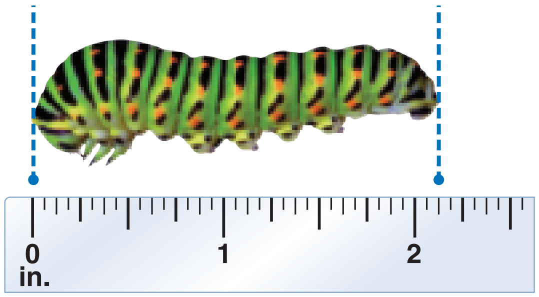
A 0.4 **D** 1.6
B 0.6 **E** 2
C 1.5

76. GRIDDED RESPONSE One endpoint of $\overline{AB}$ has coordinates $(-3, 5)$. If the coordinates of the midpoint of $\overline{AB}$ are $(2, -6)$, what is the approximate length of $\overline{AB}$?

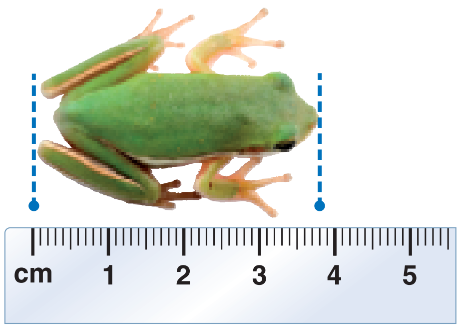
Spiral Review

Find the length of each object. (Lesson 1-2)

77.



78.



Draw and label a figure for each relationship. (Lesson 1-1)

79. $\overleftrightarrow{FG}$ lies in plane M and contains point H.

80. Lines r and s intersect at point W.

81. TRUCKS A sport-utility vehicle has a maximum load limit of 75 pounds for its roof. You want to place a 38-pound cargo carrier and 4 pieces of luggage on top of the roof. Write and solve an inequality to find the average allowable weight for each piece of luggage. (Lesson 0-6)

Skills Review

Solve each equation.

82. $8x - 15 = 5x$

83. $5y - 3 + y = 90$

84. $16a + 21 = 20a - 9$

85. $9k - 7 = 21 - 3k$

86. $11z - 13 = 3z + 17$

87. $15 + 6n = 4n + 23$

LESSON 1-4

Angle Measure

Then

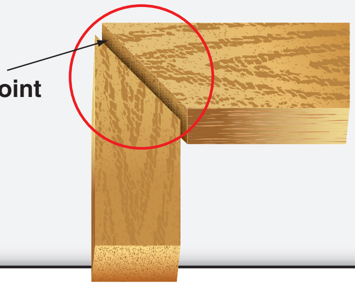
- You measured line segments.

Now

1. Measure and classify angles.
2. Identify and use congruent angles and the bisector of an angle.

Why?

- One of the skills Dale must learn in carpentry class is how to cut a *miter* joint. This joint is created when two boards are cut at an angle to each other. He has learned that one miscalculation in angle measure can result in mitered edges that do not fit together.



Dennis Halliman/Hulton Archive/Getty Images

NewVocabulary

ray
opposite rays
angle
side
vertex
interior
exterior
degree
right angle
acute angle
obtuse angle
angle bisector

Common Core State Standards

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

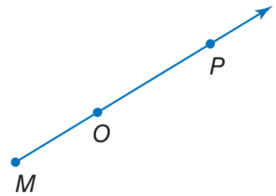
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices

5 Use appropriate tools strategically.
6 Attend to precision.

1 Measure and Classify Angles

A **ray** is a part of a line. It has one endpoint and extends indefinitely in one direction. Rays are named by stating the endpoint first and then any other point on the ray. The ray shown cannot be named as $\overrightarrow{OM}$ because O is not the endpoint of the ray.

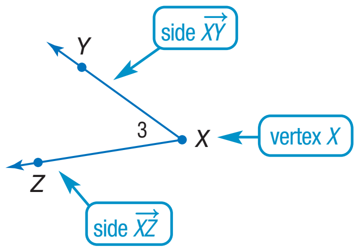


ray MP, $\overrightarrow{MP}$, ray MO, or $\overrightarrow{MO}$

If you choose a point on a line, that point determines exactly two rays called **opposite rays**. Since both rays share a common endpoint, opposite rays are collinear

$\overrightarrow{JH}$ and $\overrightarrow{JK}$ are opposite rays.

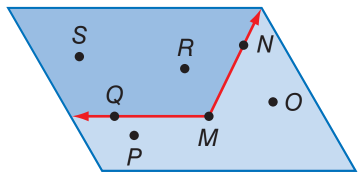
An **angle** is formed by two *noncollinear* rays that have a common endpoint. The rays are called **sides** of the angle. The common endpoint is the **vertex**.



When naming angles using three letters, the vertex must be the second of the three letters. You can name an angle using a single letter only when there is exactly one angle located at that vertex. The angle shown can be named as $\angle X$, $\angle YXZ$, $\angle ZXY$, or $\angle 3$.

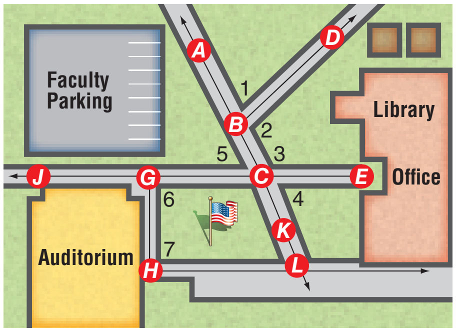
An angle divides a plane into three distinct parts.

- Points Q, M, and N lie on the angle.
- Points S and R lie in the **interior** of the angle.
- Points P and O lie in the **exterior** of the angle.



Real-World Example 1 Angles and Their Parts

MAPS **Use the map of a high school shown.**



a. Name all angles that have *B* as a vertex.

$\angle 1$ or $\angle ABD$, and $\angle 2$ or $\angle DBC$

b. Name the sides of $\angle 3$.

$\overrightarrow{CA}$ and $\overrightarrow{CE}$ or $\overrightarrow{CB}$ and $\overrightarrow{CE}$

c. What is another name for $\angle GHL$?

$\angle 7$, $\angle H$, or $\angle LHG$

d. Name a point in the interior of $\angle DBK$.

Point *E*

> **StudyTip**
>
> **Segments as Sides** Because a ray can contain a line segment, the side of an angle can be a segment.

GuidedPractice

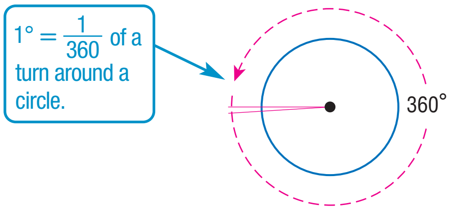
1A. What is the vertex of $\angle 5$?

1B. Name the sides of $\angle 5$.

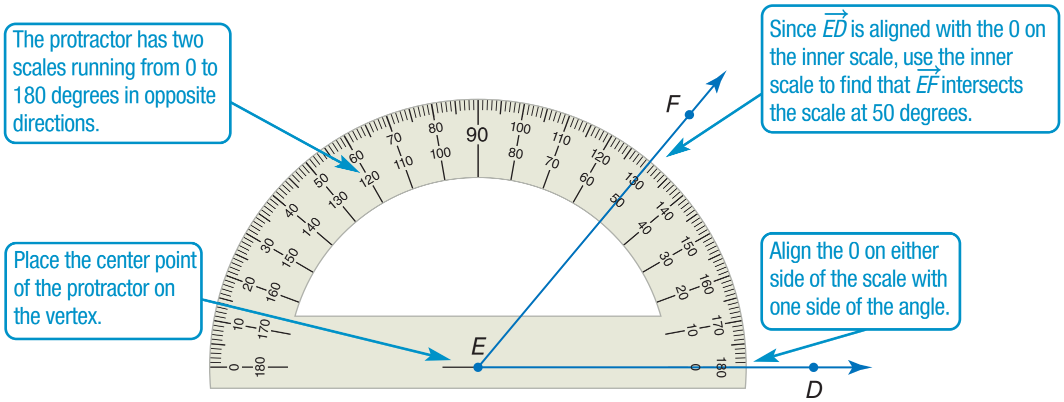
1C. Write another name for $\angle ECL$.

1D. Name a point in the exterior of $\angle CLH$.

Angles are measured in units called degrees. The **degree** results from dividing the distance around a circle into 360 parts.



To measure an angle, you can use a *protractor*. Angle *DEF* below is a 50 degree (50°) angle. We say that the *degree measure* of $\angle DEF$ is 50, or $m\angle DEF = 50$.

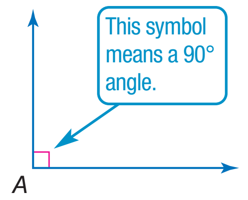
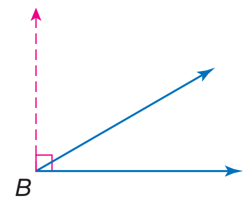
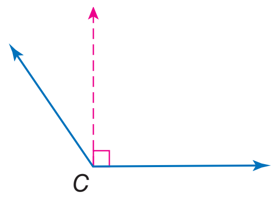


ReadingMath

Straight Angle Opposite rays with the same vertex form a *straight angle*. Its measure is 180. Unless otherwise specified in this book, however, the term *angle* means a nonstraight angle.

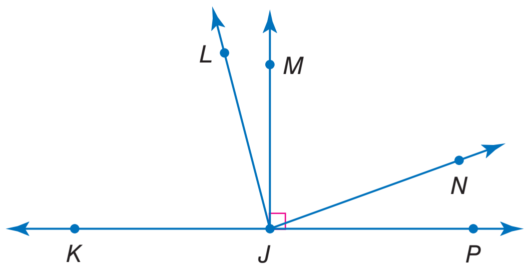
Angles can be classified by their measures as shown below.

KeyConcept Classify Angles

right angle	acute angle	obtuse angle
		
$m\angle A = 90$	$m\angle B < 90$	$180 > m\angle C > 90$

Example 2 Measure and Classify Angles

Copy the diagram below, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



WatchOut!

Classify Before Measuring Classifying an angle before measuring it can prevent you from choosing the wrong scale on your protractor. In Example 2b, you must decide whether $\angle LJP$ measures 75 or 105. Since $\angle LJP$ is an obtuse angle, you can reason that the correct measure must be 105.

a. $\angle MJP$

$\angle MJP$ is marked as a right angle, so $m\angle MJP = 90$.

b. $\angle LJP$

Point L on angle $\angle LJP$ lies on the exterior of right angle $\angle MJP$, so $\angle LJP$ is an obtuse angle. Use a protractor to find that $m\angle LJP = 105$

CHECK Since $105 > 90$, $\angle LJP$ is an obtuse angle. ✓

c. $\angle NJP$

Point N on angle $\angle NJP$ lies on the interior of right angle $\angle MJP$, so $\angle NJP$ is an acute angle. Use a protractor to find that $m\angle NJP = 20$.

CHECK Since $20 < 90$, $\angle NJP$ is an acute angle. ✓

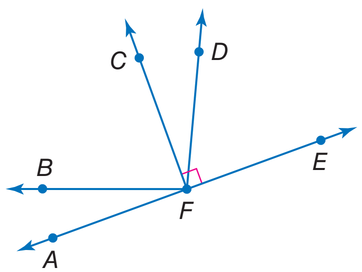
GuidedPractice

2A. $\angle AFB$

2B. $\angle CFA$

2C. $\angle AFD$

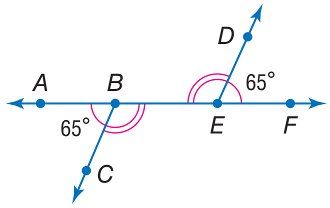
2D. $\angle CFD$



2 Congruent Angles

Just as segments that have the same measure are congruent segments, angles that have the same measure are *congruent angles*.

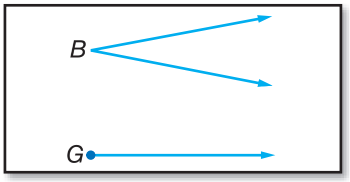
In the figure, since $m\angle ABC = m\angle FED$, then $\angle ABC \cong \angle FED$. Matching numbers of arcs on a figure also indicate congruent angles, so $\angle CBE \cong \angle DEB$.



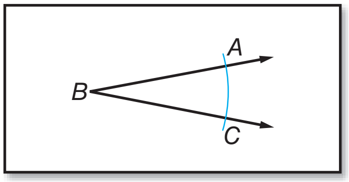
You can produce an angle congruent to a given angle using a construction.

Construction Copy an Angle

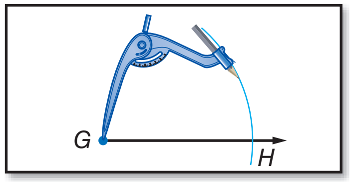
Step 1 Draw an angle like $\angle B$ on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint *G*.



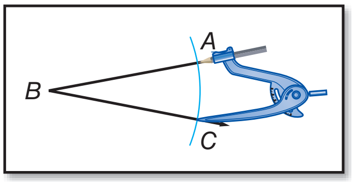
Step 2 Place the tip of the compass at point *B* and draw a large arc that intersects both sides of $\angle B$. Label the points of intersection *A* and *C*.



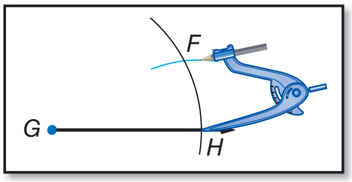
Step 3 Using the same compass setting, put the compass at point *G* and draw a large arc that starts above the ray and intersects the ray. Label the point of intersection *H*.



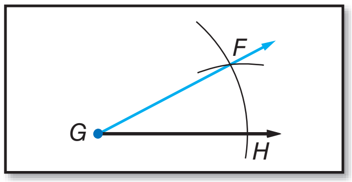
Step 4 Place the point of your compass on *C* and adjust so that the pencil tip is on *A*.



Step 5 Without changing the setting, place the compass at point *H* and draw an arc to intersect the larger arc you drew in Step 4. Label the point of intersection *F*.



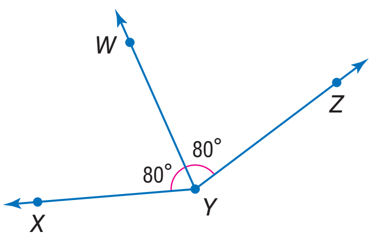
Step 6 Use a straightedge to draw $\overrightarrow{GF}$. $\angle ABC \cong \angle FGH$



StudyTip

Segments A line segment can also bisect an angle.

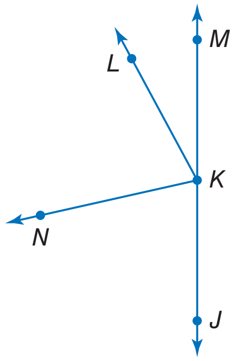
A ray that divides an angle into two congruent angles is called an **angle bisector**. If $\overrightarrow{YW}$ is the angle bisector of $\angle XYZ$, then point *W* lies in the interior of $\angle XYZ$ and $\angle XYW \cong \angle WYZ$.



Just as with segments, when a line, segment, or ray divides an angle into smaller angles, the sum of the measures of the smaller angles equals the measure of the largest angle. So in the figure, $m\angle XYW + m\angle WYZ = m\angle XYZ$.

Example 3 Measure and Classify Angles

ALGEBRA **In the figure, $\overrightarrow{KJ}$ and $\overrightarrow{KM}$ are opposite rays, and $\overrightarrow{KN}$ bisects $\angle JKL$. If $m\angle JKN = 8x - 13$ and $m\angle NKL = 6x + 11$, find $m\angle JKN$.**



Step 1 Solve for x.

Since $\overrightarrow{KN}$ bisects $\angle JKL$, $\angle JKN \cong \angle NKL$.

$m\angle JKN = m\angle NKL$	Definition of congruent angles
$8x - 13 = 6x + 11$	Substitution
$8x = 6x + 24$	Add 13 to each side.
$2x = 24$	Subtract 6x from each side.
$x = 12$	Divide each side by 2.

Step 2 Use the value of x to find $m\angle JKN$.

$m\angle JKN = 8x - 13$	Given
$= 8(12) - 13$	$x = 12$
$= 96 - 13$ or 83	Simplify.

StudyTip

Checking Solutions Check that you have computed the value of x correctly by substituting the value into the expression for $\angle NKL$. If you don't get the same measure as $\angle JKN$, you have made an error.

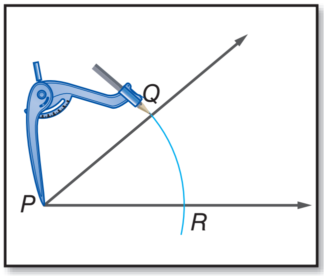
GuidedPractice

3. Suppose $m\angle JKL = 9y + 15$ and $m\angle JKN = 5y + 2$. Find $m\angle JKL$.

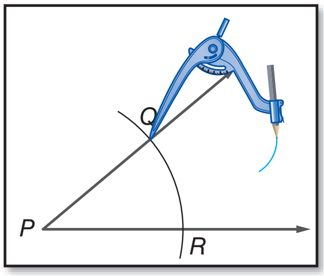
You can produce the angle bisector of any angle without knowing the measure of the angle.

Construction Bisect an Angle

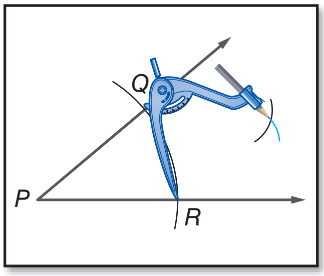
Step 1 Draw an angle on your paper. Label the vertex as P. Put your compass at point P and draw a large arc that intersects both sides of $\angle P$. Label the points of intersection Q and R.



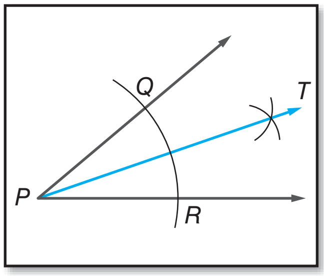
Step 2 With the compass at point Q, draw an arc in the interior of the angle.



Step 3 Keeping the same compass setting, place the compass at point R and draw an arc that intersects the arc drawn in Step 2. Label the point of intersection T.



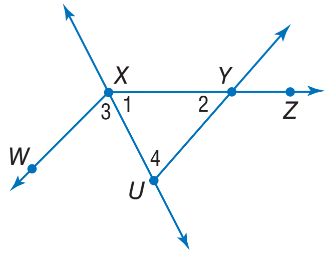
Step 4 Draw $\overrightarrow{PT}$. $\overrightarrow{PT}$ is the bisector of $\angle P$.



Check Your Understanding

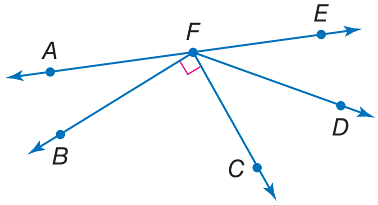
= Step-by-Step Solutions begin on page R14.

Example 1 **Use the figure at the right.**



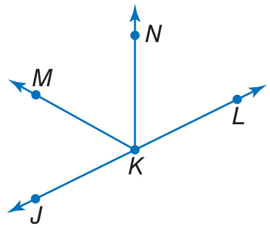
1. Name the vertex of ∠4.
2. Name the sides of ∠3.
3. What is another name for ∠2?
4. What is another name for ∠*UXY*?

Example 2 **Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.**

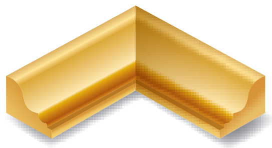


5. ∠*CFD*
6. ∠*AFD*
7. ∠*BFC*
8. ∠*AFB*

Example 3 **ALGEBRA** **In the figure, $\overrightarrow{KJ}$ and $\overrightarrow{KL}$ are opposite rays. $\overrightarrow{KN}$ bisects ∠*LKM*.**



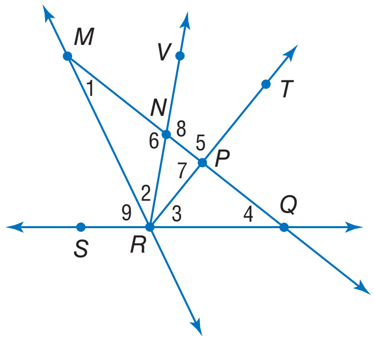
9. If $m\angle LKM = 7x - 5$ and $m\angle NKM = 3x + 9$, find $m\angle LKM$.
10. If $m\angle NKL = 7x - 9$ and $m\angle JKM = x + 3$, find $m\angle JKN$.
11. **CCSS PRECISION** A miter cut is used to build picture frames with corners that meet at right angles.
 a. José miters the ends of some wood for a picture frame at congruent angles. What is the degree measure of his cut? Explain and classify the angle.
 b. What does the joint represent in relation to the angle formed by the two pieces?



Practice and Problem Solving

Extra Practice is on page R1.

Example 1 **For Exercises 12–29, use the figure at the right.**



Name the vertex of each angle.

12. ∠4
13. ∠7
14. ∠2
15. ∠1

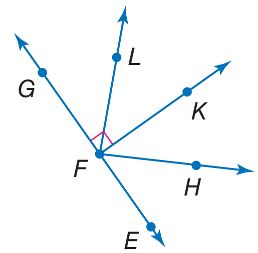
Name the sides of each angle.

16. ∠*TPQ*
17. ∠*VNM*
18. ∠6
19. ∠3

Write another name for each angle.

20. ∠9
21. ∠*QPT*
22. ∠*MQS*
23. ∠5
24. Name an angle with vertex *N* that appears obtuse.
25. Name an angle with vertex *Q* that appears acute.
26. Name a point in the interior of ∠*VRQ*.
27. Name a point in the exterior of ∠*MRT*.
28. Name a pair of angles that share exactly one point.
29. Name a pair of angles that share more than one point.

Example 2 **Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.**



30. $\angle GFK$

31. $\angle EFK$

32. $\angle LFK$

33. $\angle EFH$

34. $\angle GFH$

35. $\angle EFL$

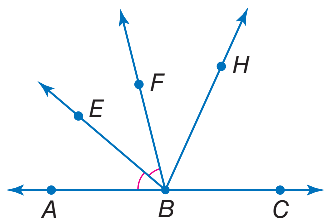
36. **CLOCKS** Determine at least three different times during the day when the hands on a clock form each of the following angles. Explain.

a. right angle

b. obtuse angle

c. congruent acute angles

Example 3 **ALGEBRA** **In the figure, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. $\overrightarrow{BH}$ bisects $\angle EBC$.**



37. If $m\angle ABE = 2n + 7$ and $m\angle EBF = 4n - 13$, find $m\angle ABE$.

38. If $m\angle EBH = 6x + 12$ and $m\angle HBC = 8x - 10$, find $m\angle EBH$.

39. If $m\angle ABF = 7b - 24$ and $m\angle ABE = 2b$, find $m\angle EBF$.

40. If $m\angle EBC = 31a - 2$ and $m\angle EBH = 4a + 45$, find $m\angle HBC$.

41. If $m\angle ABF = 8s - 6$ and $m\angle ABE = 2(s + 11)$, find $m\angle EBF$.

42. If $m\angle EBC = 3r + 10$ and $m\angle ABE = 2r - 20$, find $m\angle EBF$.

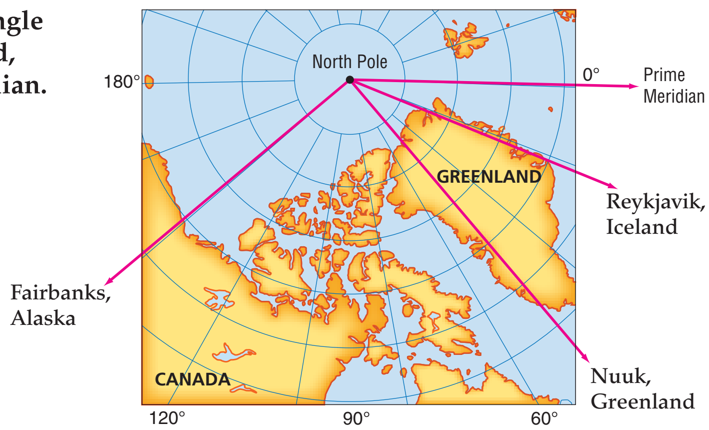
43. **MAPS** **Estimate the measure of the angle formed by each city or location listed, the North Pole, and the Prime Meridian.**

a. Nuuk, Greenland

b. Fairbanks, Alaska

c. Reykjavik, Iceland

d. Prime Meridian



44. **CCSS** **TOOLS** A compass rose is a design on a map that shows directions. In addition to the directions of north, south, east, and west, a compass rose can have as many as 32 markings.

Dimitri Vervits/ImageState

a. With the center of the compass as its vertex, what is the measure of the angle between due west and due north?

b. What is the measure of the angle between due north and north-west?

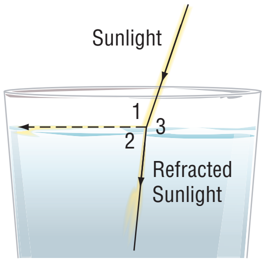
c. How does the north-west ray relate to the angle in part **a**?

Plot the points in a coordinate plane and sketch $\angle XYZ$. Then classify it as *right*, *acute*, or *obtuse*.

45. $X(5, -3)$, $Y(4, -1)$, $Z(6, -2)$

46. $X(6, 7)$, $Y(2, 3)$, $Z(4, 1)$

47. **PHYSICS** When you look at a pencil in water, it looks bent. This illusion is due to *refraction*, or the bending of light when it moves from one substance to the next.

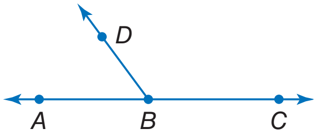


a. What is $m\angle 1$? Classify this angle as *acute*, *right*, or *obtuse*.

b. What is $m\angle 2$? Classify this angle as *acute*, *right*, or *obtuse*.

c. Without measuring, determine how many degrees the path of the light changes after it enters the water. Explain your reasoning.

48. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship of angles that compose opposite rays.



a. Geometric Draw four lines, each with points A, B, and C. Draw $\overrightarrow{BD}$ for each line, varying the placement of point D. Use a protractor to measure $\angle ABD$ and $\angle DBC$ for each figure.

b. Tabular Organize the measures for each figure into a table. Include a row in your table to record the sum of these measures.

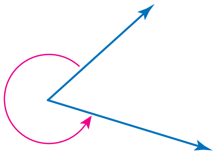
c. Verbal Make a conjecture about the sum of the measures of the two angles. Explain your reasoning.

d. Algebraic If x is the measure of $\angle ABD$ and y is the measure of $\angle DBC$, write an equation that relates the two angle measures.

H.O.T. Problems Use Higher-Order Thinking Skills

49. **OPEN ENDED** Draw an obtuse angle named ABC. Measure $\angle ABC$. Construct an angle bisector $\overrightarrow{BD}$ of $\angle ABC$. Explain the steps in your construction and justify each step. Classify the two angles formed by the angle bisector.

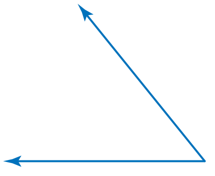
50. **CHALLENGE** Describe how you would use a protractor to measure the angle shown.



51. **CCSS ARGUMENTS** The sum of two acute angles is *sometimes*, *always*, or *never* an obtuse angle. Explain.

52. **CHALLENGE** $\overrightarrow{MP}$ bisects $\angle LMN$, $\overrightarrow{MQ}$ bisects $\angle LMP$, and $\overrightarrow{MR}$ bisects $\angle QMP$. If $m\angle RMP = 21$, find $m\angle LMN$. Explain your reasoning.

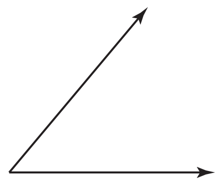
53. **WRITING IN MATH** Rashid says that he can estimate the measure of an acute angle using a piece of paper to within six degrees of accuracy. Explain how this would be possible. Then use this method to estimate the measure of the angle shown.



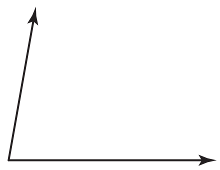
Standardized Test Practice

54. Which of the following angles measures closest to 60°?

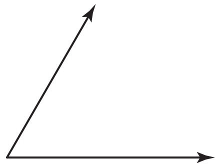
A



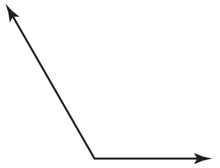
C



B

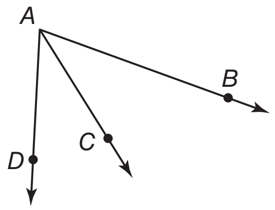


D



55. SHORT RESPONSE Leticia surveyed 50 English majors at a university to see if the school should play jazz music in the cafeteria during lunch. The school has 75 different majors and a total of 2000 students. Explain why the results of Leticia's survey are or are not representative of the entire student body.

56. In the figure below, if $m\angle BAC = 38$, what must be the measure of $\angle BAD$ in order for $\overrightarrow{AC}$ to be an angle bisector?



F 142 **H** 52

G 76 **J** 38

57. SAT/ACT If n is divisible by 2, 5, and 14, which of the following is also divisible by these numbers?

A $n + 7$ **D** $n + 20$

B $n + 10$ **E** $n + 70$

C $n + 14$

Spiral Review

Find the distance between each pair of points. Round to the nearest hundredth. (Lesson 1-3)

58. $A(-1, -8), B(3, 4)$ **59.** $C(0, 1), D(-2, 9)$ **60.** $E(-3, -12), F(5, 4)$

61. $G(4, -10), H(9, -25)$ **62.** $J\left(1, \frac{1}{4}\right), K\left(-3, \frac{7}{4}\right)$ **63.** $L\left(-5, \frac{8}{5}\right), M\left(5, \frac{2}{5}\right)$

Find the value of the variable and ST if S is between R and T. (Lesson 1-2)

64. $RS = 7a, ST = 12a, RT = 76$ **65.** $RS = 12, ST = 2x, RT = 34$

66. PHOTOGRAPHY Photographers often place their cameras on tripods. In the diagram, the tripod is placed on an inclined surface, and the length of each leg is adjusted so that the camera remains level with the horizon. Are the feet of the tripod coplanar? Explain your reasoning. (Lesson 1-1)

Complete each sentence. (Lesson 0-1)

67. 54 in. = __?__ ft **68.** 275 mm = __?__ m **69.** 7 gal = __?__ pt

Skills Review

Solve each equation.

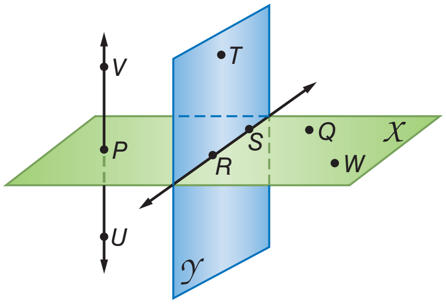
70. $(90 - x) - x = 18$ **71.** $(5x + 3) + 7x = 180$ **72.** $(13x + 10) + 2x = 90$

73. $(180 - x) - 4x = 56$ **74.** $(4n + 17) + (n - 2) = 180$ **75.** $(8a - 23) + (9 - 2a) = 90$

Mid-Chapter Quiz

Lessons 1-1 through 1-4

Use the figure to complete each of the following. (Lesson 1-1)



1. Name another point that is collinear with points *U* and *V*.

2. What is another name for plane *Y*?

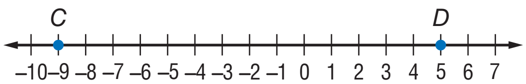
3. Name a line that is coplanar with points *P*, *Q*, and *W*.

Find the value of *x* and *AC* if *B* is between points *A* and *C*. (Lesson 1-2)

4. $AB = 12$, $BC = 8x - 2$, $AC = 10x$

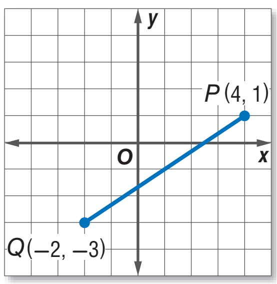
5. $AB = 5x$, $BC = 9x - 2$, $AC = 11x + 7.6$

6. Find *CD* and the coordinate of the midpoint of $\overline{CD}$.

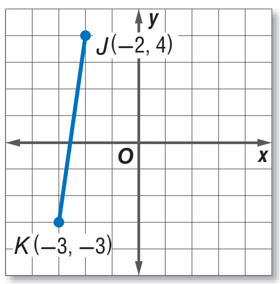


Find the coordinates of the midpoint of each segment. Then find the length of each segment. (Lesson 1-3)

7.



8.



Find the coordinates of the midpoint of a segment with the given endpoints. Then find the distance between each pair of points. (Lesson 1-3)

9. *P*(26, 12) and *Q*(8, 42)

10. *M*(6, −41) and *N*(−18, −27)

11. **MAPS** A map of a town is drawn on a coordinate grid. The high school is found at point (3, 1) and town hall is found at (−5, 7). (Lesson 1-3)

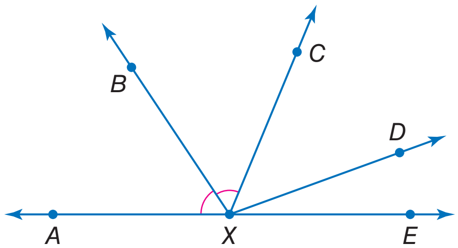
    a. If the high school is at the midpoint between the town hall and the town library, at which ordered pair should you find the library?

    b. If one unit on the grid is equivalent to 50 meters, how far is the high school from town hall?

12. **MULTIPLE CHOICE** The vertex of $\angle ABC$ is located at the origin. Point *A* is located at (5, 0) and Point *C* is located at (0, 2). How can $\angle ABC$ be classified?

    A acute
    B obtuse
    C right
    D scalene

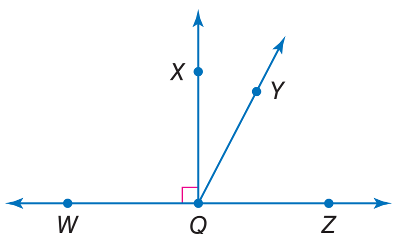
In the figure, $\overrightarrow{XA}$ and $\overrightarrow{XE}$ are opposite rays, and $\angle AXC$ is bisected by $\overrightarrow{XB}$. (Lesson 1-4)



13. If $m\angle AXC = 8x - 7$ and $m\angle AXB = 3x + 10$, find $m\angle AXC$.

14. If $m\angle CXD = 4x + 6$, $m\angle DXE = 3x + 1$, and $m\angle CXE = 8x - 2$, find $m\angle DXE$.

Classify each angle as *acute*, *right*, or *obtuse*. (Lesson 1-4)



15. $\angle WQY$

16. $\angle YQZ$

LESSON 1-5

Angle Relationships

Then

- You measured and classified angles.

Now

- **1** Identify and use special pairs of angles.
- **2** Identify perpendicular lines.

Why?

- Cheerleaders position their arms and legs at specific angles to create various formations when performing at games and at competitions. Certain pairs of angles have special names and share specific relationships.

Zia Soleil/Iconica/Getty Images

NewVocabulary

adjacent angles
linear pair
vertical angles
complementary angles
supplementary angles
perpendicular

CCSS Common Core State Standards

Content Standards

Preparation for G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

Mathematical Practices

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

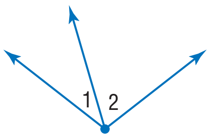
1 Pairs of Angles

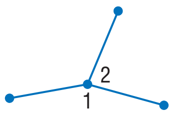
Some pairs of angles are special because of how they are positioned in relationship to each other. Three of these angle pairs are described below.

KeyConcept Special Angle Pairs

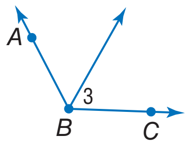
Adjacent angles are two angles that lie in the same plane and have a common vertex and a common side, but no common interior points.

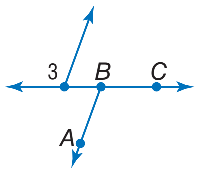
Examples ∠1 and ∠2 are adjacent angles.





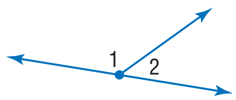
Nonexamples ∠3 and ∠*ABC* are nonadjacent angles



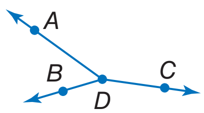


A **linear pair** is a pair of adjacent angles with noncommon sides that are opposite rays.

Example ∠1 and ∠2

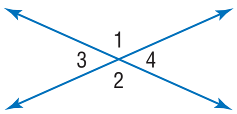


Nonexample ∠*ADB* and ∠*ADC*

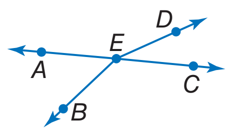


Vertical angles are two nonadjacent angles formed by two intersecting lines.

Examples ∠1 and ∠2; ∠3 and ∠4

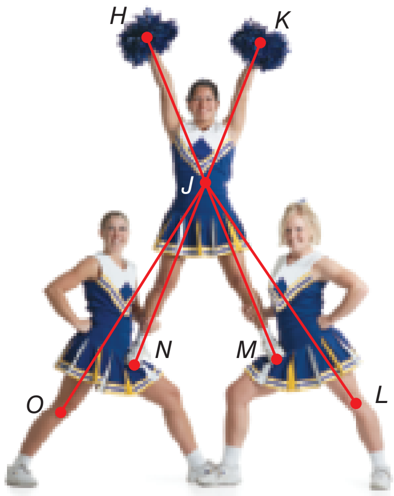


Nonexample ∠*AEB* and ∠*DEC*



Real-World Example 1 Identify Angle Pairs

CHEERLEADING **Name an angle pair that satisfies each condition.**



a. two acute adjacent angles

∠*HJK*, ∠*LJM*, ∠*MJN*, and ∠*NJO* are acute angles.

∠*LJM* and ∠*MJN* are acute adjacent angles, and ∠*MJN* and ∠*NJO* are acute adjacent angles.

b. two obtuse vertical angles

∠*HJN* and ∠*KJM* are obtuse vertical angles.

GuidedPractice

1A. a linear pair

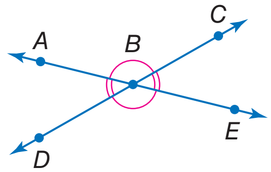
1B. two acute vertical angles

Some pairs of angles are special because of the relationship between their angle measures.

KeyConcept Angle Pair Relationships

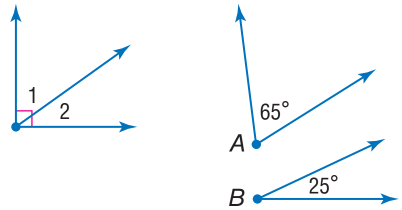
Vertical angles are congruent.

Examples ∠*ABC* ≅ ∠*DBE* and ∠*ABD* ≅ ∠*CBE*



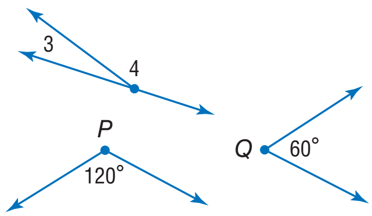
Complementary angles are two angles with measures that have a sum of 90.

Examples ∠1 and ∠2 are complementary.
∠*A* is complementary to ∠*B*.



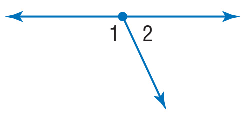
Supplementary angles are two angles with measures that have a sum of 180.

Examples ∠3 and ∠4 are supplementary.
∠*P* and ∠*Q* are supplementary.



The angles in a linear pair are supplementary.

Example $m\angle 1 + m\angle 2 = 180$



StudyTip
Linear Pair vs. Supplementary Angles While the angles in a linear pair are always supplementary, some supplementary angles do not form a linear pair.

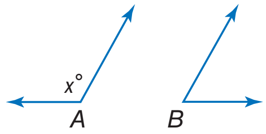
Remember that angle measures are real numbers. So the operations for real numbers and algebra can be used with angle measures.

Example 2 Angle Measure

ALGEBRA Find the measures of two supplementary angles if the difference in the measures of the two angles is 18.

Understand The problem relates the measures of two supplementary angles. You know that the sum of the measures of supplementary angles is 180. You need to find the measure of each angle.

Plan Draw two figures to represent the angles. Let the measure of one angle be x. If $m\angle A = x$, then because $\angle A$ and $\angle B$ are supplementary, $m\angle B + x = 180$ or $m\angle B = 180 - x$.



The problem states that the difference of the two angle measures is 18, or $m\angle B - m\angle A = 18$.

Solve

$m\angle B - m\angle A = 18$	Given
$(180 - x) - x = 18$	$m\angle A = x$, $m\angle B = 180 - x$
$180 - 2x = 18$	Simplify.
$-2x = -162$	Subtract 180 from each side.
$x = 81$	Divide each side by −2.

Use the value of x to find each angle measure.

$m\angle A = x$ $m\angle B = 180 - x$

$= 81$ $= 180 - 81$ or 99

Check Add the angle measures to verify that the angles are supplementary.

$m\angle A + m\angle B \stackrel{?}{=} 180$

$81 + 99 = 180$ ✓

Guided Practice

2. Find the measures of two complementary angles if the measure of the larger angle is 12 more than twice the measure of the smaller angle.

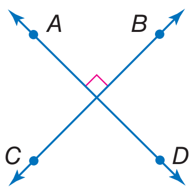
> **Problem-Solving Tip**
>
> **Write an Equation** While you could use the guess-and-check strategy to find two measures with a sum of 180 and a difference of 18, writing an equation is a more efficient approach to this problem.

2 Perpendicular Lines

Lines, segments, or rays that form right angles are **perpendicular.**

Key Concept Perpendicular Lines

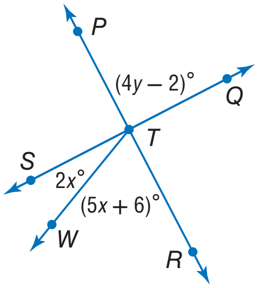
- Perpendicular lines intersect to form four right angles.
- Perpendicular lines intersect to form congruent adjacent angles.
- Segments and rays can be perpendicular to lines or other line segments and rays.
- The right angle symbol in the figure indicates that the lines are perpendicular.



Symbol $\perp$ is read *is perpendicular to.*   **Example** $\overleftrightarrow{AD} \perp \overleftrightarrow{CB}$

Example 3 Perpendicular Lines

ALGEBRA Find x and y so that $\overleftrightarrow{PR}$ and $\overleftrightarrow{SQ}$ are perpendicular.



If $\overleftrightarrow{PR} \perp \overleftrightarrow{SQ}$, then $m\angle STR = 90$ and $m\angle PTQ = 90$.

To find x, use $\angle STW$ and $\angle WTR$.

$m\angle STR = m\angle STW + m\angle WTR$	Sum of parts = whole
$90 = 2x + (5x + 6)$	Substitution
$90 = 7x + 6$	Combine like terms.
$84 = 7x$	Subtract 6 from each side.
$12 = x$	Divide each side by 7.

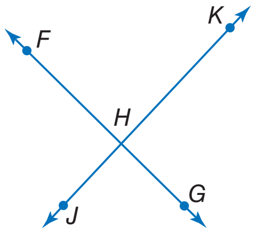
To find y, use $m\angle PTQ$.

$m\angle PTQ = 4y - 2$	Given
$90 = 4y - 2$	Substitution
$92 = 4y$	Add 2 to each side.
$23 = y$	Divide each side by 4.

GuidedPractice

3. Suppose $m\angle D = 3x - 12$. Find x so that $\angle D$ is a right angle.

In the figure at the right, it *appears* that $\overleftrightarrow{FG} \perp \overleftrightarrow{JK}$. However, you cannot assume this is true unless other information, such as $m\angle FHJ = 90$, is given.



In geometry, figures are sketches used to depict a situation. They are not drawn to reflect total accuracy. There are certain relationships that you can assume to be true, but others you cannot. Study the figure and the lists below.

KeyConcept Interpreting Diagrams

CAN be Assumed

All points shown are coplanar.

G, H, and J are collinear.

$\overrightarrow{HM}$, $\overrightarrow{HL}$, $\overrightarrow{HK}$, and $\overleftrightarrow{GJ}$ intersect at H.

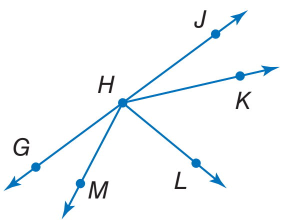
H is between G and J.

L is in the interior of $\angle MHK$.

$\angle GHM$ and $\angle MHL$ are adjacent angles.

$\angle GHL$ and $\angle LHJ$ are a linear pair.

$\angle JHK$ and $\angle KHG$ are supplementary.



CANNOT be Assumed

Perpendicular lines: $\overrightarrow{HM} \perp \overrightarrow{HL}$

Congruent angles: $\angle JHK \cong \angle GHM$

$\angle JHK \cong \angle KHL$

$\angle KHL \cong \angle LHM$

Congruent segments: $\overline{GH} \cong \overline{HJ}$

$\overline{HJ} \cong \overline{HK}$

$\overline{HK} \cong \overline{HL}$

$\overline{HL} \cong \overline{HG}$

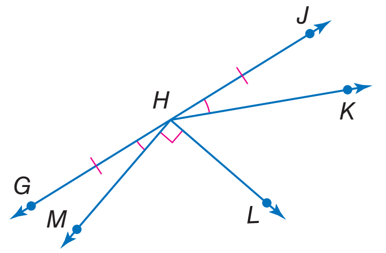
The list of statements that can be assumed is not a complete list. There are more special pairs of angles than those listed.

StudyTip

Additional Information Additional information for a figure may be given using congruent angle markings, congruent segment markings, or right angle symbols.

Example 4 Interpret Figures

Determine whether each statement can be assumed from the figure. Explain.



a. **$\angle KHJ$ and $\angle GHM$ are complementary.**

No; they are congruent, but we do not know anything about their exact measures.

b. **$\angle GHK$ and $\angle JHK$ are a linear pair.**

Yes; they are adjacent angles whose noncommon sides are opposite rays.

c. **$\overrightarrow{HL}$ is perpendicular to $\overrightarrow{HM}$.**

Yes; the right angle symbol in the figure indicates that $\overrightarrow{HL} \perp \overrightarrow{HM}$.

GuidedPractice

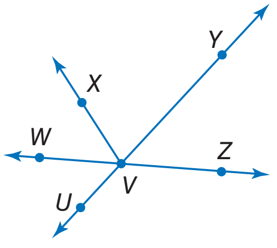
4A. $\angle GHL$ and $\angle LHJ$ are supplementary.

4B. $\angle GHM$ and $\angle MHK$ are adjacent angles.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

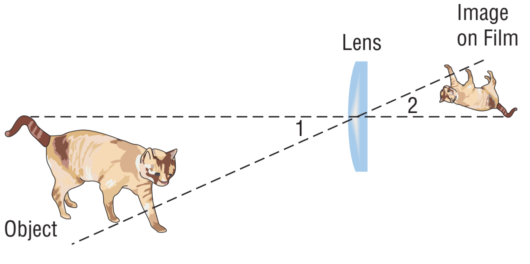
Example 1 **Name an angle pair that satisfies each condition.**



1. two acute vertical angles

2. two obtuse adjacent angles

Examples 1–2 **3.** **CAMERAS** Cameras use lenses and light to capture images.



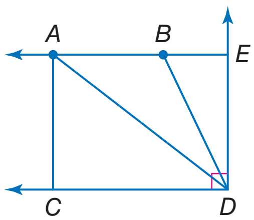
a. What type of angles are formed by the object and its image?

b. If the measure of ∠2 is 15, what is the measure of ∠1?

Examples 2–3 **4.** **ALGEBRA** The measures of two complementary angles are $7x + 17$ and $3x - 20$. Find the measures of the angles.

5 **ALGEBRA** Lines x and y intersect to form adjacent angles 2 and 3. If $m\angle 2 = 3a - 27$ and $m\angle 3 = 2b + 14$, find the values of a and b so that x is perpendicular to y.

Example 4 **Determine whether each statement can be assumed from the figure. Explain.**

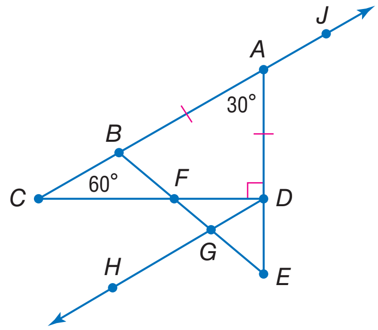


6. $\angle CAD$ and $\angle DAB$ are complementary.
7. $\angle EDB$ and $\angle BDA$ are adjacent, but they are neither complementary nor supplementary.

Practice and Problem Solving

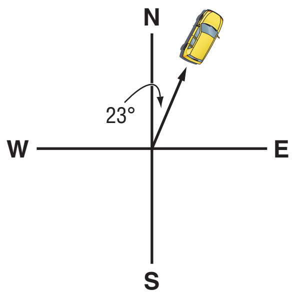
Extra Practice is on page R1.

Examples 1–2 **Name an angle or angle pair that satisfies each condition.**



8. two adjacent angles
9. two acute vertical angles
10. two obtuse vertical angles
11. two complementary adjacent angles
12. two complementary nonadjacent angles
13. two supplementary adjacent angles
14. a linear pair whose vertex is F
15. an angle complementary to $\angle FDG$
16. an angle supplementary to $\angle CBF$
17. an angle supplementary to $\angle JAE$

18. CCSS **REASONING** You are using a compass to drive 23° east of north. Express your direction in another way using an acute angle and two of the four directions: north, south, east, and west. Explain your reasoning.



Example 2 **Find the value of each variable.**

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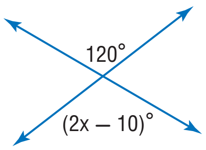

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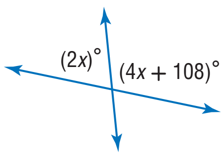

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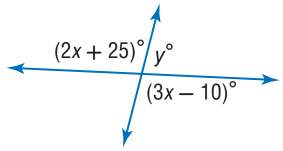

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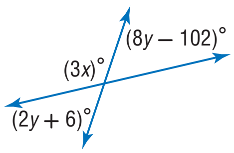

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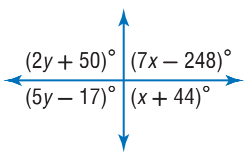

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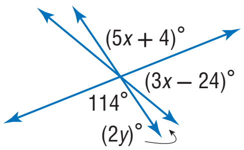


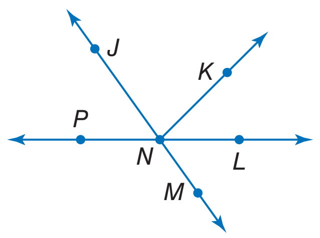
25. **ALGEBRA** $\angle E$ and $\angle F$ are supplementary. The measure of $\angle E$ is 54 more than the measure of $\angle F$. Find the measures of each angle.

26. **ALGEBRA** The measure of an angle's supplement is 76 less than the measure of the angle. Find the measure of the angle and its supplement.

27. **ALGEBRA** The measure of the supplement of an angle is 40 more than two times the measure of the complement of the angle. Find the measure of the angle.

28. **ALGEBRA** $\angle 3$ and $\angle 4$ form a linear pair. The measure of $\angle 3$ is four more than three times the measure of $\angle 4$. Find the measure of each angle.

Example 3 **ALGEBRA** **Use the figure at the right.**



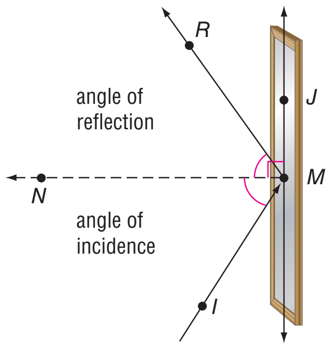
29. If $m\angle KNL = 6x - 4$ and $m\angle LNM = 4x + 24$, find the value of x so that $\angle KNM$ is a right angle.

30. If $m\angle JNP = 3x - 15$ and $m\angle JNL = 5x + 59$, find the value of x so that $\angle JNP$ and $\angle JNL$ are supplements of each other.

31. If $m\angle LNM = 8x + 12$ and $m\angle JNL = 12x - 32$, find $m\angle JNP$.

32. If $m\angle JNP = 2x + 3$, $m\angle KNL = 3x - 17$, and $m\angle KNJ = 3x + 34$, find the measure of each angle.

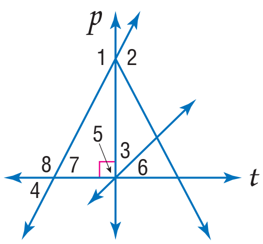
33. **PHYSICS** As a ray of light meets a mirror, the light is reflected. The angle at which the light strikes the mirror is the *angle of incidence*. The angle at which the light is reflected is the *angle of reflection*. The angle of incidence and the angle of reflection are congruent. In the diagram at the right, if $m\angle RMI = 106$, find the angle of reflection and $m\angle RMJ$.



34. **ALGEBRA** Rays AB and BC are perpendicular. Point D lies in the interior of $\angle ABC$. If $m\angle ABD = 3r + 5$ and $m\angle DBC = 5r - 27$, find $m\angle ABD$ and $m\angle DBC$.

35. **ALGEBRA** $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ intersect at point V. If $m\angle WVY = 4a + 58$ and $m\angle XVY = 2b - 18$, find the values of a and b so that $\overleftrightarrow{WX}$ is perpendicular to $\overleftrightarrow{YZ}$.

Example 4 **Determine whether each statement can be assumed from the figure. Explain.**



36. $\angle 4$ and $\angle 7$ are vertical angles.

37. $\angle 4$ and $\angle 8$ are supplementary.

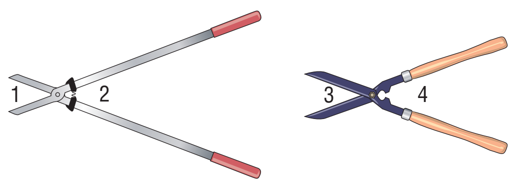
38. $p \perp t$

39. $\angle 3 \cong \angle 6$

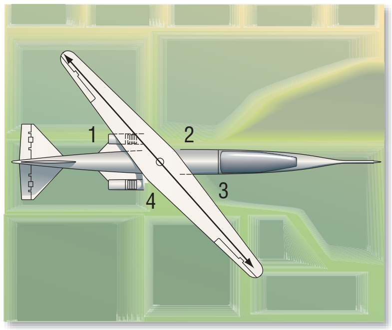
40. $\angle 5 \cong \angle 3 + \angle 6$

41. $\angle 5$ and $\angle 7$ form a linear pair.

42. CCSS **ARGUMENTS** In the diagram of the pruning shears shown, $m\angle 1 = m\angle 3$. What conclusion can you reach about the relationship between $\angle 4$ and $\angle 2$? Explain.



FLIGHT **The wing of the aircraft shown can pivot up to 60° in either direction from the perpendicular position.**



43. Identify a pair of vertical angles.

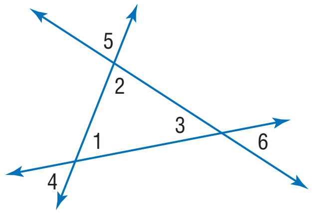
44. Identify two pairs of supplementary angles.

45 If $m\angle 1 = 110$, what is $m\angle 3$? $m\angle 4$?

46. What is the minimum possible value for $m\angle 2$? the maximum?

47. Is there a wing position in which none of the angles are obtuse? Explain.

48. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the sum of the interior angles of a triangle and the angles vertical to them.



a. **Geometric** Draw three sets of three intersecting lines and label each as shown.

b. **Tabular** For each set of lines, measure and record $m\angle 1$, $m\angle 2$, and $m\angle 3$ in a table. Record $m\angle 1 + m\angle 2 + m\angle 3$ in a separate column.

c. **Verbal** Explain how you can find $m\angle 4$, $m\angle 5$, and $m\angle 6$ when you know $m\angle 1$, $m\angle 2$, and $m\angle 3$.

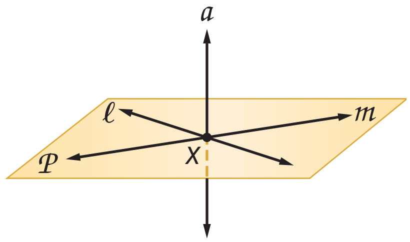
d. **Algebraic** Write an equation that relates $m\angle 1 + m\angle 2 + m\angle 3$ to $m\angle 4 + m\angle 5 + m\angle 6$. Then use substitution to write an equation that relates $m\angle 4 + m\angle 5 + m\angle 6$ to an integer.

H.O.T. Problems Use Higher-Order Thinking Skills

49. CCSS **REASONING** Are there angles that do not have a complement? Explain.

50. **OPEN ENDED** Draw a pair of intersecting lines that forms a pair of complementary angles. Explain your reasoning.

51. **CHALLENGE** If a line, line segment, or ray is perpendicular to a plane, it is perpendicular to every line, line segment, or ray in the plane that intersects it.



a. If a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them. If line a is perpendicular to line ℓ and line m at point X, what must also be true?

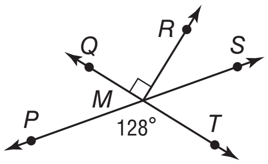
b. If a line is perpendicular to a plane, then any line perpendicular to the given line at the point of intersection with the given plane is in the given plane. If line a is perpendicular to plane $\mathcal{P}$ and line m at point X, what must also be true?

c. If a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane. If line a is perpendicular to plane $\mathcal{P}$, what must also be true?

52. **WRITING IN MATH** Describe three different ways you can determine that an angle is a right angle.

Standardized Test Practice

53. What is $m\angle RMS$ in the figure below?



A 26

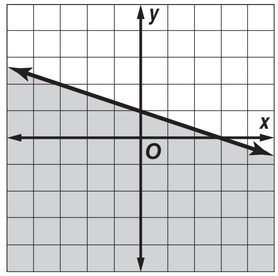
B 38

C 52

D 128

54. EXTENDED RESPONSE For a fundraiser, a theater club is making 400 cookies. They want to make twice as many chocolate chip as peanut butter cookies and three times as many peanut butter as oatmeal raisin cookies. Determine how many of each type of cookie the theater club will make. Show your work.

55. ALGEBRA Which inequality is graphed below?



F $y > -\frac{1}{3}x + 1$

G $y < -\frac{1}{3}x + 1$

H $y \geq -\frac{1}{3}x + 1$

J $y \leq -\frac{1}{3}x + 1$

56. SAT/ACT One third of a number is three more than one fourth the same number. What is the number?

A 3

B 12

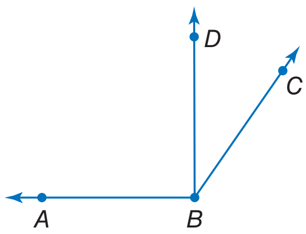
C 36

D 42

E 48

Spiral Review

Copy the diagram shown and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree. (Lesson 1-4)



57. $\angle ABC$

58. $\angle DBC$

59. $\angle ABD$

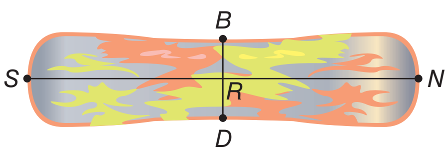
Find the coordinates of the midpoint of a segment with the given endpoints. (Lesson 1-3)

60. $P(3, -7)$, $Q(9, 6)$

61. $A(-8, -5)$, $B(1, 7)$

62. $J(-7, 4)$, $K(3, 1)$

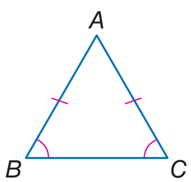
63. SNOWBOARDING In the design on the snowboard shown, $\overline{BD}$ bisects $\overline{SN}$ at R. If $SN = 163$ centimeters, find RN. (Lesson 1-2)



Skills Review

Name the congruent sides and angles in each figure.

64.



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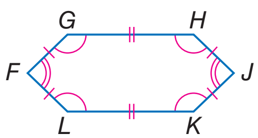

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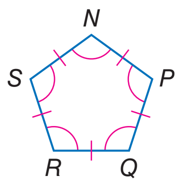
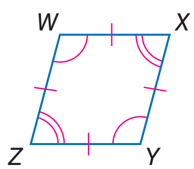


67.



EXTEND 1-5

Geometry Lab
Constructing Perpendiculars

You can use a compass and a straightedge to construct a line perpendicular to a given line through a point on the line, or through a point *not* on the line.

CCSS Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Activity Construct a Perpendicular

a. Construct a line perpendicular to line ℓ and passing through point P on ℓ.

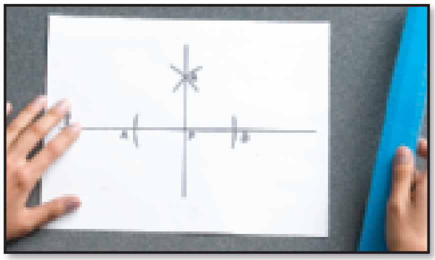
Step 1

Place the compass at P. Draw arcs to the right and left of P that intersect line ℓ using the same compass setting. Label the points of intersection A and B.

Step 2

With the compass at A, draw an arc above line ℓ using a setting greater than AP. Using the same compass setting, draw an arc from B that intersects the previous arc. Label the intersection Q.

Step 3



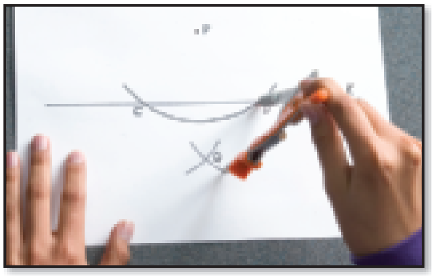
Use a straightedge to draw $\overleftrightarrow{QP}$.

b. Construct a line perpendicular to line k and passing through point P *not* on k.

Step 1

Place the compass at P. Draw an arc that intersects line k in two different places. Label the points of intersection C and D.

Step 2



With the compass at C, draw an arc below line k using a setting greater than $\frac{1}{2}CD$. Using the same compass setting, draw an arc from D that intersects the previous arc. Label the intersection Q.

Step 3

Use a straightedge to draw $\overleftrightarrow{PQ}$.

Model and Analyze the Results

1. Draw a line and construct a line perpendicular to it through a point on the line.
2. Draw a line and construct a line perpendicular to it through a point not on the line.
3. How is the second construction similar to the first one?

LESSON 1-6 Two-Dimensional Figures

Then

- You measured one-dimensional figures.

Now

1. Identify and name polygons.
2. Find perimeter, circumference, and area of two-dimensional figures.

Why?

- Mosaics are patterns or pictures created using small bits of colored glass or stone. They are usually set into a wall or floor and often make use of polygons.

NewVocabulary
polygon
vertex of a polygon
concave
convex
n-gon
equilateral polygon
equiangular polygon
regular polygon
perimeter
circumference
area

Common Core State Standards

Content Standards
G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Mathematical Practices
2 Reason abstractly and quantitatively.
6 Attend to precision.

1 Identify Polygons

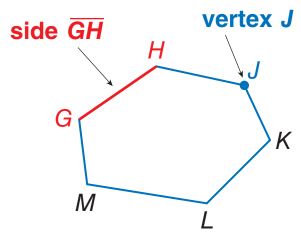
Most of the closed figures shown in the mosaic are polygons. The term *polygon* is derived from a Greek word meaning *many angles.*

KeyConcept Polygons

A **polygon** is a closed figure formed by a finite number of coplanar segments called *sides* such that

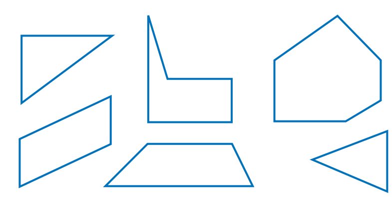
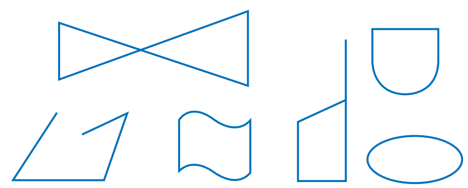
- the sides that have a common endpoint are noncollinear, and
- each side intersects exactly two other sides, but only at their endpoints.

The vertex of each angle is a **vertex of the polygon**. A polygon is named by the letters of its vertices, written in order of consecutive vertices.

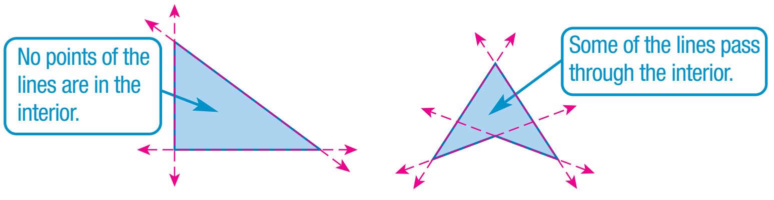


polygon *GHJKLM*

The table below shows some additional examples of polygons and some examples of figures that are not polygons.

Polygons	Not Polygons
	

Polygons can be **concave** or **convex**. Suppose the line containing each side is drawn. If any of the lines contain any point in the interior of the polygon, then it is concave. Otherwise it is convex.



convex polygon   concave polygon

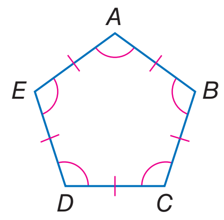
> **Study Tip**
>
> **Naming Polygons** The Greek prefixes used to name polygons are also used to denote number. For example a *bi*cycle has two wheels, and a *tri*pod has three legs.

In general, a polygon is classified by its number of sides. The table lists some common names for various categories of polygon. A polygon with n sides is an **n-gon**. For example, a polygon with 15 sides is a 15-gon.

Number of Sides	Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
11	hendecagon
12	dodecagon
n	n-gon

An **equilateral polygon** is a polygon in which all sides are congruent. An **equiangular polygon** is a polygon in which all angles are congruent.

A convex polygon that is both equilateral and equiangular is called a **regular polygon**. An *irregular polygon* is a polygon that is *not* regular.



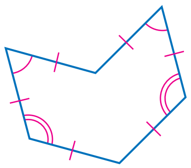
regular pentagon *ABCDE*

> **Reading Math**
>
> **Simple Closed Curves** Polygons and circles are examples of *simple closed curves*. Such a curve begins and ends at the same point without crossing itself. The figures below are *not* simple closed curves.
>
> 

Example 1 Name and Classify Polygons

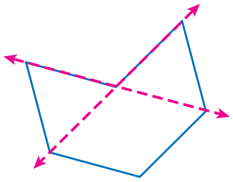
Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

a.



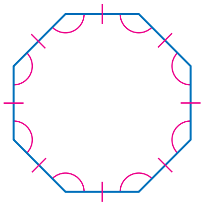
The polygon has 6 sides, so it is a hexagon.

Two of the lines containing the sides of the polygon will pass through the interior of the hexagon, so it is concave.



Only convex polygons can be regular, so this is an irregular hexagon.

b.



There are 8 sides, so this is an octagon.

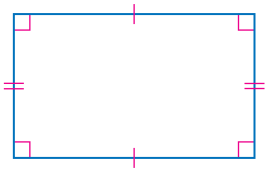
No line containing any of the sides will pass through the interior of the octagon, so it is convex.

All of the sides are congruent, so it is equilateral. All of the angles are congruent, so it is equiangular.

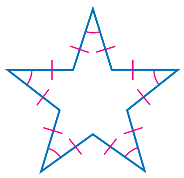
Since the polygon is convex, equilateral, and equiangular, it is regular. So this is a regular octagon.

Guided Practice

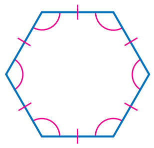
1A.



1B.



1C.

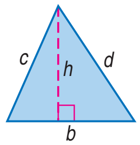
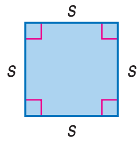
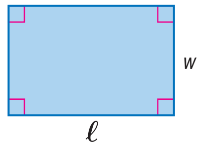
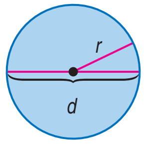


2 Perimeter, Circumference, and Area

The **perimeter** of a polygon is the sum of the lengths of the sides of the polygon. Some shapes have special formulas for perimeter, but all are derived from the basic definition of perimeter. You will derive these formulas in Chapter 11. The **circumference** of a circle is the distance around the circle.

The **area** of a figure is the number of square units needed to cover a surface. Review the formulas for the perimeter and area of three common polygons and circle given below.

KeyConcept Perimeter, Circumference, and Area

Triangle	Square	Rectangle	Circle
			
$P = b + c + d$	$P = s + s + s + s$ $= 4s$	$P = \ell + w + \ell + w$ $= 2\ell + 2w$	$C = 2\pi r$ or $C = \pi d$
$A = \frac{1}{2}bh$	$A = s^2$	$A = \ell w$	$A = \pi r^2$

P = perimeter of polygon; A = area of figure; C = circumference
b = base, h = height; ℓ = length, w = width; r = radius, d = diameter

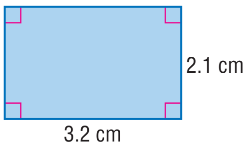
ReadingMath

Pi The symbol π is read *pi*. This is not a variable but an irrational number. The most accurate way to perform a calculation with π is to use a calculator. If no calculator is available, 3.14 is a good estimate for π.

Example 2 Find Perimeter and Area

Find the perimeter or circumference and area of each figure.

a.



$P = 2\ell + 2w$ — Perimeter of rectangle

$= 2(3.2) + 2(2.1)$ — $\ell = 3.2$, $w = 2.1$

$= 10.6$ — Simplify.

The perimeter is 10.6 centimeters.

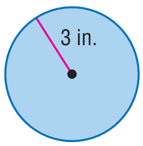
$A = \ell w$ — Area of rectangle

$= (3.2)(2.1)$ — $\ell = 3.2$, $w = 2.1$

$= 6.72$ — Simplify.

The area is about 6.7 square centimeters.

b.



$C = 2\pi r$ — Circumference

$= 2\pi(3)$ — $r = 3$

≈ 18.85 — Use a calculator.

The circumference is about 18.9 inches.

$A = \pi r^2$ — Area of circle

$= \pi(3)^2$ — $r = 3$

≈ 28.3 — Use a calculator.

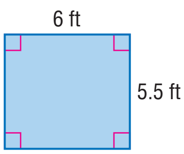
The area is about 28.3 square inches.

StudyTip

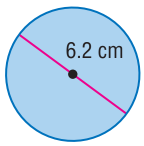
Perimeter vs. Area Since calculating the area of a figure involves multiplying two dimensions (unit × unit), *square units* are used. There is only one dimension used when finding the perimeter (the distance around), thus, it is given simply in *units.*

GuidedPractice

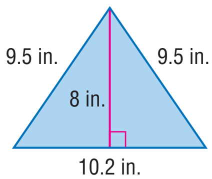
2A.



2B.



2C.



Standardized Test Example 3 Largest Area

Yolanda has 26 centimeters of cording to frame a photograph in her scrapbook. Which of these shapes would use *most* or all of the cording and enclose the *largest* area?

A right triangle with each leg about 7 centimeters long

B circle with a radius of about 4 centimeters

C rectangle with a length of 8 centimeters and a width of 4.5 centimeters

D square with a side length of 6 centimeters

> **Test-Taking Tip**
> **Mental Math** When you are asked to compare measures for varying figures, it can be helpful to use mental math. Estimate the perimeter or area of each figure, and then check your calculations.

Read the Test Item

You are asked to compare the area and perimeter of four different shapes.

Solve the Test Item

Find the perimeter and area of each shape.

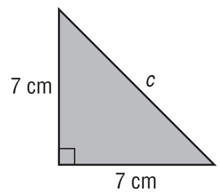
Right Triangle

Use the Pythagorean Theorem to find the length of the hypotenuse.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 7^2 + 7^2$ or 98 $a = 7, b = 7$

$c = \sqrt{98}$ or about 9.9 Simplify.



$P = a + b + c$ Perimeter of a triangle

$\approx 7 + 7 + 9.9$ or about 23.9 cm Substitution

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(7)$ or 24.5 cm^2 Substitution

> **Study Tip**
> **Irrational Measures** Notice that the triangle perimeter given in Example 3 is only an approximation. Because the length of the hypotenuse is an irrational number, the actual perimeter of the triangle is the irrational measure $(14 + \sqrt{98})$ centimeters.

Circle	Rectangle	Square
$C = 2\pi r$	$P = 2\ell + 2w$	$P = 4s$
$= 2\pi(4)$	$= 2(8) + 2(4.5)$	$= 4(6)$
≈ 25.1 cm	$= 25$ cm	$= 24$ cm
$A = \pi r^2$	$A = \ell w$	$A = s^2$
$= \pi(4)^2$	$= (8)(4.5)$	$= 6^2$
$\approx 50.3\ \text{cm}^2$	$= 36\ \text{cm}^2$	$= 36\ \text{cm}^2$

The shape that uses the most cording and encloses the largest area is the circle. The answer is B.

Guided Practice

3. Dasan has 32 feet of fencing to fence in a play area for his dog. Which shape of play area uses *most* or all of the fencing and encloses the *largest* area?

F circle with radius of about 5 feet

G rectangle with length 5 feet and width 10 feet

H right triangle with legs of length 10 feet each

J square with side length 8 feet

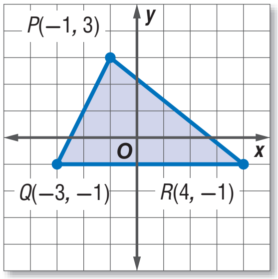
You can use the Distance Formula to find the perimeter of a polygon graphed on a coordinate plane.

Example 4 Perimeter and Area on the Coordinate Plane

COORDINATE GEOMETRY **Find the perimeter and area of △*PQR* with vertices *P*(−1, 3), *Q*(−3, −1), and *R*(4, −1).**

Step 1 Find the perimeter of △*PQR*.

Graph △*PQR*.



To find the perimeter of △*PQR*, first find the lengths of each side. Counting the squares on the grid, we find that *QR* = 7 units. Use the Distance Formula to find the lengths of $\overline{PQ}$ and $\overline{PR}$.

$\overline{PQ}$ has endpoints at *P*(−1, 3) and *Q*(−3, −1).

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[-1 - (-3)]^2 + [3 - (-1)]^2}$ Substitute.

$= \sqrt{2^2 + 4^2}$ Subtract.

$= \sqrt{20}$ or about 4.5 Simplify.

$\overline{PR}$ has endpoints at *P*(−1, 3) and *R*(4, −1).

$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-1 - 4)^2 + [3 - (-1)]^2}$ Substitute.

$= \sqrt{(-5)^2 + 4^2}$ Subtract.

$= \sqrt{41}$ or about 6.4 Simplify.

The perimeter of △*PQR* is $7 + \sqrt{20} + \sqrt{41}$ or about 17.9 units.

Study Tip

Linear and Square Units Remember to use linear units with perimeter and square units with area.

Step 2 Find the area of △*PQR*.

To find the area of the triangle, find the lengths of the height and base. The height is the perpendicular distance from *P* to $\overline{QR}$. Counting squares on the graph, the height is 4 units. The length of $\overline{QR}$ is 7 units.

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(4)$ or 14 Substitute and simplify.

The area of △*PQR* is 14 square units.

Guided Practice

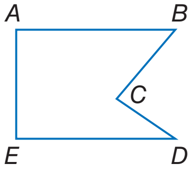
4. Find the perimeter and area of △*ABC* with vertices *A*(−1, 4), *B*(−1, −1), and *C*(6, −1).

Check Your Understanding

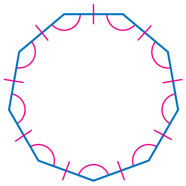
= Step-by-Step Solutions begin on page R14.

Example 1 **Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.**

1.



2.

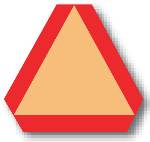


SIGNS **Identify the shape of each traffic sign and classify it as *regular* or *irregular*.**

3. stop

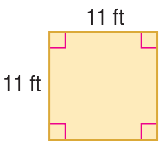
4. caution or warning

5. slow moving vehicle

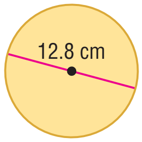


Example 2 **Find the perimeter or circumference and area of each figure. Round to the nearest tenth.**

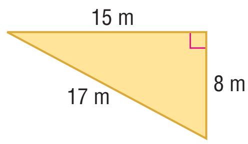
6.



7.



8.



Example 3

9. MULTIPLE CHOICE Vanesa is making a banner for the game. She has 20 square feet of fabric. What shape will use *most* or all of the fabric?

A a square with a side length of 4 feet

B a rectangle with a length of 4 feet and a width of 3.5 feet

C a circle with a radius of about 2.5 feet

D a right triangle with legs of about 5 feet each

Example 4

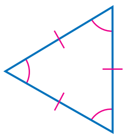
10. CCSS **REASONING** Find the perimeter and area of $\triangle ABC$ with vertices $A(-1, 2)$, $B(3, 6)$, and $C(3, -2)$.

Practice and Problem Solving

Extra Practice is on page R1.

Example 1 **Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.**

11.



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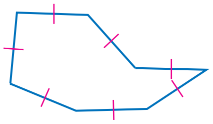

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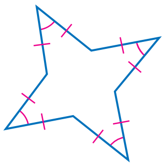

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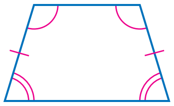

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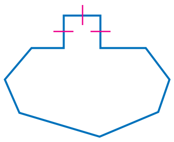

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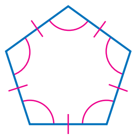
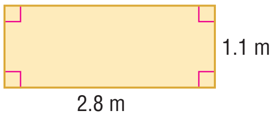
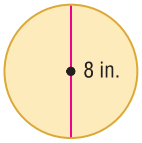


Examples 2–3 **Find the perimeter or circumference and area of each figure. Round to the nearest tenth.**

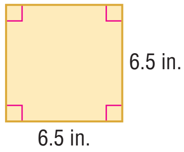
17.



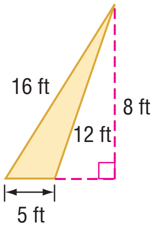
18.



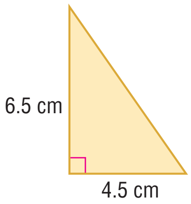
19.



20.



21



22.

23. **CRAFTS** Joy has a square picture that is 4 inches on each side. The picture is framed with a length of ribbon. She wants to use the same piece of ribbon to frame a circular picture. What is the maximum radius of the circular frame?

24. **LANDSCAPING** Mr. Jackson has a circular garden with a diameter of 10 feet surrounded by edging. Using the same length of edging, he is going to create a square garden. What is the maximum side length of the square?

Example 4 CCSS **REASONING** **Graph each figure with the given vertices and identify the figure. Then find the perimeter and area of the figure.**

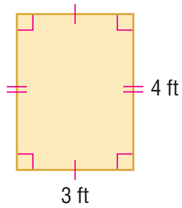
25. $D(-2, -2)$, $E(-2, 3)$, $F(2, -1)$

26. $J(-3, -3)$, $K(3, 2)$, $L(3, -3)$

27. $P(-1, 1)$, $Q(3, 4)$, $R(6, 0)$, $S(2, -3)$

28. $T(-2, 3)$, $U(1, 6)$, $V(5, 2)$, $W(2, -1)$

29. **CHANGING DIMENSIONS** Use the rectangle at the right.



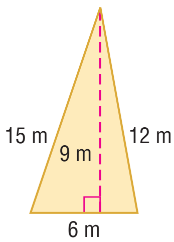
a. Find the perimeter of the rectangle.

b. Find the area of the rectangle.

c. Suppose the length and width of the rectangle are doubled. What effect would this have on the perimeter? the area? Justify your answer.

d. Suppose the length and width of the rectangle are halved. What effect does this have on the perimeter? the area? Justify your answer.

30. **CHANGING DIMENSIONS** Use the triangle at the right.



a. Find the perimeter of the triangle.

b. Find the area of the triangle.

c. Suppose the side lengths and height of the triangle were doubled. What effect would this have on the perimeter? the area? Justify your answer.

d. Suppose the side lengths and height of the triangle were divided by three. What effect would this have on the perimeter? the area? Justify your answer.

31. **ALGEBRA** A rectangle of area 360 square yards is 10 times as long as it is wide. Find its length and width.

32. **ALGEBRA** A rectangle of area 350 square feet is 14 times as wide as it is long. Find its length and width.

33. **DISC GOLF** The diameter of the most popular brand of flying disc used in disc golf measures between 8 and 10 inches. Find the range of possible circumferences and areas for these flying discs to the nearest tenth.

ALGEBRA **Find the perimeter or circumference for each figure described.**

34. The area of a square is 36 square units.

35. The length of a rectangle is half the width. The area is 25 square meters.

36. The area of a circle is 25π square units.

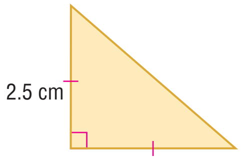
37. The area of a circle is 32π square units.

38. A rectangle's length is 3 times its width. The area is 27 square inches.

39. A rectangle's length is twice its width. The area is 48 square inches.

CCSS PRECISION **Find the perimeter and area of each figure in inches. Round to the nearest hundredth, if necessary.**

40.



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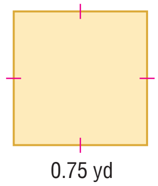
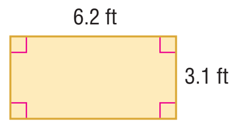


42.



43. **MULTIPLE REPRESENTATIONS** Collect and measure the diameter and circumference of ten round objects using a millimeter measuring tape.

Object	d	C	$\frac{C}{d}$
1			
2			
3			
⋮			
10			

a. **Tabular** Record the measures in a table as shown.

b. **Algebraic** Compute the value of $\frac{C}{d}$ to the nearest hundredth for each object and record the result.

c. **Graphical** Make a scatter plot of the data with d-values on the horizontal axis and C-values on the vertical axis.

d. **Verbal** Find an equation for a line of best fit for the data. What does this equation represent? What does the slope of the line represent?

H.O.T. Problems Use Higher-Order Thinking Skills

44. **WHICH ONE DOESN'T BELONG?** Identify the term that does not belong with the other three. Explain your reasoning.

square | circle | triangle | pentagon

45. **CHALLENGE** The vertices of a rectangle with side lengths of 10 and 24 units are on a circle of radius 13 units. Find the area between the figures.

46. **REASONING** Name a polygon that is always regular and a polygon that is sometimes regular. Explain your reasoning.

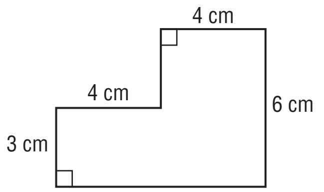
47. **OPEN ENDED** Draw a pentagon. Is your pentagon *convex* or *concave*? Is your pentagon *regular* or *irregular*? Justify your answers.

48. **CHALLENGE** A rectangular room measures 20 feet by 12.5 feet. How many 5-inch square tiles will it take to cover the floor of this room? Explain.

49. **WRITING IN MATH** Describe two possible ways that a polygon can be equiangular but not a regular polygon.

Standardized Test Practice

50. Find the perimeter of the figure.



A 17 cm **C** 28 cm
B 25 cm **D** 31 cm

51. PROBABILITY In three successive rolls of a fair number cube, Matt rolls a 6. What is the probability of Matt rolling a 6 if the number cube is rolled a fourth time?

F $\frac{1}{6}$ **H** $\frac{1}{3}$
G $\frac{1}{4}$ **J** 1

52. SHORT RESPONSE Miguel is planning a party for 80 guests. According to the pattern in the table, how many gallons of ice cream should Miguel buy?

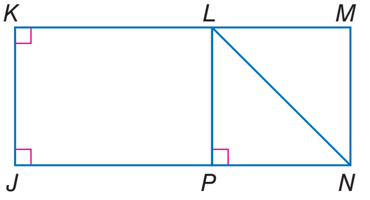
Number of Guests	Gallons of Ice Cream
8	2
16	4
24	6
32	8

53. SAT/ACT A frame 2 inches wide surrounds a painting that is 18 inches wide and 14 inches tall. What is the area of the frame?

A 68 in^2 **D** 252 in^2
B 84 in^2 **E** 396 in^2
C 144 in^2

Spiral Review

Determine whether each statement can be assumed from the figure. Explain. (Lesson 1-5)



54. $\angle KJN$ is a right angle.

55. $\angle PLN \cong \angle NLM$

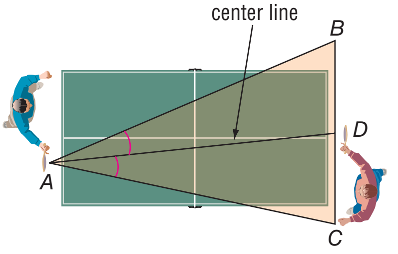
56. $\angle PNL$ and $\angle MNL$ are complementary.

57. $\angle KLN$ and $\angle MLN$ are supplementary.

58. TABLE TENNIS The diagram shows the angle of play for a table tennis player. If a right-handed player has a strong forehand, he should stand to the left of the center line of his opponent's angle of play. (Lesson 1-4)

a. What geometric term describes the center line?

b. If the angle of play shown in the diagram measures 43°, what is $m\angle BAD$?



Name an appropriate method to solve each system of equations. Then solve the system. (Lesson 0-8)

59. $-5x + 2y = 13$
$2x + 3y = -9$

60. $y = -5x + 7$
$y = 3x - 17$

61. $x - 8y = 16$
$7x - 4y = -18$

Skills Review

Evaluate each expression if $P = 10$, $B = 12$, $h = 6$, $r = 3$, and $\ell = 5$. Round to the nearest tenth, if necessary.

62. $\frac{1}{2}P\ell + B$ **63.** $\frac{1}{3}Bh$ **64.** $\frac{1}{3}\pi r^2 h$ **65.** $2\pi rh + 2\pi r^2$

EXTEND 1-6

Geometry Software Lab
Two-Dimensional Figures

You can use The Geometer's Sketchpad® to draw and investigate polygons.

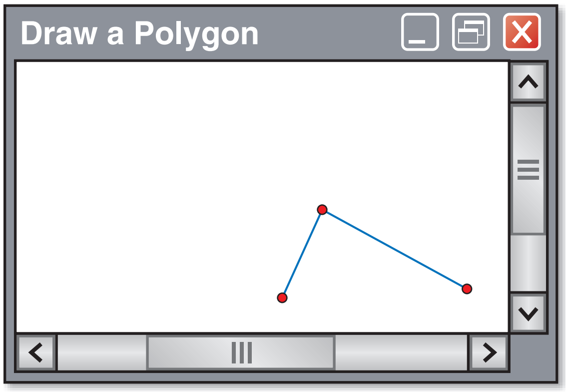
Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Activity 1 Draw a Polygon

Draw $\triangle XYZ$.

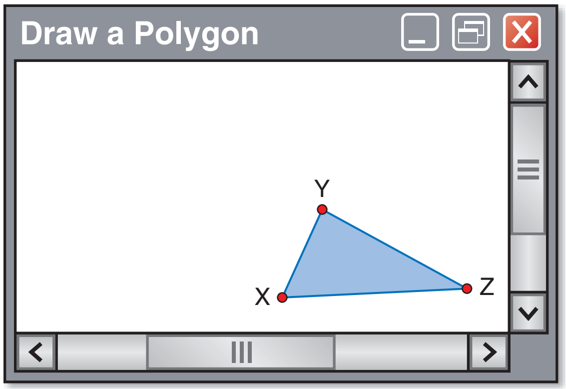
Step 1 Select the segment tool from the toolbar, and click to set the first endpoint X of side $\overline{XY}$. Then drag the cursor, and click again to set the other endpoint Y.

Step 2 Click on point Y to set the endpoint of $\overline{YZ}$. Drag the cursor and click to set point Z.



Step 3 Click on point Z to set the endpoint of $\overline{ZX}$. Then move the cursor to highlight point X. Click on X to draw $\overline{ZX}$.

Step 4 Use the pointer tool to click on points X, Y, and Z. Under the **Display** menu, select **Show Labels** to label the vertices of your triangle.



Activity 2 Measure Sides

Find XY, YZ, and ZX.

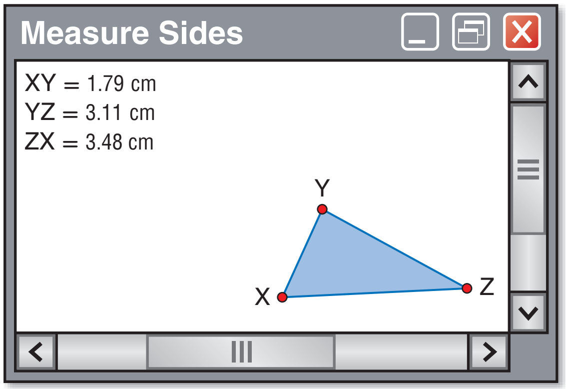
Step 1 Use the pointer tool to select $\overline{XY}$, $\overline{YZ}$, and $\overline{ZX}$.

Step 2 Select the **Length** command under the **Measure** menu to display the lengths of $\overline{XY}$, $\overline{YZ}$, and $\overline{ZX}$.

$XY = 1.79$ cm

$YZ = 3.11$ cm

$ZX = 3.48$ cm



(continued on the next page)

Geometry Software Lab
Two-Dimensional Figures *Continued*

Activity 3 Find Perimeter

Find the perimeter of $\triangle XYZ$.

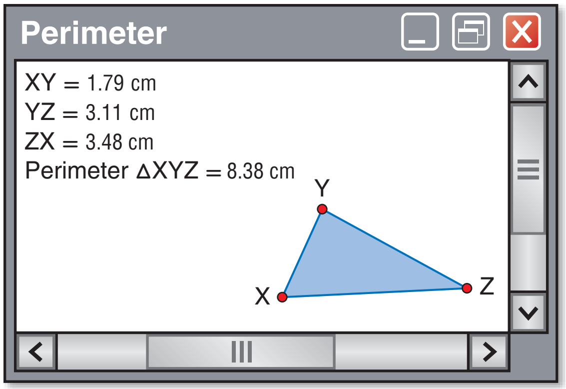
Step 1 Use the pointer tool to select points X, Y, and Z.

Step 2 Under the **Construct** menu, select **Triangle Interior**. The triangle will now be shaded.

Step 3 Select the triangle interior using the pointer.

Step 4 Choose the **Perimeter** command under the **Measure** menu to find the perimeter of $\triangle XYZ$.

The perimeter of $\triangle XYZ$ is 8.38 centimeters.



Activity 4 Measure Angles

Find $m\angle X$, $m\angle Y$, and $m\angle Z$.

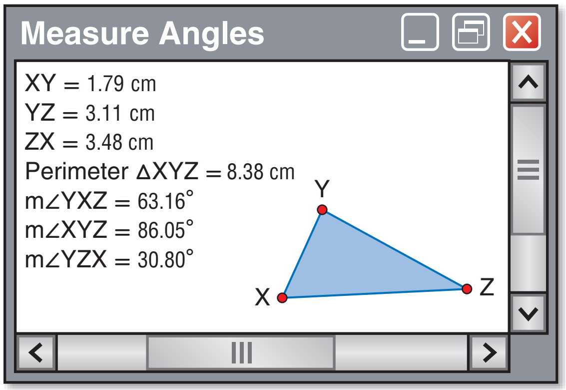
Step 1 Recall that $\angle X$ can also be named $\angle YXZ$ or $\angle ZXY$. Use the pointer to select points Y, X, and Z in order.

Step 2 Select the **Angle** command from the **Measure** menu to find $m\angle X$.

Step 3 Select points X, Y, and Z. Find $m\angle Y$.

Step 4 Select points X, Z, and Y. Find $m\angle Z$.

$m\angle X = 63.16$, $m\angle Y = 86.05$, and $m\angle Z = 30.8$.



Analyze the Results

1. Add the side measures from Activity 2. How does this compare to the result in Activity 3?
2. What is the sum of the angle measures of $\triangle XYZ$?
3. Repeat the activities for each figure.
   a. irregular quadrilateral **b.** square **c.** pentagon **d.** hexagon
4. Draw another quadrilateral and find its perimeter. Then enlarge your figure using the **Dilate** command. How does changing the sides affect the perimeter?
5. Compare your results with those of your classmates.
6. Make a conjecture about the sum of the measures of the angles in any triangle.
7. What is the sum of the measures of the angles of a quadrilateral? pentagon? hexagon?
8. How are the sums of the angles of polygons related to the number of sides?
9. Test your conjecture on other polygons. Does your conjecture hold? Explain.
10. When the sides of a polygon are changed by a common factor, does the perimeter of the polygon change by the same factor as the sides? Explain.

LESSON 1-7 Three-Dimensional Figures

Then
- You identified and named two-dimensional figures.

Now
1. Identify and name three-dimensional figures.
2. Find surface area and volume.

Why?
- Architects often provide three-dimensional models of their ideas to clients. These models give their clients a better idea of what the completed structure will look like than a two-dimensional drawing. Three-dimensional figures, or *solids*, are made up of flat or curved surfaces.

New Vocabulary
polyhedron
face
edge
vertex
prism
base
pyramid
cylinder
cone
sphere
regular polyhedron
Platonic solid
surface area
volume

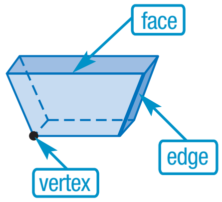
Common Core State Standards

Content Standards
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

Mathematical Practices
2 Reason abstractly and quantitatively.
6 Attend to precision.

1 Identify Three-Dimensional Figures

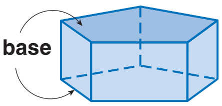
A solid with all flat surfaces that enclose a single region of space is called a **polyhedron**. Each flat surface or **face** is a polygon. The line segments where the faces intersect are called **edges**. The point where three or more edges intersect is called a **vertex**. Below are examples and definitions of polyhedrons and other types of solids.



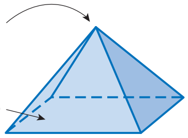
Key Concept Types of Solids

Polyhedrons

A **prism** is a polyhedron with two parallel congruent faces called **bases** connected by parallelogram faces.

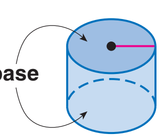


A **pyramid** is a polyhedron that has a polygonal base and three or more triangular faces that meet at a common vertex.

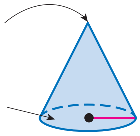


Not Polyhedrons

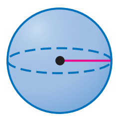
A **cylinder** is a solid with congruent parallel circular bases connected by a curved surface.



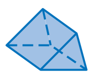
A **cone** is a solid with a circular base connected by a curved surface to a single vertex.

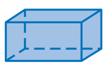


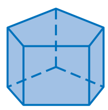
A **sphere** is a set of points in space that are the same distance from a given point. A sphere has no faces, edges, or vertices.

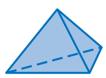


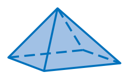
Polyhedrons or *polyhedra* are named by the shape of their bases.

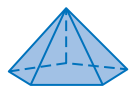

triangular prism


rectangular prism


pentagonal prism


triangular pyramid

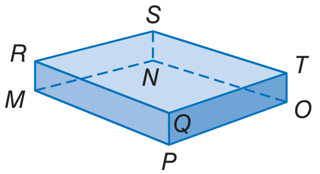

rectangular pyramid


pentagonal pyramid

Example 1 Identify Solids

Determine whether each solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

a.



The solid is formed by polygonal faces, so it is a polyhedron. There are two parallel congruent rectangular bases, so it is a rectangular prism.

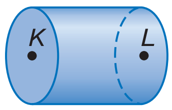
Bases: ▭*MNOP*, ▭*RSTQ*

Faces: ▭*RQPM*, ▭*RSNM*, ▭*STON*, ▭*QTOP*, ▭*RSTQ*, ▭*MNOP*

Edges: $\overline{MN}$, $\overline{NO}$, $\overline{OP}$, $\overline{PM}$, $\overline{RS}$, $\overline{ST}$, $\overline{TQ}$, $\overline{QR}$, $\overline{RM}$, $\overline{SN}$, $\overline{TO}$, $\overline{QP}$

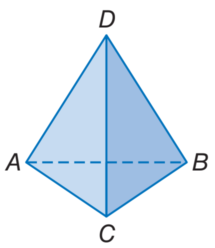
Vertices: *M*, *N*, *O*, *P*, *Q*, *R*, *S*, *T*

b.



The solid has a curved surface, so it is not a polyhedron. It has two congruent circular bases, so it is a cylinder.

c.



The solid is formed by polygonal faces, so it is a polyhedron. The base is a triangle, and the three faces meet in a vertex, so it is a triangular pyramid.

Bases: △*ABC*

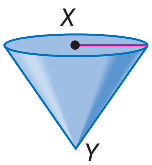
Faces: △*ABC*, △*ADC*, △*CDB*, △*BDA*

Edges: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$, $\overline{DA}$, $\overline{DB}$, $\overline{DC}$

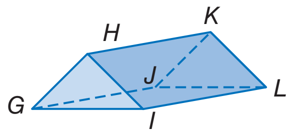
Vertices: *A*, *B*, *C*, *D*

GuidedPractice

1A.



1B.



ReadingMath

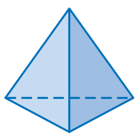
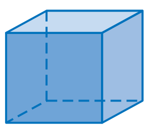
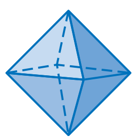
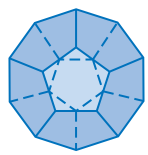
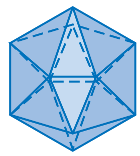
Symbols Symbols can be used in naming the focus of polyhedra. The symbol ▭ means rectangle. The symbol △ means triangle. The symbol ⊙ means circle.

Math HistoryLink

Plato (427–347 B.C.) Plato, a philosopher, mathematician, and scientist, lived in Athens, Greece. He is best known for founding a school known as "The Academy." In mathematics, he was concerned with the idea of proofs, and he insisted that definitions must be accurate and hypotheses must be clear.

A polyhedron is a **regular polyhedron** if all of its faces are regular congruent polygons and all of the edges are congruent. There are exactly five types of regular polyhedrons, called **Platonic Solids** because Plato used them extensively.

KeyConcept Platonic Solids

Tetrahedron	Hexahedron or Cube	Octahedron	Dodecahedron	Icosahedron
				
4 equilateral triangle faces	6 square faces	8 equilateral triangular faces	12 regular pentagonal faces	20 equilateral triangular faces

Study Tip

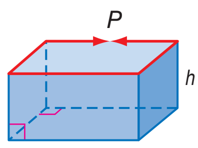
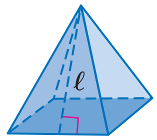
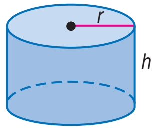
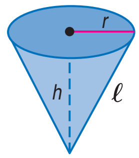
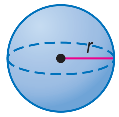
Euclidean Solids The *Euclidean solids* include the cube, the pyramid, the cylinder, the cone, and the sphere.

2 Surface Area and Volume

Surface area is a two-dimensional measurement of the surface of a solid figure. The surface area of a polyhedron is the sum of the areas of each face. **Volume** is the measure of the amount of space enclosed by a solid figure.

Review the formulas for the surface area and volume of five common solids given below. You will derive these formulas in Chapter 12.

Key Concept Surface Area and Volume

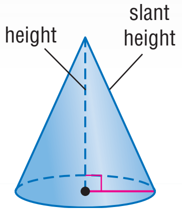
Prism	Regular Pyramid	Cylinder	Cone	Sphere
				
$S = Ph + 2B$	$S = \frac{1}{2}P\ell + B$	$S = 2\pi rh + 2\pi r^2$	$S = \pi r\ell + \pi r^2$	$S = 4\pi r^2$
$V = Bh$	$V = \frac{1}{3}Bh$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$

S = total surface area, V = volume, h = height of a solid

P = perimeter of the base, B = area of base, ℓ = slant height, r = radius

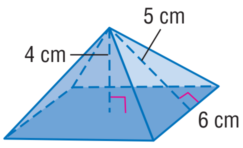
Watch Out!

Height vs. Slant Height The *height* of a pyramid or cone is not the same as its *slant height.*



Example 2 Find Surface Area and Volume

Find the surface area and volume of the square pyramid.



Surface Area

Since the base of the pyramid is a square, the perimeter P of the base is 4 • 6 or 24 centimeters. The area of the base B is 6 • 6 or 36 square centimeters. The slant height is 5 centimeters.

$S = \frac{1}{2}P\ell + B$ — Surface area of pyramid

$= \frac{1}{2}(24)(5) + 36$ or 96 — $P = 24$ cm, $\ell = 5$ cm, $B = 36$ cm^2

The surface area of the square pyramid is 96 square centimeters.

Volume

The height of the pyramid is 4 centimeters.

$V = \frac{1}{3}Bh$ — Volume of pyramid

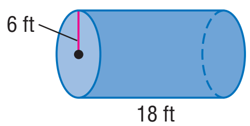
$= \frac{1}{3}(36)(4)$ or 48 — $B = 36$ cm^2, $h = 4$ cm

The volume is 48 cubic centimeters.

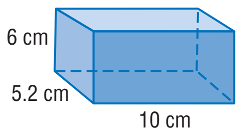
Guided Practice

Find the surface area and volume of each solid to the nearest tenth.

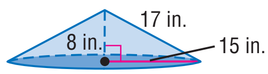
2A.



2B.



2C.

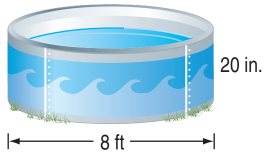


StudyTip

CCSS Precision Be sure that you have converted all units of measure to be consistent before you begin volume or surface area calculations.

Real-World Example 3 Surface Area and Volume

POOLS The diameter of the pool Mr. Sato purchased is 8 feet. The height of the pool is 20 inches. Find each measure to the nearest tenth.



a. surface area of the pool

The pool is a cylinder.

$A = 2\pi rh + \pi r^2$ — Surface area of cylinder with one base

$= 2\pi(4)\left(1\frac{2}{3}\right) + \pi(4)^2$ — $r = 4$ ft, $h = 20$ in. or $1\frac{2}{3}$ ft

≈ 92.2 — Use a calculator.

The surface area of the pool is about 92.2 square feet.

b. the volume of water needed to fill the pool to a depth of 16 inches

$V = \pi r^2 h$ — Volume of cylinder

$= \pi(4)^2\left(1\frac{1}{3}\right)$ — $r = 4$ ft, $h = 16$ in. or $1\frac{1}{3}$ ft

≈ 67.0 — Use a calculator.

The volume of water needed is approximately 67.0 cubic feet.

Guided Practice

3. CRAFTS Jessica is making spherical candles using a mold that is 10 centimeters in diameter. Find each measure to the nearest tenth.

A. the volume of wax needed to fill the mold

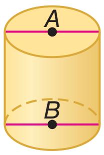
B. the surface area of the finished candle

Check Your Understanding

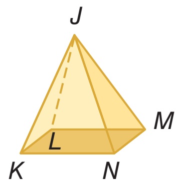
● = Step-by-Step Solutions begin on page R14.

Example 1 **Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.**

1.

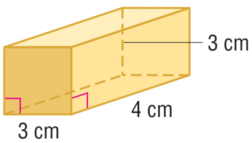


2.

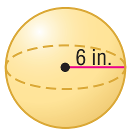


Example 2 **Find the surface area and volume of each solid to the nearest tenth.**

3



4.



Example 3

5. PARTY FAVORS Lawana is making cone-shaped hats 4 inches in diameter, 6.5 inches tall, with a slant height of 6.8 inches for party favors. Find each measure to the nearest tenth.

a. the volume of candy that will fill each cone

b. the area of material needed to make each hat assuming there is no overlap of material

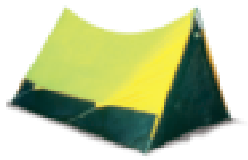
Practice and Problem Solving

Extra Practice is on page R1.

Example 1 Identify the solid modeled by each object. State whether the solid modeled is a polyhedron.

6. 

7. 

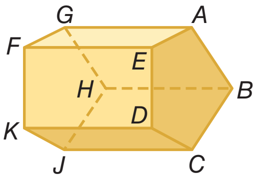
8. 

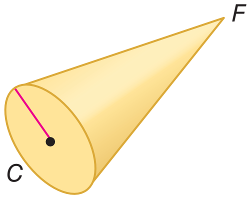
9. 

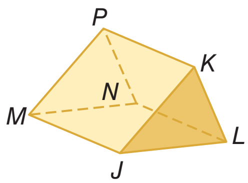
10. 

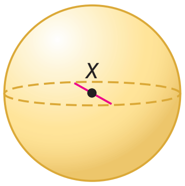
11. 

CCSS STRUCTURE Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

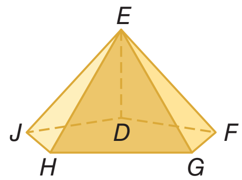
12. 

13. 

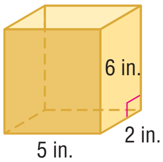
14. 

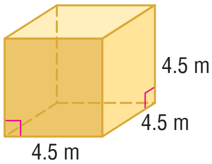
15. 

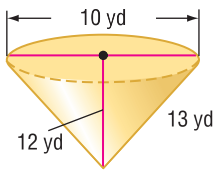
16. 

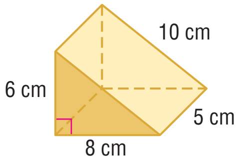
17. 

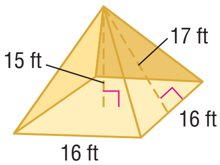
Example 2 Find the surface area and volume of each solid to the nearest tenth.

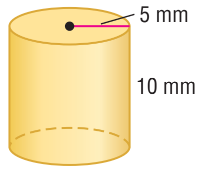
18. 

19. 

20. 

21 

22. 

23. 

Example 3 **24. SANDBOX** A rectangular sandbox is 3 feet by 4 feet. The depth of the box is 8 inches, but the depth of the sand is $\frac{3}{4}$ of the depth of the box. Find each measure to the nearest tenth.

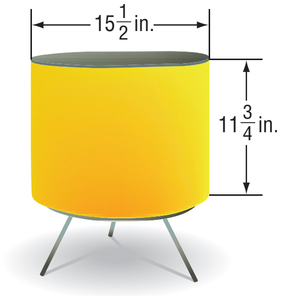
a. the surface area of the sandbox assuming there is no lid

b. the volume of sand in the sandbox

25. **ART** Fernando and Humberto Campana designed the Inflating Table shown. The diameter of the table is $15\frac{1}{2}$ inches. Suppose the height of the cylinder is $11\frac{3}{4}$ inches. Find each measure to the nearest tenth. Assume that the sides of the table are perpendicular to the bases of the table.

a. the volume of air that will fully inflate the table

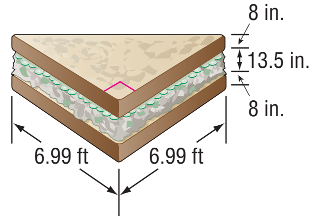
b. the surface area of the table when fully inflated



26. **CCSS SENSE-MAKING** In 1999, Marks & Spencer, a British department store, created the biggest sandwich ever made. The tuna and cucumber sandwich was in the form of a triangular prism. Suppose each slice of bread was 8 inches thick. Find each measure to the nearest tenth.

a. the surface area in square feet of the sandwich when filled

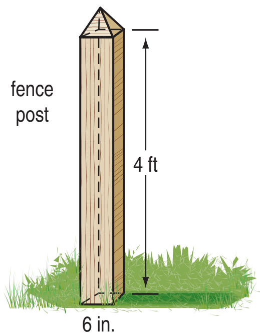
b. the volume of filling in cubic feet to the nearest tenth



27. **ALGEBRA** The surface area of a cube is 54 square inches. Find the length of each edge.

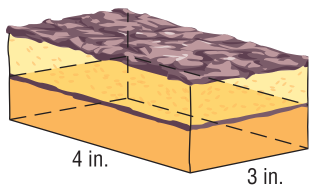
28. **ALGEBRA** The volume of a cube is 729 cubic centimeters. Find the length of each edge.

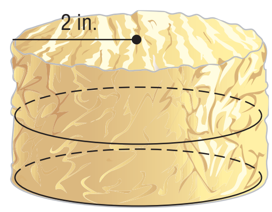
29. **PAINTING** Tara is painting her family's fence. Each post is composed of a square prism and a square pyramid. The height of the pyramid is 4 inches. Determine the surface area and volume of each post.



30. **COLLECT DATA** Use a ruler or tape measure and what you have learned in this lesson to find the surface area and volume of a soup can.

31. **CAKES** Cakes come in many shapes and sizes. Often they are stacked in two or more layers, like those in the diagrams shown below.





a. If each layer of the rectangular prism cake is 3 inches high, calculate the area of the cake that will be frosted assuming there is no frosting between layers.

b. Calculate the area of the cylindrical cake that will be frosted, if each layer is 4 inches in height.

c. If one can of frosting will cover 50 square inches of cake, how many cans of frosting will be needed for each cake?

d. If the height of each layer of cake is 5 inches, what does the radius of the cylindrical cake need to be, so the same amount of frosting is used for both cakes? Explain your reasoning.

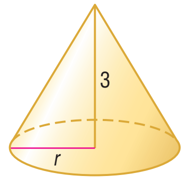
32. **CHANGING UNITS** A gift box has a surface area of 6.25 square feet. What is the surface area of the box in square inches?

33. **CHANGING UNITS** A square pyramid has a volume of 4320 cubic inches. What is the volume of this pyramid in cubic feet?

34. **EULER'S FORMULA** The number of faces F, vertices V, and edges E of a polyhedron are related by Euler's (OY luhrz) Formula: $F + V = E + 2$. Determine whether Euler's Formula is true for each of the figures in Exercises 18–23.

35. **CHANGING DIMENSIONS** A rectangular prism has a length of 12 centimeters, width of 18 centimeters, and height of 22 centimeters. Describe the effect on the volume of a rectangular prism when each dimension is doubled.

36. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate how changing the length of the radius of a cone affects the cone's volume.



a. **Tabular** Create a table showing the volume of a cone when doubling the radius. Use radius values between 1 and 8.

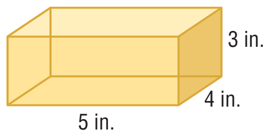
b. **Graphical** Use the values from your table to create a graph of radius versus volume.

c. **Verbal** Make a conjecture about the effect of doubling the radius of a cone on the volume. Explain your reasoning.

d. **Algebraic** If r is the radius of a cone, write an expression showing the effect doubling the radius has on the cone's volume.

H.O.T. Problems Use Higher-Order Thinking Skills

37. CCSS **CRITIQUE** Alex and Emily are calculating the surface area of the rectangular prism shown. Is either of them correct? Explain your reasoning.



Alex	Emily
$(5 \cdot 3) \cdot 6$ faces	$2(5 \cdot 4 \cdot 3)$
$= 90 \text{ in}^2$	$= 120 \text{ in}^2$

38. **REASONING** Is a cube a regular polyhedron? Explain.

39. **CHALLENGE** Describe the solid that results if the number of sides of each base increases infinitely. The bases of each solid are regular polygons inscribed in a circle.

a. pyramid **b.** prism

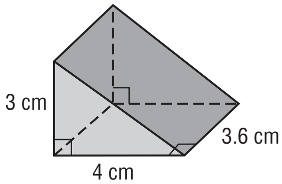
40. **OPEN ENDED** Draw an irregular 14-sided polyhedron which has two congruent bases.

41. **CHALLENGE** Find the volume of a cube that has a total surface area of 54 square millimeters.

42. **WRITING IN MATH** A reference sheet listed the formula for the surface area of a prism as $SA = Bh + 2B$. Use units of measure to explain why there must be a typographical error in this formula.

Standardized Test Practice

43. GRIDDED RESPONSE What is the surface area of the triangular prism in square centimeters?



44. ALGEBRA What is the value of $(-0.8)^2 + (-0.3)^3$?

A 0.627
B 0.613
C 0.370
D 0.327

45. The length of each side of a cube is multiplied by 5. What is the change in the volume of the cube?

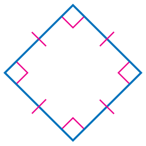
F The volume is 125 times the original volume.
G The volume is 25 times the original volume.
H The volume is 10 times the original volume.
J The volume is 5 times the original volume.

46. SAT/ACT What is the difference in surface area between a cube with an edge length of 7 inches and a cube with edge length of 4 inches?

A 18 in^2
B 33 in^2
C 66 in^2
D 99 in^2
E 198 in^2

Spiral Review

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*. (Lesson 1-6)

47.


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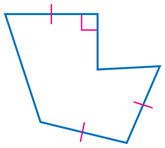

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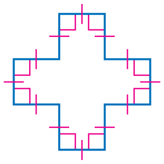

Find the value of each variable. (Lesson 1-5)

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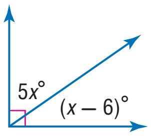

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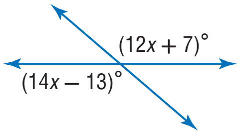

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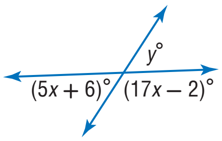
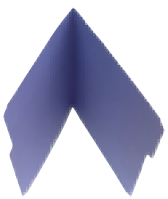


GAMES **What type of geometric intersection is modeled in each photograph?** (Lesson 1-1)

53.


54.


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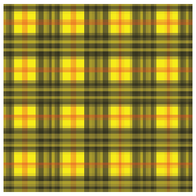
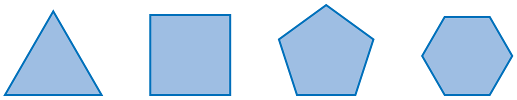


Skills Review

Sketch the next two figures in each pattern.

56.


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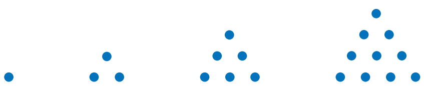

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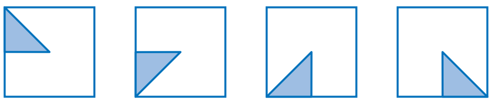

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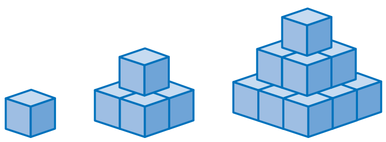
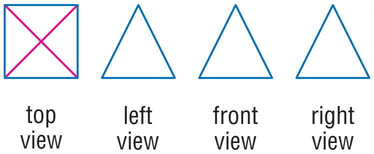


EXTEND 1-7

Geometry Lab
Two-Dimensional Representations of Three-Dimensional Objects

If you see a three-dimensional object from only one viewpoint, you may not know its true shape. Here are four views of a square pyramid.

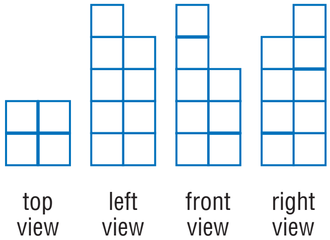
The two-dimensional views of the top, left, front, and right sides of an object are called an orthographic drawing.



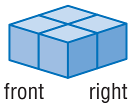
Common Core State Standards
Content Standards
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★
Mathematical Practices 5

Activity 1

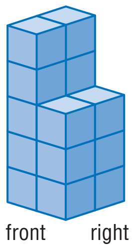
Make a model of a figure for the orthographic drawing shown.



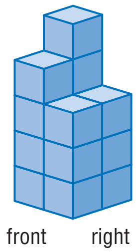
Step 1 Start with a base that matches the top view.



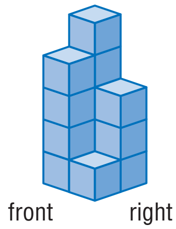
Step 2 The front view indicates that the front left side is 5 blocks high and that the right side is 3 blocks high. However, the dark segments indicate breaks in the surface.



Step 3 The break on the left side of the front view indicates that the back left column is 5 blocks high, but that the front left column is only 4 blocks high, so remove 1 block from the front left column.



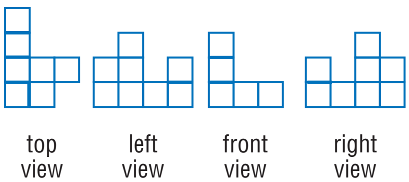
Step 4 The break on the right side of the front view indicates that the back right column is 3 blocks high, but that the front right column is only 1 block high, so remove 2 blocks from the front right column.



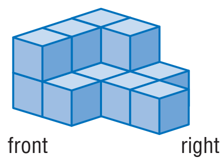
Step 5 Use the left and right views and the breaks in those views to confirm that you have made the correct figure.

Model and Analyze

1. Make a model of a figure for the orthographic drawing shown.

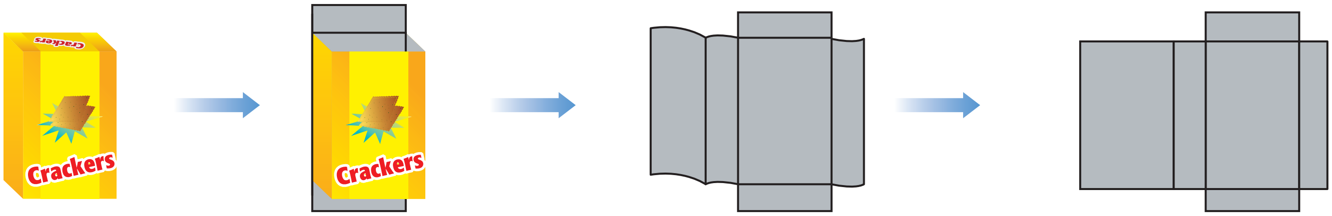


2. Make an orthographic drawing of the figure shown.



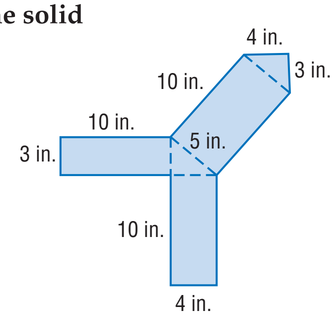
Geometry Lab
Two-Dimensional Representations of Three-Dimensional Objects *Continued*

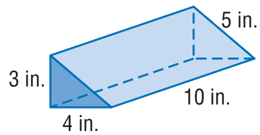
If you cut a cardboard box at the edges and lay it flat, you will have a two-dimensional diagram called a **net** that you can fold to form a three-dimensional solid.



Activity 2

Make a model of a figure for the given net. Then identify the solid formed, and find its surface area.





Use a large sheet of paper, a ruler, scissors, and tape. Draw the net on the paper. Cut along the solid lines. Fold the pattern on the dashed lines and secure the edges with tape. This is the net of a triangular prism.

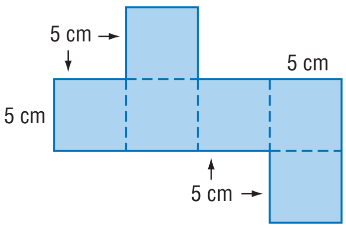
Use the net to find the surface area S.

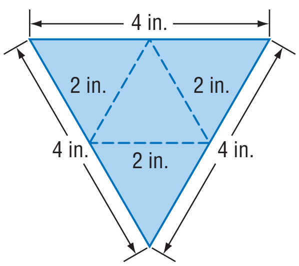
$S = 2\left[\frac{1}{2}(4)(3)\right] + 4(10) + 3(10) + 5(10)$ Area of two congruent triangles plus area of three rectangles

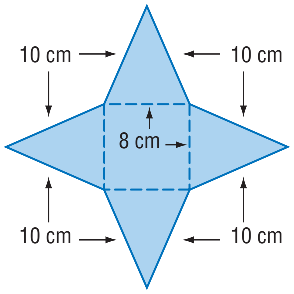
$= 12 + 40 + 30 + 50 \text{ or } 132 \text{ in}^2$ Simplify.

Model and Analyze

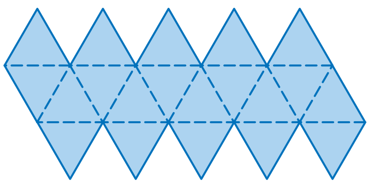
Make a model of a figure for each net. Then identify the solid formed and find its surface area. If the solid has more than one name, list both.

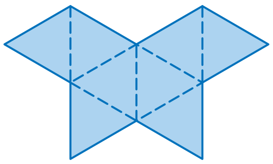
3. 

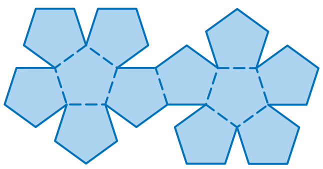
4. 

5. 

Identify the Platonic Solid that can be formed by the given net.

6. 

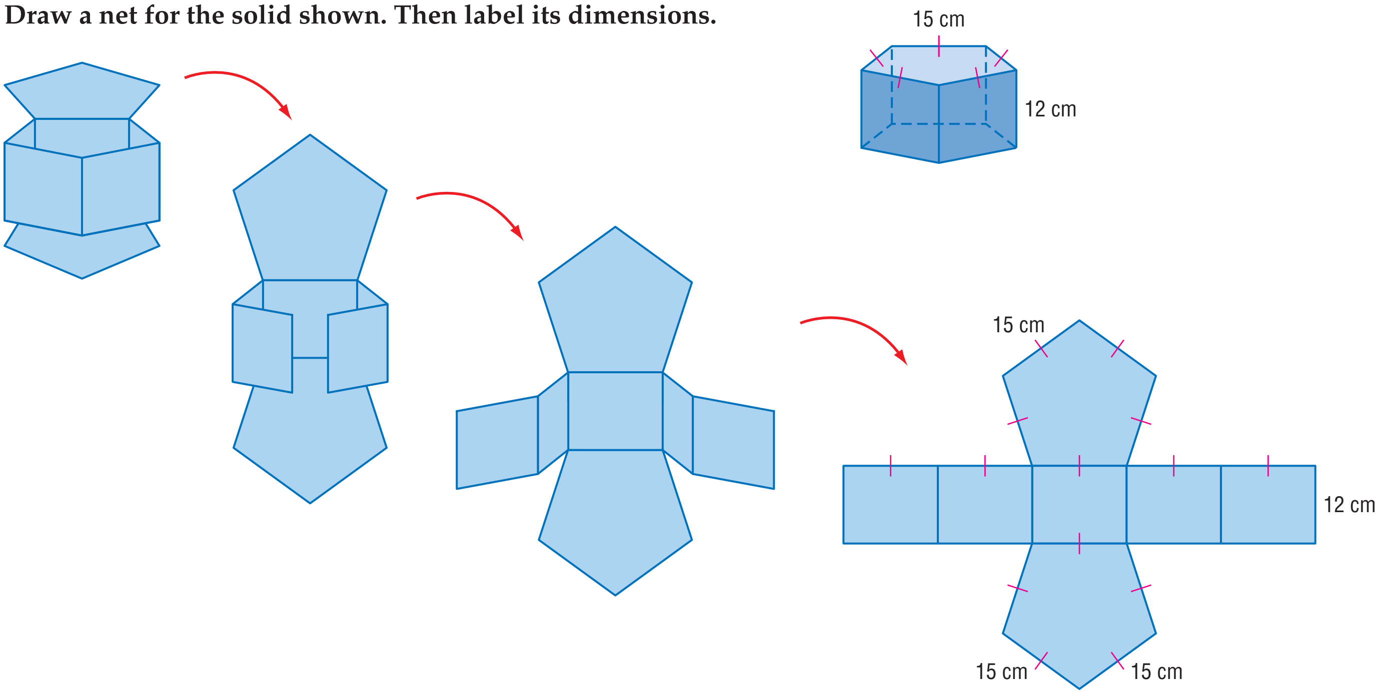
7. 

8. 

To draw the net of a three-dimensional solid, visualize cutting the solid along one or more of its edges, opening up the solid, and flattening it completely.

Activity 3

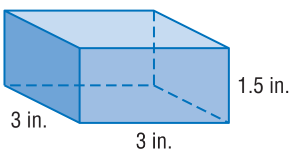
Draw a net for the solid shown. Then label its dimensions.



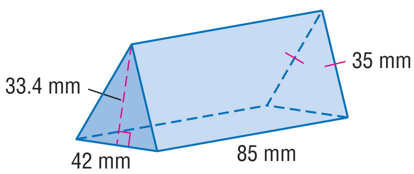
Model and Analyze

Draw a net for each solid. Then label its dimensions.

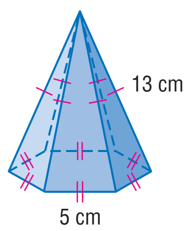
9.



10.



11.



12. **PACKAGING** A can of pineapple is shown.

a. What shape are the top and bottom of the can?

b. If you remove the top and bottom and then make a vertical cut down the side of the can, what shape will you get when you uncurl the remaining body of the can and flatten it?

c. If the diameter of the can is 3 inches and its height is 2 inches, draw a net of the can and label its dimensions. Explain your reasoning.

CHAPTER 1

Study Guide and Review

Study Guide

KeyConcepts

Points, Lines, and Planes (Lesson 1-1)

- There is exactly one line through any two points.
- There is exactly one plane through any three noncollinear points.

Distance and Midpoints (Lesson 1-3)

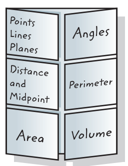
- On a number line, the measure of a segment with endpoint coordinates a and b is $|a - b|$.
- In the coordinate plane, the distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- On a number line, the coordinate of the midpoint of a segment with endpoints a and b is $\frac{a + b}{2}$.
- In the coordinate plane, the coordinates of the midpoint of a segment with endpoints that are (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Angles (Lessons 1-4 and 1-5)

- An angle is formed by two noncollinear rays that have a common endpoint, called its vertex. Angles can be classified by their measures.
- Adjacent angles are two coplanar angles that lie in the same plane and have a common vertex and a common side but no common interior points.
- Vertical angles are two nonadjacent angles formed by two intersecting lines.
- A linear pair is a pair of adjacent angles with noncommon sides that are opposite rays.
- Complementary angles are two angles with measures that have a sum of 90.
- Supplementary angles are two angles with measures that have a sum of 180.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

- **acute angle** (p. 38)
- **adjacent angles** (p. 46)
- **angle** (p. 36)
- **angle bisector** (p. 39)
- **area** (p. 58)
- **base** (p. 67)
- **between** (p. 15)
- **circumference** (p. 58)
- **collinear** (p. 5)
- **complementary angles** (p. 47)
- **concave** (p. 56)
- **cone** (p. 67)
- **congruent** (p. 16)
- **construction** (p. 17)
- **convex** (p. 56)
- **coplanar** (p. 5)
- **cylinder** (p. 67)
- **degree** (p. 37)
- **distance** (p. 25)
- **edge** (p. 67)
- **equiangular polygon** (p. 57)
- **equilateral polygon** (p. 57)
- **exterior** (p. 36)
- **face** (p. 67)
- **interior** (p. 36)
- **intersection** (p. 6)
- **line** (p. 5)
- **line segment** (p. 14)
- **linear pair** (p. 46)
- **midpoint** (p. 27)
- ***n*-gon** (p. 57)
- **obtuse angle** (p. 38)
- **opposite rays** (p. 36)
- **perimeter** (p. 58)
- **perpendicular** (p. 48)
- **plane** (p. 5)
- **Platonic solid** (p. 68)
- **point** (p. 5)
- **polygon** (p. 56)
- **polyhedron** (p. 67)
- **prism** (p. 67)
- **pyramid** (p. 67)
- **ray** (p. 36)
- **regular polygon** (p. 57)
- **regular polyhedron** (p. 68)
- **right angle** (p. 38)
- **segment bisector** (p. 29)
- **side** (p. 36)
- **space** (p. 7)
- **sphere** (p. 67)
- **supplementary angles** (p. 47)
- **surface area** (p. 69)
- **undefined term** (p. 5)
- **vertex** (pp. 36, 67)
- **vertex of a polygon** (p. 56)
- **vertical angles** (p. 46)
- **volume** (p. 69)

VocabularyCheck

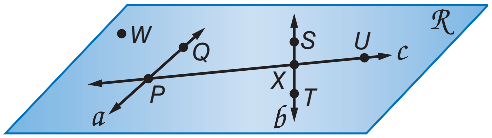
Fill in the blank in each sentence with the vocabulary term that best completes the sentence.

1. A _____ is a flat surface made up of points that extends infinitely in all directions.
2. A set of points that all lie on the same line are said to be _____.
3. If two lines intersect to form four right angles, the lines are _____.
4. If the sum of the measures of two angles is 180, then the angles are called _____ angles.

Lesson-by-Lesson Review

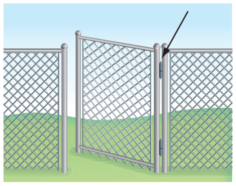
1-1 Points, Lines, and Planes

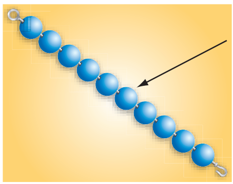
Use the figure to complete each of the following.



5. Name the intersection of lines *a* and *c*.
6. Give another name for line *b*.
7. Name a point that is not contained in any of the three lines *a*, *b*, or *c*.
8. Give another name for plane *WPX*.

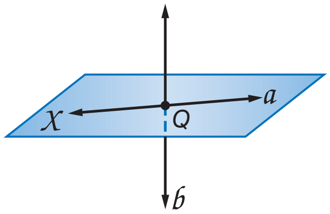
Name the geometric term that is best modeled by each item.

9. 

10. 

Example 1

Draw and label a figure for the relationship below.



Plane $\mathcal{X}$ contains line *a*, line *b* intersects line *a* at point *Q*, but line *b* is not in plane $\mathcal{X}$.

Draw a surface to represent plane $\mathcal{X}$ and label it.

Draw a line in plane $\mathcal{X}$ and label it line *a*.

Draw a line *b* intersecting both the plane and line *a* and label the point of intersection *Q*.

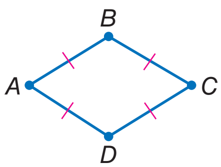
1-2 Linear Measure

Find the value of the variable and *XP*, if *X* is between *P* and *Q*.

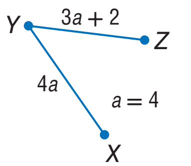
11. $XQ = 13$, $XP = 5x - 3$, $PQ = 40$
12. $XQ = 3k$, $XP = 7k - 2$, $PQ = 6k + 16$

Determine whether each pair of segments is congruent.

13. $\overline{AB}$, $\overline{CD}$



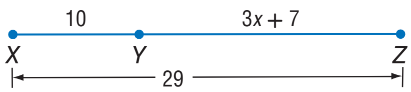
14. $\overline{XY}$, $\overline{YZ}$



15. **DISTANCE** The distance from Salvador's job to his house is 3 times greater than the distance from his house to school. If his house is between his job and school and the distance from his job to school is 6 miles, how far is it from Salvador's house to school?

Example 2

Use the figure to find the value of the variable and the length of $\overline{YZ}$.



$XZ = XY + YZ$	Betweenness of points
$29 = 10 + 3x + 7$	Substitution
$29 = 3x + 17$	Simplify.
$12 = 3x$	Subtract 17 from each side.
$4 = x$	Divide each side by 3.
$YZ = 3x + 7$	Given
$= 3(4) + 7$ or 19	Substitution

So, $x = 4$ and $YZ = 19$.

1-3 Distance and Midpoints

Find the distance between each pair of points.

16. $A(-3, 1)$, $B(7, 13)$

17. $P(2, -1)$, $Q(10, -7)$

Find the coordinates of the midpoint of a segment with the given endpoints.

18. $L(-3, 16)$, $M(17, 4)$

19. $C(32, -1)$, $D(0, -12)$

Find the coordinates of the missing endpoint if M is the midpoint of $\overline{XY}$.

20. $X(-11, -6)$, $M(15, 4)$

21. $M(-4, 8)$, $Y(19, 0)$

22. **HIKING** Carol and Marita are hiking in a state park and decide to take separate trails. The map of the park is set up on a coordinate grid. Carol's location is at the point (7, 13) and Marita is at (3, 5).

a. Find the distance between them.

b. Find the coordinates of the point midway between their locations.

Example 3

Find the distance between $X(5, 7)$ and $Y(-7, 2)$.

Let $(x_1, y_1) = (5, 7)$ and $(x_2, y_2) = (-7, 2)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-7 - 5)^2 + (2 - 7)^2} \\ &= \sqrt{(-12)^2 + (-5)^2} \\ &= \sqrt{169} \text{ or } 13 \end{aligned}$$

The distance from X to Y is 13 units.

Example 4

Find the coordinates of the midpoint between $P(-4, 13)$ and $Q(6, 5)$.

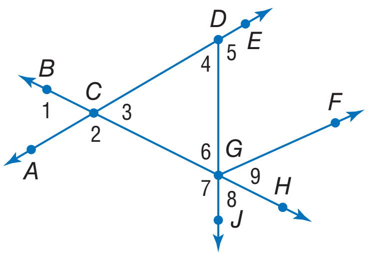
Let $(x_1, y_1) = (-4, 13)$ and $(x_2, y_2) = (6, 5)$.

$$\begin{aligned} M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= M\left(\frac{-4 + 6}{2}, \frac{13 + 5}{2}\right) \\ &= M(1, 9) \end{aligned}$$

The coordinates of the midpoint are (1, 9).

1-4 Angle Measure

For Exercises 23–26, refer to the figure below.



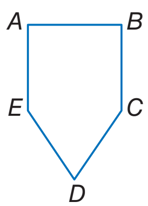
23. Name the vertex of $\angle 7$.

24. Write another name for $\angle 4$.

25. Name the sides of $\angle 2$.

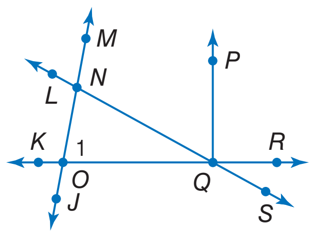
26. Name a pair of opposite rays.

27. **SIGNS** A sign at West High School has the shape shown. Measure each of the angles and classify them as *right*, *acute*, or *obtuse*.



Example 5

Refer to the figure below. Name all angles that have Q as a vertex.



$\angle OQN$, $\angle NQP$, $\angle PQR$, $\angle RQS$, $\angle SQO$, $\angle OQP$, $\angle NQR$, $\angle PQS$, $\angle OQR$

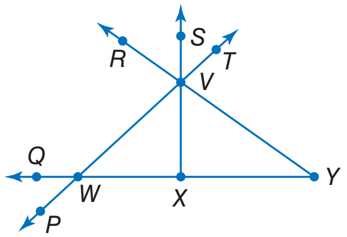
Example 6

In the figure above, list all other names for $\angle 1$.

$\angle NOQ$, $\angle QON$, $\angle MOQ$, $\angle QOM$, $\angle MOR$, $\angle ROM$, $\angle NOR$, $\angle RON$

1-5 Angle Relationships

For Exercises 28–30, refer to the figure below.



28. Name an angle supplementary to $\angle TVY$.

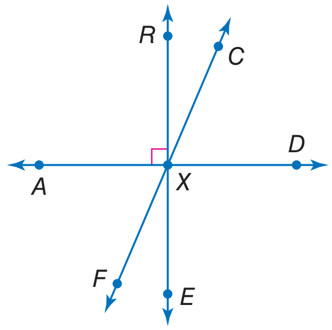
29. Name a pair of vertical angles with vertex W.

30. If $m\angle SXW = 5x - 16$, find the value of x so that $\overline{SX} \perp \overline{WY}$.

31. **PARKING** The parking arm shown below rests in a horizontal position and opens to a vertical position. After the arm has moved 24°, how many more degrees does it have to move so that it is vertical?

Example 7

Name a pair of supplementary angles and a pair of complementary angles in the figure below.



Sample answers:

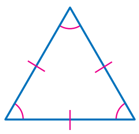
Supplementary angles: $\angle RXA$ and $\angle RXD$

Complementary angles: $\angle RXC$ and $\angle CXD$

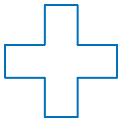
1-6 Two-Dimensional Figures

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

32.



33.

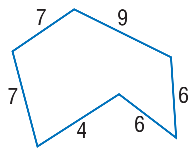


34. Find the perimeter of quadrilateral *ABCD* with vertices $A(-3, 5)$, $B(0, 5)$, $C(2, 0)$, and $D(-5, 0)$.

35. **PARKS** Westside Park received 440 feet of chain-link fencing as a donation to build an enclosed play area for dogs. The park administrators need to decide what shape the area should have. They have three options: (1) a rectangle with length of 100 feet and width of 120 feet, (2) a square with sides of length 110 feet, or (3) a circle with radius of approximately 70 feet. Find the areas of all three enclosures and determine which would provide the largest area for the dogs.

Example 8

Name the polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.



There are 6 sides, so this is a hexagon. If two of the sides are extended to make lines, they will pass through the interior of the hexagon, so it is concave. Since it is concave, it cannot be regular.

Example 9

Find the perimeter of the polygon in the figure above.

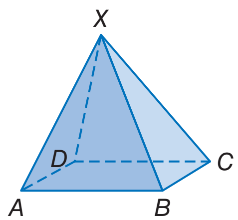
$P = s_1 + s_2 + s_3 + s_4 + s_5 + s_6$	Definition of perimeter
$= 7 + 7 + 9 + 6 + 6 + 4$	Substitution
$= 39$	Simplify.

The perimeter of the polygon is 39 units.

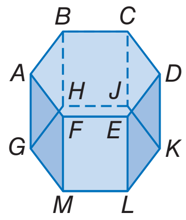
1-7 Three-Dimensional Figures

Identify each solid. Name the bases, faces, edges, and vertices.

36.

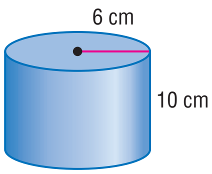


37.

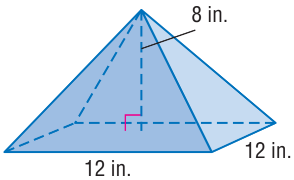


Find the surface area and volume of each solid.

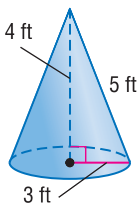
38.



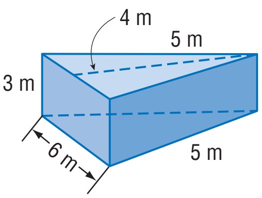
39.



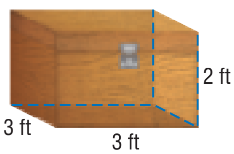
40.



41.



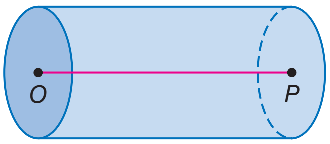
42. **BUILDING** Chris is building a trunk like the one shown below. His design is a square prism. What is the volume of the trunk?



43. **HOCKEY** A regulation hockey puck is a cylinder made of vulcanized rubber 1 inch thick and 3 inches in diameter. Find the surface area and volume of a hockey puck.

Example 10

Identify the solid below. Name the bases, faces, edges, and vertices.



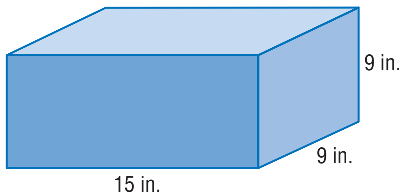
This solid has congruent circular bases in a pair of parallel planes. So, it is a cylinder.

Bases: circle O and circle P

A cylinder has no faces, edges, or vertices.

Example 11

Find the surface area and volume of the rectangular prism below.



$T = Ph + 2B$	Surface area of a prism
$= (48)(9) + 2(135)$	Substitution
$= 702$	Simplify.

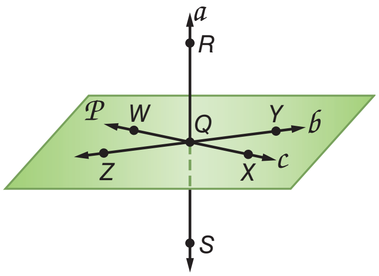
The surface area is 702 square inches.

$V = Bh$	Volume of a prism
$= (135)(9)$	Substitution
$= 1215$	Simplify.

The volume is 1215 cubic inches.

CHAPTER 1 Practice Test

Use the figure to name each of the following.



1. the line that contains points Q and Z
2. two points that are coplanar with points W, X, and Y
3. the intersection of lines a and b

Find the value of the variable if P is between J and K.

4. $JP = 2x$, $PK = 7x$, $JK = 27$
5. $JP = 3y + 1$, $PK = 12y - 4$, $JK = 75$
6. $JP = 8z - 17$, $PK = 5z + 37$, $JK = 17z - 4$

Find the coordinates of the midpoint of a segment with the given endpoints.

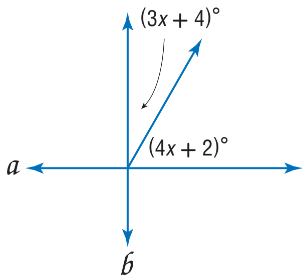
7. $(16, 5)$ and $(28, -13)$
8. $(-11, 34)$ and $(47, 0)$
9. $(-4, -14)$ and $(-22, 9)$

Find the distance between each pair of points.

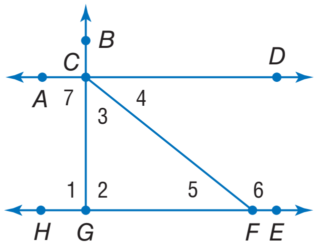
10. $(43, -15)$ and $(29, -3)$
11. $(21, 5)$ and $(28, -1)$
12. $(0, -5)$ and $(18, -10)$

13. **ALGEBRA** The measure of $\angle X$ is 18 more than three times the measure of its complement. Find the measure of $\angle X$.

14. Find the value of x that will make lines a and b perpendicular in the figure below.

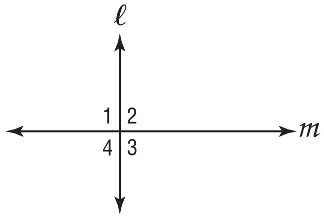


For Exercises 15–18, use the figure below.



15. Name the vertex of $\angle 3$.
16. Name the sides of $\angle 1$.
17. Write another name for $\angle 6$.
18. Name a pair of angles that share exactly one point.

19. **MULTIPLE CHOICE** If $m\angle 1 = m\angle 2$, which of the following statements is true?



A $\angle 2 \cong \angle 4$

B $\angle 2$ is a right angle.

C $\ell \perp m$

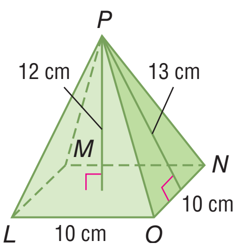
D All of the above

Find the perimeter of each polygon.

20. triangle XYZ with vertices $X(3, 7)$, $Y(-1, -5)$, and $Z(6, -4)$
21. rectangle $PQRS$ with vertices $P(0, 0)$, $Q(0, 7)$, $R(12, 7)$, and $S(12, 0)$

22. **SAFETY** A severe weather siren in a local city can be heard within a radius of 1.3 miles. If the mayor of the city wants a new siren that will cover double the area of the old siren, what should the radius of the coverage area of the new siren be? Round to the nearest tenth of a mile.

Refer to the figure at the right.



23. Name the base.
24. Find the surface area.
25. Find the volume.

CHAPTER 1

Preparing for Standardized Tests

Solving Math Problems

Strategies for Solving Math Problems

Sean Justice/Stone/Getty Images

The first step to solving any math problem is to read the problem. When reading a math problem to get the information you need to solve, it is helpful to use special reading strategies.

Step 1

Read the problem to determine what information is given.

- **Analyze:** Determine what exactly the problem is asking you to solve.
- **Underline:** If you are able to write in your test book, underline any important information.

Step 2

Reread the problem to determine what information is needed to solve the problem.

- **Think:** How does the information fit together?
- **Key Words:** Are there any key words, variables or mathematical terms in the problem?
- **Diagrams:** Do you need to use a diagram, list or table?
- **Formulas:** Do you need a formula or an equation to solve the problem?

Step 3

Devise a plan and solve the problem. Use the information you found in Steps 1 and 2.

- **Question:** What problem are you solving?
- **Estimate:** Estimate an answer.
- **Eliminate:** Eliminate all answers that do not make sense and/or vary greatly from your estimate.

Step 4

Check your answer.

- **Reread:** Quickly reread the problem to make sure you solved the whole problem.
- **Reasonableness:** Is your answer reasonable?
- **Units:** Make sure your answer has the correct units of measurement.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Carmen is using a coordinate grid to make a map of her backyard. She plots the swing set at point $S(2, 5)$ and the big oak tree at point $O(-3, -6)$. If each unit on the grid represents 5 feet, what is the distance between the swing set and the oak tree? Round your answer to the nearest whole foot.

A 12 ft **B** 25 ft **C** 60 ft **D** 74 ft

Determine what exactly the problem is asking you to solve. Underline any important information.

Carmen is using a coordinate grid to make a map of her backyard. She plots the swing set at point $S(2, 5)$ and the big oak tree at point $O(-3, -6)$. If each unit on the grid represents 5 feet, what is the distance between the swing set and the oak tree? Round your answer to the nearest whole foot.

The problem is asking for the distance between the swing set and the oak tree. The key word is distance, so you know you will need to use the Distance Formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-3 - 2)^2 + (-6 - 5)^2}$ $(x_1, y_1) = (2, 5)$ and $(x_2, y_2) = (-3, -6)$

$= \sqrt{(-5)^2 + (-11)^2}$ Subtract.

$= \sqrt{25 + 121}$ or $\sqrt{146}$ Simplify.

The distance between swing set and the oak tree is $\sqrt{146}$ units. Use a calculator to find that $\sqrt{146}$ units is approximately 12.08 units.

Since each unit on the grid represents 5 feet, the distance is $(12.08) \cdot (5)$ or 60.4 ft. Therefore, the correct answer is C.

Check your answer to make sure it is reasonable, and that you have used the correct units.

Exercises

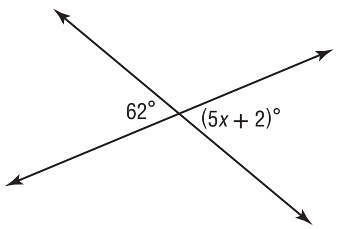
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A regular pentagon has a perimeter of 24 inches. What is the measure of each side?

A 3 inches **C** 4 inches

B 3.8 inches **D** 4.8 inches

2. What is the value of x in the figure at the right?



F 10

G 12

H 14

J 15

CHAPTER 1

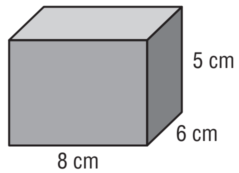
Standardized Test Practice

Cumulative, Chapter 1

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. If the dimensions of the prism below were doubled, by what factor would the volume of the prism increase?



A 2 **C** 8

B 4 **D** 16

2. Find the distance between $M(-3, 1)$ and $N(2, 8)$ on a coordinate plane.

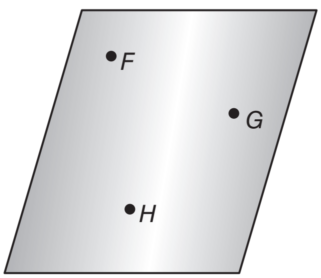
F 6.1 units

G 6.9 units

H 7.3 units

J 8.6 units

3. Which of the following terms best describes points F, G, and H?



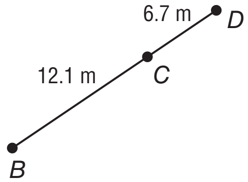
A collinear **C** coplanar

B congruent **D** skew

Test-Taking Tip

Question 3 Understanding the terms of geometry can help you solve problems. The term *congruent* refers to geometric figures, and *skew* refers to lines, therefore both answers can be eliminated.

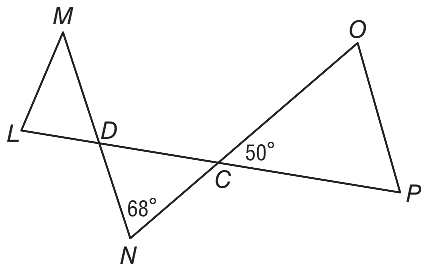
4. What is the length of segment BD?



F 17.4 m **H** 18.8 m

G 18.3 m **J** 19.1 m

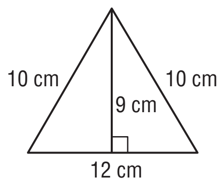
5. In the figure below, what is the measure of angle CDN?



A 58° **C** 68°

B 62° **D** 70°

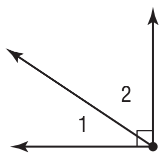
6. Find the perimeter of the figure below.



F 20 cm **H** 32 cm

G 29 cm **J** 41 cm

7. What is the relationship of $\angle 1$ and $\angle 2$?



A complementary angles

B linear pair

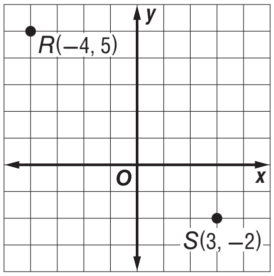
C supplementary angles

D vertical angles

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. Find the distance between points R and S on the coordinate grid below. Round to the nearest tenth.



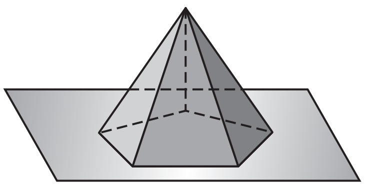
9. **SHORT RESPONSE** Find the value of x and AB if B is between A and C, $AB = 2x$, $AC = 6x - 5$, and $BC = 7$.

10. Suppose two lines intersect in a plane.

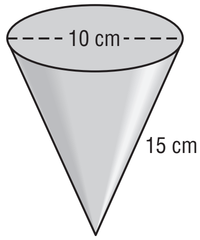
a. What do you know about the two pairs of vertical angles formed?

b. What do you know about the pairs of adjacent angles formed?

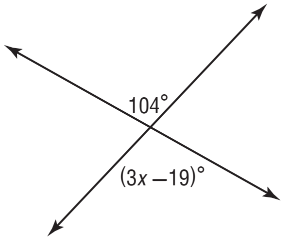
11. **GRIDDED RESPONSE** How many planes are shown in the figure below?



12. **GRIDDED RESPONSE** What is the total surface area of the cone? Round your answer to the nearest square centimeter.



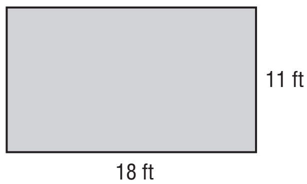
13. **GRIDDED RESPONSE** What is the value of x in the figure?



Extended Response

Record your answers on a sheet of paper. Show your work.

14. Julie's room has the dimensions shown in the figure.



a. Find the perimeter of her room.

b. Find the area of her room.

c. If the length and width doubled, what effect would it have on the perimeter?

d. What effect would it have on the area?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Go to Lesson...	1-7	1-3	1-1	1-2	1-4	1-6	1-5	1-3	1-2	1-5	1-1	1-7	1-4	1-6