CHAPTER 5

Relationships in Triangles

Then

You learned how to classify triangles.

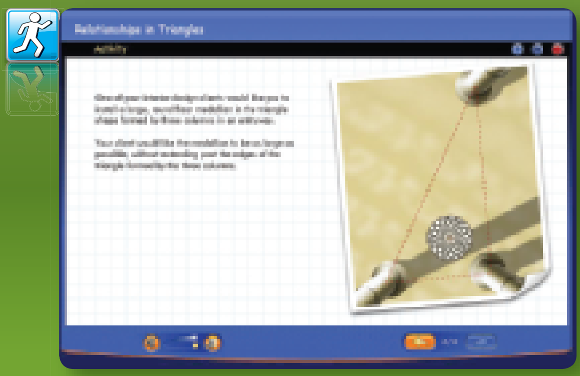
Now

In this chapter, you will:

- Learn about special segments and points related to triangles.
- Learn about relationships between the sides and angles of triangles.
- Learn to write indirect proofs.

Why? ▲

INTERIOR DESIGN Triangle relationships are used to find and compare angle measures and distances. Interior designers use the relationships in triangles to maximize efficiency and create balance in their designs.



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Cusp/SuperStock

Get Ready for the Chapter

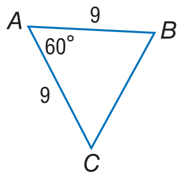
Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

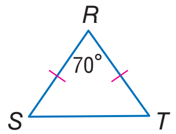
QuickCheck

Find each measure.

1. BC



2. $m\angle RST$



3. GARDENS Bronson is creating a right triangular flower bed. If two of the sides of the flower bed are 7 feet long each, what is the length of the third side to the nearest foot?

Make a conjecture based on the given information.

4. $\angle 3$ and $\angle 4$ are a linear pair.

5. $JKLM$ is a square.

6. $\overrightarrow{BD}$ is an angle bisector of $\angle ABC$.

7. REASONING Determine whether the following conjecture is *always*, *sometimes*, or *never* true based on the given information. Justify your reasoning.

Given: collinear points D, E, and F

Conjecture: $DE + EF = DF$

Solve each inequality.

8. $x + 13 < 41$

9. $x - 6 > 2x$

10. $6x + 9 < 7x$

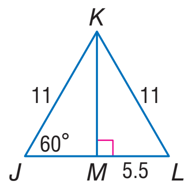
11. $8x + 15 > 9x - 26$

12. MUSIC Nina added 15 more songs to her MP3 player, resulting in a total of more than 120 songs. How many songs were originally on the player?

QuickReview

Example 1

Find each measure.



a. JM

$m\angle J = m\angle L$, so $m\angle L = 60$ and $\overline{JM} \cong \overline{LM}$ by the Converse of the Isosceles Triangle Theorem. Since $LM = 5.5$, $JM = 5.5$ by substitution.

b. $m\angle JKL$

$m\angle J + m\angle JKL + m\angle L = 180$	△ Sum Theorem
$60 + m\angle JKL + 60 = 180$	$m\angle J = m\angle L = 60$
$120 + m\angle JKL = 180$	Simplify.
$m\angle JKL = 60$	Subtract.

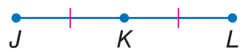
Example 2

K is the midpoint of $\overline{JL}$. Make a conjecture based on the given information and draw a figure to illustrate your conjecture.

Given: K is the midpoint of $\overline{JL}$. J, K, and L are collinear points, and K lies an equal distance between J and L.

Conjecture: $\overline{JK} \cong \overline{KL}$

Check: Draw $\overline{JL}$. This illustrates the conjecture.



Example 3

Solve $3x + 5 > 2x$.

$3x + 5 > 2x$	Given
$3x - 3x + 5 > 2x - 3x$	Subtract.
$5 > -x$	Simplify.
$-5 < x$	Divide.

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

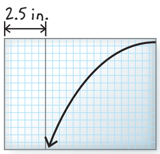
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 5. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

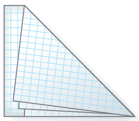
FOLDABLES® StudyOrganizer

Relationships in Triangles Make this Foldable to help you organize your Chapter 5 notes about relationships in triangles. Begin with seven sheets of grid paper.

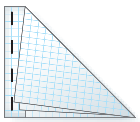
1 **Stack** the sheets. Fold the top right corner to the bottom edge to form an isosceles right triangle.



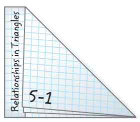
2 **Fold** the rectangular part in half.



3 **Staple** the sheets along the rectangular fold in four places.



4 **Label** each sheet with a lesson number and the rectangular tab with the chapter title.



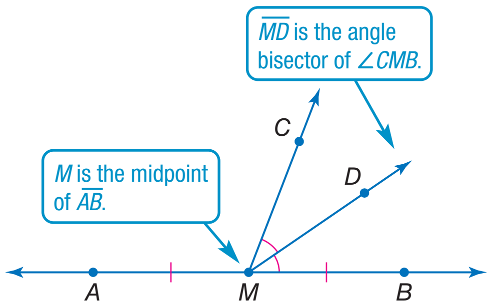
NewVocabulary

English		Español
perpendicular bisector	p. 324	mediatriz
concurrent lines	p. 325	rectas concurrentes
point of concurrency	p. 325	punto de concurrencia
circumcenter	p. 325	circuncentro
incenter	p. 328	incentro
median	p. 335	mediana
centroid	p. 335	baricentro
altitude	p. 337	altura
orthocenter	p. 337	ortocentro
indirect reasoning	p. 355	razonamiento indirecto
indirect proof	p. 355	demostración indirecta
proof by contradiction	p. 355	demostración por contradicción

ReviewVocabulary

angle bisector bisectriz de un ángulo a ray that divides an angle into two congruent angles (Lesson 1-4)

midpoint punto medio the point on a segment exactly halfway between the endpoints of the segment (Lesson 1-3)



EXPLORE 5-1

Geometry Lab
Constructing Bisectors

Paper folding can be used to construct special segments in triangles.

CCSS Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Construction Perpendicular Bisector

Construct a perpendicular bisector of the side of a triangle.

Step 1

Draw, label, and cut out $\triangle MPQ$.

Step 2

Fold the triangle in half along $\overline{MQ}$ so that vertex M touches vertex Q.

Step 3

Use a straightedge to draw $\overline{AB}$ along the fold. $\overline{AB}$ is the perpendicular bisector of $\overline{MQ}$.

An angle bisector in a triangle is a line containing a vertex of the triangle and bisecting that angle.

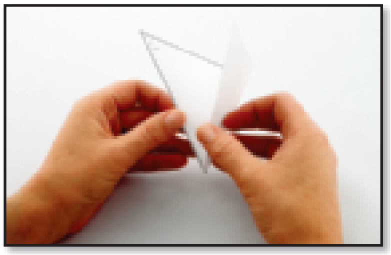
Construction Angle Bisector

Construct an angle bisector of a triangle.

Step 1

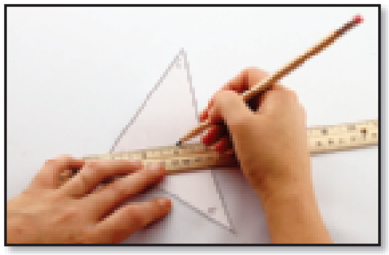
Draw, label, and cut out $\triangle ABC$.

Step 2



Fold the triangle in half through vertex A, such that sides $\overline{AC}$ and $\overline{AB}$ are aligned.

Step 3



Label point L at the crease along edge $\overline{BC}$. Use a straightedge to draw $\overline{AL}$ along the fold. $\overline{AL}$ is an angle bisector of $\triangle ABC$.

Model and Analyze

1. Construct the perpendicular bisectors of the other two sides of $\triangle MPQ$. Construct the angle bisectors of the other two angles of $\triangle ABC$. What do you notice about their intersections?

Repeat the two constructions for each type of triangle.

2. acute

3. obtuse

4. right

LESSON 5-1 Bisectors of Triangles

Then
- You used segment and angle bisectors.

Now
1. Identify and use perpendicular bisectors in triangles.
2. Identify and use angle bisectors in triangles.

Why?
- Creating a work triangle in a kitchen can make food preparation more efficient by cutting down on the number of steps you have to take. To locate the point that is equidistant from the sink, stove, and refrigerator, you can use the perpendicular bisectors of the triangle.

NewVocabulary
perpendicular bisector
concurrent lines
point of concurrency
circumcenter
incenter

Common Core State Standards

Content Standards

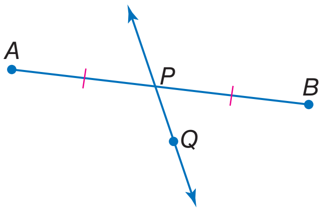
G.CO.10 Prove theorems about triangles.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

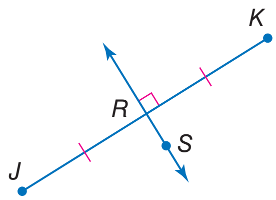
Mathematical Practices
1 Make sense of problems and persevere in solving them.
3 Construct viable arguments and critique the reasoning of others.

1 Perpendicular Bisectors

In Lesson 1-3, you learned that a segment bisector is any segment, line, or plane that intersects a segment at its midpoint. If a bisector is also perpendicular to the segment, it is called a **perpendicular bisector**.



$\overleftrightarrow{PQ}$ is a bisector of $\overline{AB}$.



$\overleftrightarrow{RS}$ is a perpendicular bisector of $\overline{JK}$.

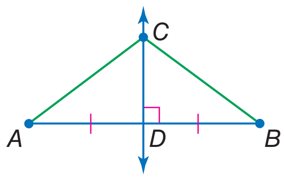
Recall that a *locus* is a set of points that satisfies a particular condition. The perpendicular bisector of a segment is the locus of points in a plane equidistant from the endpoints of the segment. This leads to the following theorems.

Theorems Perpendicular Bisectors

5.1 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

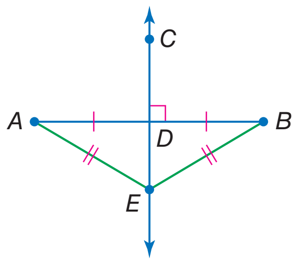
Example: If $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$, then $AC = BC$.



5.2 Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

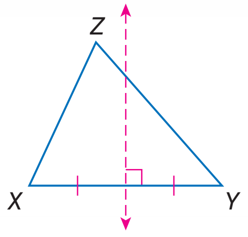
Example: If $AE = BE$, then E lies on $\overline{CD}$, the $\perp$ bisector of $\overline{AB}$.



You will prove Theorems 5.1 and 5.2 in Exercises 39 and 37, respectively.

StudyTip

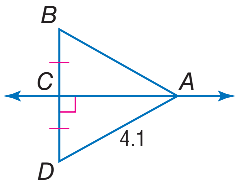
Perpendicular Bisectors The perpendicular bisector of a side of a triangle does not necessarily pass through a vertex of the triangle. For example, in $\triangle XYZ$ below, the perpendicular bisector of $\overline{XY}$ does not pass through point Z.



Example 1 Use the Perpendicular Bisector Theorems

Find each measure.

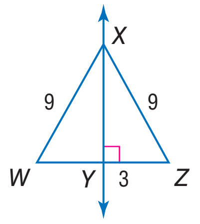
a. AB



From the information in the diagram, we know that $\overleftrightarrow{CA}$ is the perpendicular bisector of $\overline{BD}$.

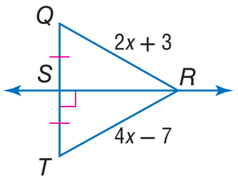
$AB = AD$	Perpendicular Bisector Theorem
$AB = 4.1$	Substitution

b. WY



Since $WX = ZX$ and $\overleftrightarrow{XY} \perp \overline{WZ}$, $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{WZ}$ by the Converse of the Perpendicular Bisector Theorem. By the definition of segment bisector, $WY = YZ$. Since $YZ = 3$, $WY = 3$.

c. RT

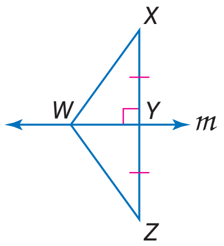


$\overleftrightarrow{SR}$ is the perpendicular bisector of $\overline{QT}$.

$RT = RQ$	Perpendicular Bisector Theorem
$4x - 7 = 2x + 3$	Substitution
$2x - 7 = 3$	Subtract $2x$ from each side.
$2x = 10$	Add 7 to each side.
$x = 5$	Divide each side by 2.

So $RT = 4(5) - 7$ or 13.

GuidedPractice

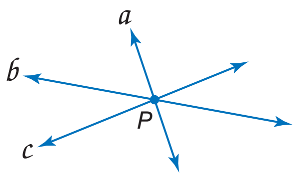


1A. If $WX = 25.3$, $YZ = 22.4$, and $WZ = 25.3$, find XY.

1B. If m is the perpendicular bisector of $\overline{XZ}$ and $WZ = 14.9$, find WX.

1C. If m is the perpendicular bisector of $\overline{XZ}$, $WX = 4a - 15$, and $WZ = a + 12$, find WX.

When three or more lines intersect at a common point, the lines are called **concurrent lines**. The point where concurrent lines intersect is called the **point of concurrency**.



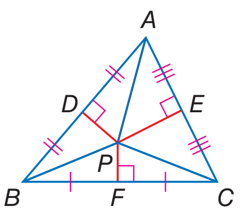
Lines a, b, and c are concurrent at P.

A triangle has three sides, so it also has three perpendicular bisectors. These bisectors are concurrent lines. The point of concurrency of the perpendicular bisectors is called the **circumcenter** of the triangle.

Theorem 5.3 Circumcenter Theorem

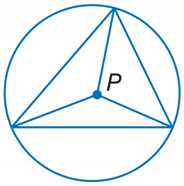
Words The perpendicular bisectors of a triangle intersect at a point called the *circumcenter* that is equidistant from the vertices of the triangle.

Example If P is the circumcenter of $\triangle ABC$, then $PB = PA = PC$.

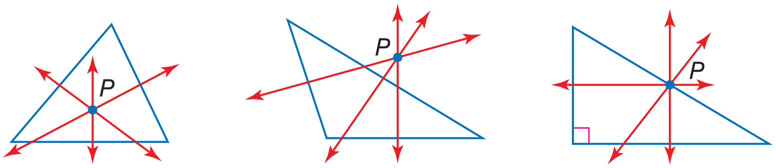


ReadingMath

Circum– The prefix *circum–* means *about* or *around*. The circumcenter is the center of a circle around a triangle that contains the vertices of the triangle.



The circumcenter can be on the interior, exterior, or side of a triangle.

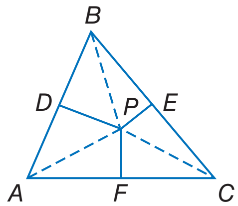


acute triangle | obtuse triangle | right triangle

Proof Circumcenter Theorem

Given: $\overline{PD}$, $\overline{PF}$, and $\overline{PE}$ are perpendicular bisectors of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, respectively.

Prove: $AP = CP = BP$



Paragraph Proof:

Since P lies on the perpendicular bisector of $\overline{AC}$, it is equidistant from A and C. By the definition of equidistant, $AP = CP$. The perpendicular bisector of $\overline{BC}$ also contains P. Thus, $CP = BP$. By the Transitive Property of Equality, $AP = BP$. Thus, $AP = CP = BP$.

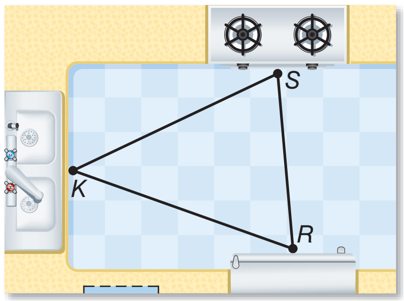
Real-WorldLink

Some basic rules of thumb for a kitchen work triangle are that the sides of the triangle should be no greater than 9 feet and no less than 4 feet. Also, the perimeter of the triangle should be no more than 26 feet and no less than 12 feet.

Source: Merillat

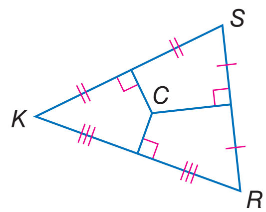
Real-World Example 2 Use the Circumcenter Theorem

INTERIOR DESIGN A stove *S*, sink *K*, and refrigerator *R* are positioned in a kitchen as shown. Find the location for the center of an island work station so that it is the same distance from these three points.



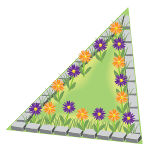
By the Circumcenter Theorem, a point equidistant from three points is found by using the perpendicular bisectors of the triangle formed by those points.

Copy $\triangle SKR$, and use a ruler and protractor to draw the perpendicular bisectors. The location for the center of the island is C, the circumcenter of $\triangle SKR$.



GuidedPractice

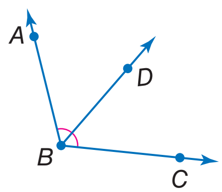
2. To water his triangular garden, Alex needs to place a sprinkler equidistant from each vertex. Where should Alex place the sprinkler?



2 Angle Bisectors

Recall from Lesson 1-4 that an angle bisector divides an angle into two congruent angles. The angle bisector can be a line, segment, or ray.

The bisector of an angle can be described as the locus of points in the interior of the angle equidistant from the sides of the angle. This description leads to the following theorems.



$\overrightarrow{BD}$ is the angle bisector of $\angle ABC$.

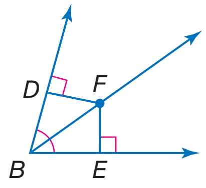
Masterfile

Theorems Angle Bisectors

5.4 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

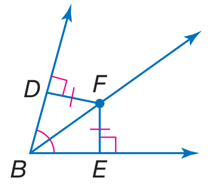
Example: If $\overrightarrow{BF}$ bisects $\angle DBE$, $\overline{FD} \perp \overrightarrow{BD}$, and $\overline{FE} \perp \overrightarrow{BE}$, then $DF = FE$.



5.5 Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.

Example: If $\overline{FD} \perp \overrightarrow{BD}$, $\overline{FE} \perp \overrightarrow{BE}$, and $DF = FE$, then $\overrightarrow{BF}$ bisects $\angle DBE$.

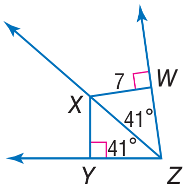


You will prove Theorems 5.4 and 5.5 in Exercises 43 and 40.

Example 3 Use the Angle Bisector Theorems

Find each measure.

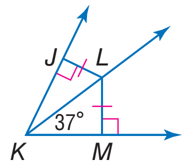
a. XY



$XY = XW$	Angle Bisector Theorem
$XY = 7$	Substitution

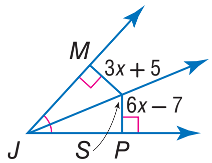
b. $m\angle JKL$

Since $\overline{LJ} \perp \overrightarrow{KJ}$, $\overline{LM} \perp \overrightarrow{KM}$, $\overline{LJ} \cong \overline{LM}$, L is equidistant from the sides of $\angle JKM$. By the Converse of the Angle Bisector Theorem, $\overrightarrow{KL}$ bisects $\angle JKM$.



$\angle JKL \cong \angle LKM$	Definition of angle bisector
$m\angle JKL = m\angle LKM$	Definition of congruent angles
$m\angle JKL = 37$	Substitution

c. SP



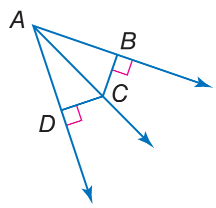
$SP = SM$	Angle Bisector Theorem
$6x - 7 = 3x + 5$	Substitution
$3x - 7 = 5$	Subtract $3x$ from each side.
$3x = 12$	Add 7 to each side.
$x = 4$	Divide each side by 3.

So, $SP = 6(4) - 7$ or 17

StudyTip

Angle Bisector For part b, only that $JL = LM$ would not be enough information to conclude that $\overrightarrow{KL}$ bisects $\angle JKM$.

GuidedPractice



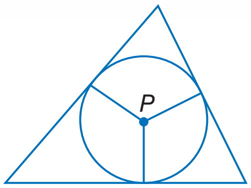
3A. If $m\angle BAC = 38$, $BC = 5$, and $DC = 5$, find $m\angle DAC$.

3B. If $m\angle BAC = 40$, $m\angle DAC = 40$, and $DC = 10$, find BC.

3C. If $\overrightarrow{AC}$ bisects $\angle DAB$, $BC = 4x + 8$, and $DC = 9x - 7$, find BC.

ReadingMath

Incenter The incenter is the center of a circle that intersects each side of the triangle at one point. For this reason, the incenter always lies in the interior of a triangle.

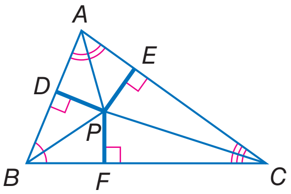


Similar to perpendicular bisectors, since a triangle has three angles, it also has three angle bisectors. The angle bisectors of a triangle are concurrent, and their point of concurrency is called the **incenter** of a triangle.

Theorem 5.6 Incenter Theorem

Words The angle bisectors of a triangle intersect at a point called the *incenter* that is equidistant from the sides of the triangle.

Example If P is the incenter of $\triangle ABC$, then $PD = PE = PF$.

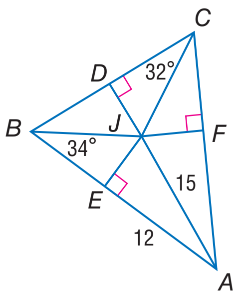


You will prove Theorem 5.6 in Exercise 38.

PT

Example 4 Use the Incenter Theorem

Find each measure if J is the incenter of $\triangle ABC$.



a. JF

By the Incenter Theorem, since J is equidistant from the sides of $\triangle ABC$, $JF = JE$. Find JF by using the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$JE^2 + 12^2 = 15^2$	Substitution
$JE^2 + 144 = 225$	$12^2 = 144$ and $15^2 = 225$.
$JE^2 = 81$	Subtract 144 from each side.
$JE = \pm 9$	Take the square root of each side.

Since length cannot be negative, use only the positive square root, 9. Since $JE = JF$, $JF = 9$.

b. $m\angle JAC$

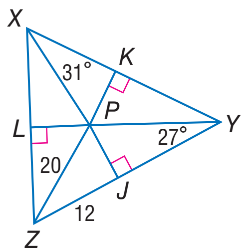
Since $\overrightarrow{BJ}$ bisects $\angle CBE$, $m\angle CBE = 2m\angle JBE$. So $m\angle CBE = 2(34)$ or 68. Likewise, $m\angle DCF = 2m\angle DCJ$, so $m\angle DCF = 2(32)$ or 64.

$m\angle CBE + m\angle DCF + m\angle FAE = 180$	Triangle Angle Sum Theorem
$68 + 64 + m\angle FAE = 180$	$m\angle CBE = 68$, $m\angle DCF = 64$
$132 + m\angle FAE = 180$	Simplify.
$m\angle FAE = 48$	Subtract 132 from each side.

Since $\overrightarrow{AJ}$ bisects $\angle FAE$, $2m\angle JAC = m\angle FAE$. This means that $m\angle JAC = \frac{1}{2}m\angle FAE$, so $m\angle JAC = \frac{1}{2}(48)$ or 24.

GuidedPractice

If P is the incenter of $\triangle XYZ$, find each measure.



4A. PK

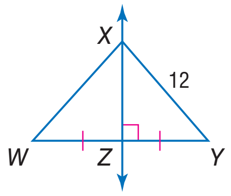
4B. $m\angle LZP$

Check Your Understanding

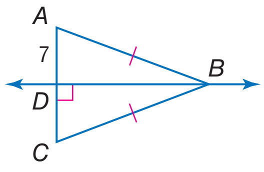
= Step-by-Step Solutions begin on page R14.

Example 1 **Find each measure.**

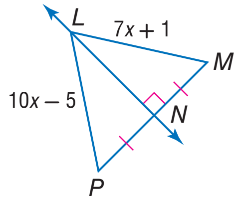
1. *XW*



2. *AC*

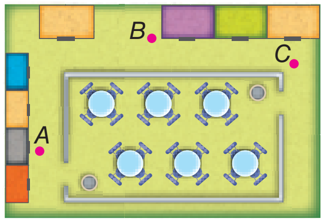


3. *LP*



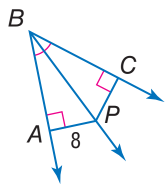
Example 2

4. **ADVERTISING** Four friends are passing out flyers at a mall food court. Three of them take as many flyers as they can and position themselves as shown. The fourth one keeps the supply of additional flyers. Copy the positions of points *A*, *B*, and *C*. Then position the fourth friend at *D* so that she is the same distance from each of the other three friends.

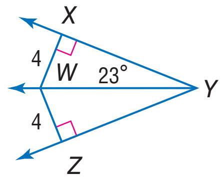


Example 3 **Find each measure.**

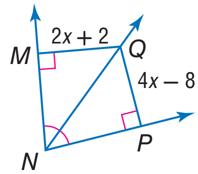
5. *CP*



6. $m\angle WYZ$

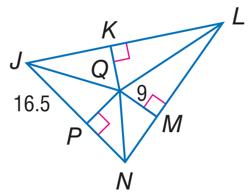


7. *QM*



Example 4

8. CCSS **SENSE-MAKING** Find *JQ* if *Q* is the incenter of $\triangle JLN$.

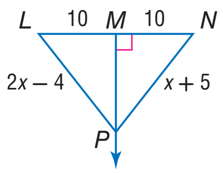


Practice and Problem Solving

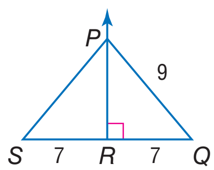
Extra Practice is on page R5.

Example 1 **Find each measure.**

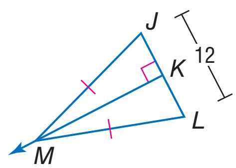
9. *NP*



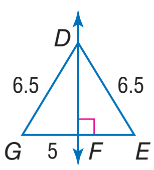
10. *PS*



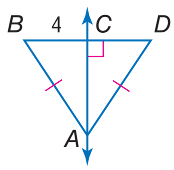
11. *KL*



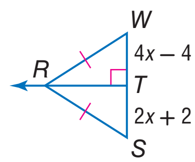
12. *EG*



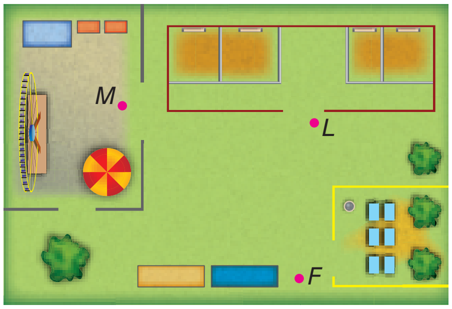
13. *CD*



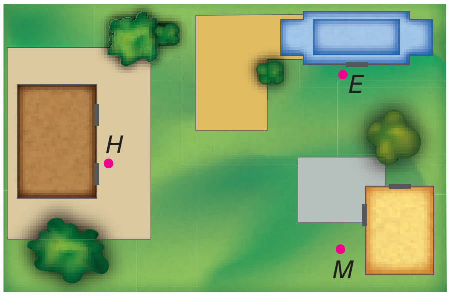
14. *SW*



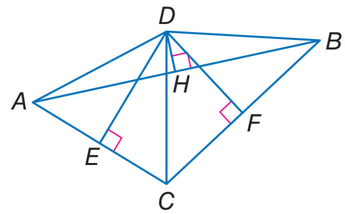
Example 2 **15. STATE FAIR** The state fair has set up the location of the midway, livestock competition, and food vendors. The fair planners decide that they want to locate the portable restrooms the same distance from each location. Copy the positions of points *M*, *L*, and *F*. Then find the location for the restrooms and label it *R*.



16. SCHOOL A school system has built an elementary, middle, and high school at the locations shown in the diagram. Copy the positions of points *E*, *M*, and *H*. Then find the location for the bus yard *B* that will service these schools so that it is the same distance from each school.



Point *D* is the circumcenter of △*ABC*. List any segment(s) congruent to each segment.



17. $\overline{AD}$

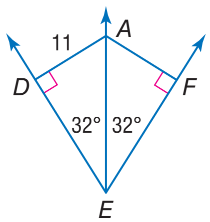
18. $\overline{BF}$

19. $\overline{AH}$

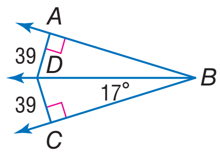
20. $\overline{DC}$

Example 3 **Find each measure.**

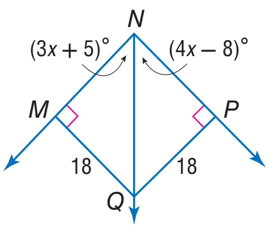
21. *AF*



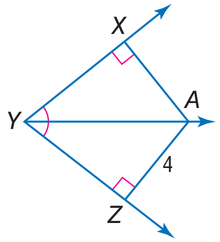
22. *m*∠*DBA*



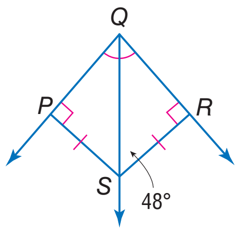
23 *m*∠*PNM*



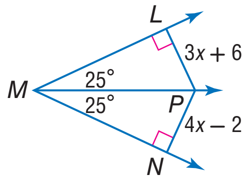
24. *XA*



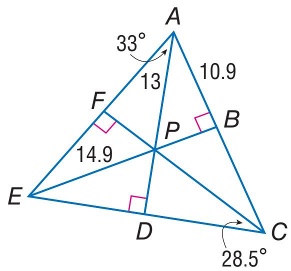
25. *m*∠*PQS*



26. *PN*



Example 4 **CCSS SENSE-MAKING** **Point *P* is the incenter of △*AEC*. Find each measure below.**



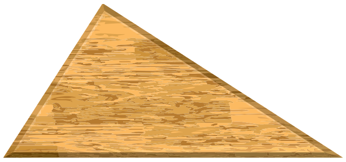
27. *PB*

28. *DE*

29. *m*∠*DAC*

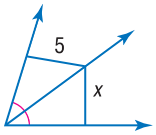
30. *m*∠*DEP*

31 **INTERIOR DESIGN** You want to place a centerpiece on a corner table so that it is located the same distance from each edge of the table. Make a sketch to show where you should place the centerpiece. Explain your reasoning.

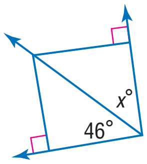


Determine whether there is enough information given in each diagram to find the value of x. Explain your reasoning.

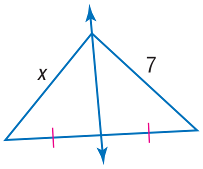
32.



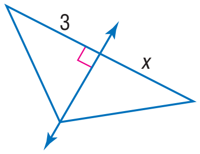
33.



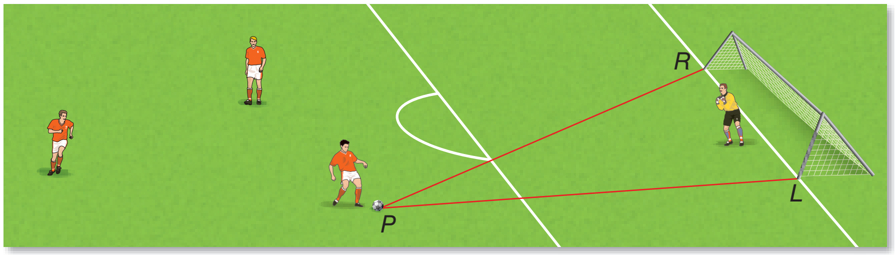
34.



35.



36. **SOCCER** A soccer player P is approaching the opposing team's goal as shown in the diagram. To make the goal, the player must kick the ball between the goal posts at L and R. The goalkeeper faces the kicker. He then tries to stand so that if he needs to dive to stop a shot, he is as far from the left-hand side of the shot angle as the right-hand side.



a. Describe where the goalkeeper should stand. Explain your reasoning.

b. Copy $\triangle PRL$. Use a compass and a straightedge to locate a point G where the goalkeeper should stand.

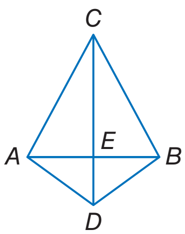
c. If the ball is kicked so it follows the path from P to R, construct the shortest path the goalkeeper should take to block the shot. Explain your reasoning.

PROOF **Write a two-column proof.**

37. Theorem 5.2

Given: $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$

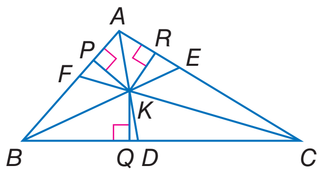
Prove: C and D are on the perpendicular bisector of $\overline{AB}$.



38. Theorem 5.6

Given: $\triangle ABC$, angle bisectors $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$
$\overline{KP} \perp \overline{AB}$, $\overline{KQ} \perp \overline{BC}$,
$\overline{KR} \perp \overline{AC}$

Prove: $KP = KQ = KR$



CCSS ARGUMENTS **Write a paragraph proof of each theorem.**

39. Theorem 5.1

40. Theorem 5.5

COORDINATE GEOMETRY **Write an equation in slope-intercept form for the perpendicular bisector of the segment with the given endpoints. Justify your answer.**

41. $A(-3, 1)$ and $B(4, 3)$

42. $C(-4, 5)$ and $D(2, -2)$

43. **PROOF** Write a two-column proof of Theorem 5.4.

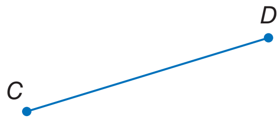
44. **GRAPHIC DESIGN** Mykia is designing a pennant for her school. She wants to put a picture of the school mascot inside a circle on the pennant. Copy the outline of the pennant and locate the point where the center of the circle should be to create the largest circle possible. Justify your drawing.

COORDINATE GEOMETRY **Find the coordinates of the circumcenter of the triangle with the given vertices. Explain.**

45. $A(0, 0)$, $B(0, 6)$, $C(10, 0)$

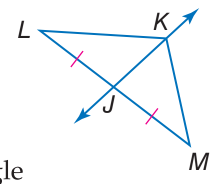
46. $J(5, 0)$, $K(5, -8)$, $L(0, 0)$

47. **LOCUS** Consider $\overline{CD}$. Describe the set of all points in space that are equidistant from C and D.



H.O.T. Problems Use Higher-Order Thinking Skills

48. **ERROR ANALYSIS** Claudio says that from the information supplied in the diagram, he can conclude that K is on the perpendicular bisector of $\overline{LM}$. Caitlyn disagrees. Is either of them correct? Explain your reasoning.



49. **OPEN ENDED** Draw a triangle with an incenter located inside the triangle but a circumcenter located outside. Justify your drawing by using a straightedge and a compass to find both points of concurrency.

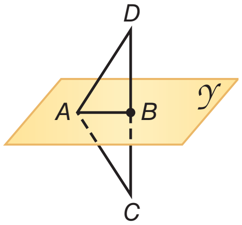
CCSS ARGUMENTS **Determine whether each statement is *sometimes*, *always*, or *never* true. Justify your reasoning using a counterexample or proof.**

50. The angle bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

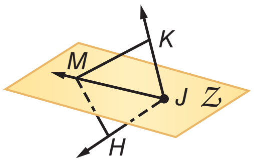
51. In an isosceles triangle, the perpendicular bisector of the base is also the angle bisector of the opposite vertex.

CHALLENGE **Write a two-column proof for each of the following.**

52. **Given:** Plane $\mathcal{Y}$ is a perpendicular bisector of $\overline{DC}$.
Prove: $\angle ADB \cong \angle ACB$



53. **Given:** Plane $\mathcal{Z}$ is an angle bisector of $\angle KJH$, $\overline{KJ} \cong \overline{HJ}$
Prove: $\overline{MH} \cong \overline{MK}$



54. **WRITING IN MATH** Compare and contrast the perpendicular bisectors and angle bisectors of a triangle. How are they alike? How are they different? Be sure to compare their points of concurrency.

Standardized Test Practice

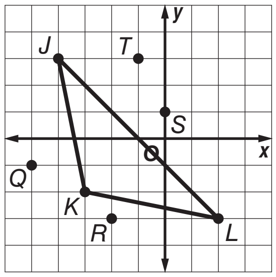
55. ALGEBRA An object is projected straight upward with initial velocity v meters per second from an initial height of s meters. The height h in meters of the object after t seconds is given by $h = -10t^2 + vt + s$. Sherise is standing at the edge of a balcony 54 meters above the ground and throws a ball straight up with an initial velocity of 12 meters per second. After how many seconds will it hit the ground?

A 3 seconds
B 4 seconds
C 6 seconds
D 9 seconds

56. SAT/ACT For $x \neq -3$, $\frac{3x+9}{x+3} =$

F $x + 12$
G $x + 9$
H $x + 3$
J x
K 3

57. A line drawn through which of the following points would be a perpendicular bisector of $\triangle JKL$?



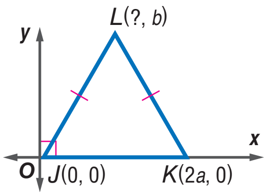
A T and K
B L and Q
C J and R
D S and K

58. SHORT RESPONSE Write an equation in slope-intercept form that describes the line containing the points $(-1, 0)$ and $(2, 4)$.

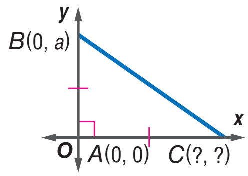
Spiral Review

Name the missing coordinate(s) of each triangle. (Lesson 4-8)

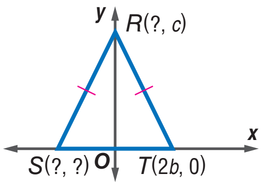
59.



60.



61.



COORDINATE GEOMETRY **Graph each pair of triangles with the given vertices. Then identify the transformation and verify that it is a congruence transformation.** (Lesson 4-7)

62. $A(-2, 4)$, $B(-2, -2)$, $C(4, 1)$;
$R(12, 4)$, $S(12, -2)$, $T(6, 1)$

63. $J(-3, 3)$, $K(-3, 1)$, $L(1, 1)$;
$X(-3, -1)$, $Y(-3, -3)$, $Z(1, -3)$

Find the distance from the line to the given point. (Lesson 3-6)

64. $y = 5$, $(-2, 4)$

65. $y = 2x + 2$, $(-1, -5)$

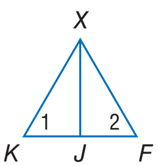
66. $2x - 3y = -9$, $(2, 0)$

67. AUDIO ENGINEERING A studio engineer charges a flat fee of $450 for equipment rental and $42 an hour for recording and mixing time. Write the equation that shows the cost to hire the studio engineer as a function of time. How much would it cost to hire the studio engineer for 17 hours? (Lesson 3-4)

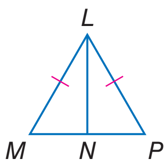
Skills Review

PROOF **Write a two-column proof for each of the following.**

68. Given: $\triangle XKF$ is equilateral.
$\overline{XJ}$ bisects $\angle X$.
Prove: J is the midpoint of $\overline{KF}$.



69. Given: $\triangle MLP$ is isosceles.
N is the midpoint of $\overline{MP}$.
Prove: $\overline{LN} \perp \overline{MP}$



EXPLORE 5-2

Geometry Lab
Constructing Medians and Altitudes

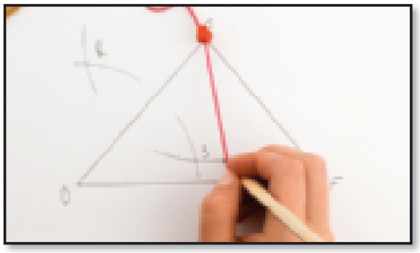
A *median* of a triangle is a segment with endpoints that are a vertex and the midpoint of the side opposite that vertex. You can use the construction for the midpoint of a segment to construct a median.

Wrap the end of string around a pencil. Use a thumbtack to fix the string to a vertex.

CCSS Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

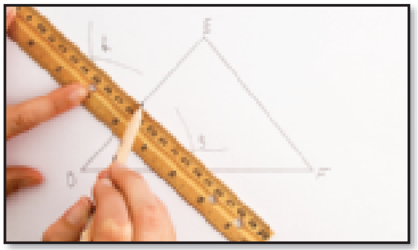
Construction 1 Median of a Triangle

Step 1



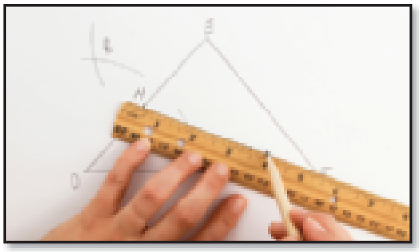
Place the thumbtack on vertex D and then on vertex E to draw intersecting arcs above and below $\overline{DE}$. Label the points of intersection R and S.

Step 2



Use a straightedge to find the point where $\overline{RS}$ intersects $\overline{DE}$. Label the point M. This is the midpoint of $\overline{DE}$.

Step 3

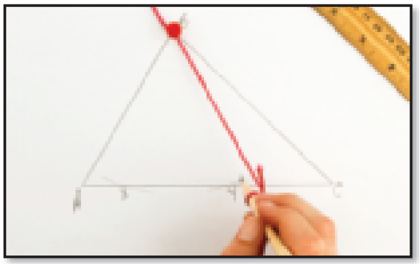


Draw a line through F and M. $\overline{FM}$ is a median of $\triangle DEF$.

An *altitude* of a triangle is a segment from a vertex of the triangle to the opposite side and is perpendicular to the opposite side.

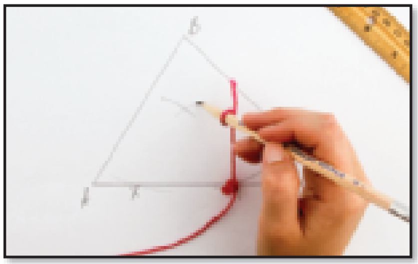
Construction 2 Altitude of a Triangle

Step 1



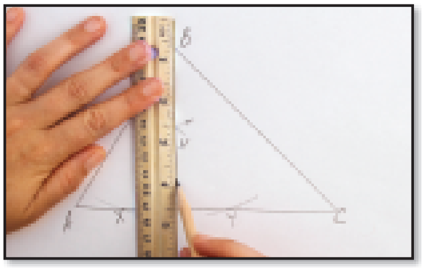
Place the thumbtack on vertex B and draw two arcs intersecting $\overline{AC}$. Label the points where the arcs intersect the sides as X and Y.

Step 2



Adjust the length of the string so that it is greater than $\frac{1}{2}XY$. Place the tack on X and draw an arc above $\overline{AC}$. Use the same length of string to draw an arc from Y. Label the points of intersection of the arcs H.

Step 3



Use a straightedge to draw $\overleftrightarrow{BH}$. Label the point where $\overleftrightarrow{BH}$ intersects $\overline{AC}$ as D. $\overline{BD}$ is an altitude of $\triangle ABC$ and is perpendicular to $\overline{AC}$.

Model and Analyze

1. Construct the medians of the other two sides of $\triangle DEF$. What do you notice about the medians of a triangle?
2. Construct the altitudes to the other two sides of $\triangle ABC$. What do you observe?

LESSON 5-2 Medians and Altitudes of Triangles

Then

- You identified and used perpendicular and angle bisectors in triangles.

Now

1. Identify and use medians in triangles.
2. Identify and use altitudes in triangles.

Why?

- A mobile is a *kinetic* or moving sculpture that uses the principles of balance and equilibrium. Simple mobiles consist of several rods attached by strings from which objects of varying weights hang. The hanging objects balance each other and can rotate freely. To ensure that a triangle in a mobile hangs parallel to the ground, artists have to find the triangle's balancing point.

NewVocabulary

median
centroid
altitude
orthocenter

Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

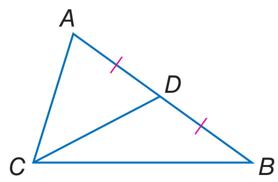
Mathematical Practices

6 Attend to precision.
3 Construct viable arguments and critique the reasoning of others.

1 Medians

A **median** of a triangle is a segment with endpoints being a vertex of a triangle and the midpoint of the opposite side.

Every triangle has three medians that are concurrent. The point of concurrency of the medians of a triangle is called the **centroid** and is always inside the triangle.



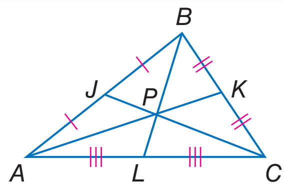
$\overline{CD}$ is a median of $\triangle ABC$.

Theorem 5.7 Centroid Theorem

The medians of a triangle intersect at a point called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.

Example If P is the centroid of $\triangle ABC$, then

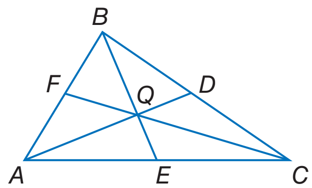
$AP = \frac{2}{3}AK$, $BP = \frac{2}{3}BL$, and $CP = \frac{2}{3}CJ$.



You will prove Theorem 5.7 in Exercise 36.

Example 1 Use the Centroid Theorem

In $\triangle ABC$, Q is the centroid and $BE = 9$. Find BQ and QE.



$BQ = \frac{2}{3}BE$	Centroid Theorem
$= \frac{2}{3}(9)$ or 6	$BE = 9$
$BQ + QE = 9$	Segment Addition
$6 + QE = 9$	$BQ = 6$
$QE = 3$	Subtract 6 from each side.

GuidedPractice **In $\triangle ABC$ above, $FC = 15$. Find each length.**

1A. FQ

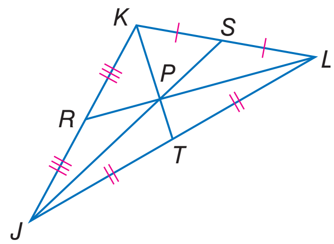
1B. QC

StudyTip

CCSS Reasoning In Example 2, you can also use number sense to find *KP*. Since $KP = \frac{2}{3}KT$, $PT = \frac{1}{3}KT$ and $KP = 2PT$. Therefore, if $PT = 2$, then $KP = 2(2)$ or 4.

Example 2 Use the Centroid Theorem

In $\triangle JKL$, $PT = 2$. Find KP.



Since $\overline{JR} \cong \overline{RK}$, R is the midpoint of $\overline{JK}$ and $\overline{LR}$ is a median of $\triangle JKL$. Likewise, S and T are the midpoints of $\overline{KL}$ and $\overline{LJ}$ respectively, so $\overline{JS}$ and $\overline{KT}$ are also medians of $\triangle JKL$. Therefore, point P is the centroid of $\triangle JKL$.

$KP = \frac{2}{3}KT$	Centroid Theorem
$KP = \frac{2}{3}(KP + PT)$	Segment Addition and Substitution
$KP = \frac{2}{3}(KP + 2)$	$PT = 2$
$KP = \frac{2}{3}KP + \frac{4}{3}$	Distributive Property
$\frac{1}{3}KP = \frac{4}{3}$	Subtract $\frac{2}{3}KP$ from each side.
$KP = 4$	Multiply each side by 3.

GuidedPractice

In $\triangle JKL$ above, $RP = 3.5$ and $JP = 9$. Find each measure.

2A. PL

2B. PS

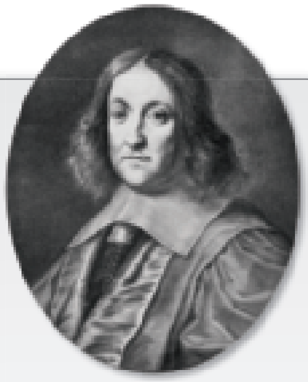
All polygons have a balance point or centroid. The centroid is also the balancing point or *center of gravity* for a triangular region. The center of gravity is the point at which the region is stable under the influence of gravity.

Real-World Example 3 Find the Centroid on Coordinate Plane

PERFORMANCE ART **A performance artist plans to balance triangular pieces of metal during her next act. When one such triangle is placed on the coordinate plane, its vertices are located at (1, 10), (5, 0), and (9, 5). What are the coordinates of the point where the artist should support the triangle so that it will balance?**

Understand You need to find the centroid of the triangle with the given coordinates. This is the point at which the triangle will balance.

Plan Graph and label the triangle with vertices $A(1, 10)$, $B(5, 0)$, and $C(9, 5)$. Since the centroid is the point of concurrency of the medians of a triangle, use the Midpoint Theorem to find the midpoint of one of the sides of the triangle. The centroid is two-thirds the distance from the opposite vertex to that midpoint.



Math HistoryLink

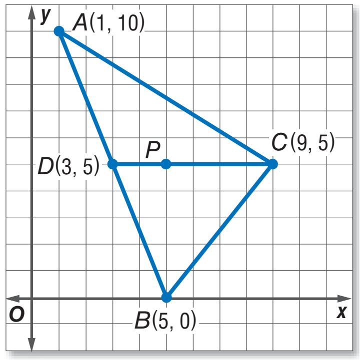
Pierre de Fermat (1601–1665) Another triangle center is the Fermat point, which minimizes the sum of the distances from the three vertices. Fermat is one of the best-known mathematicians for writing proofs.

Solve Graph $\triangle ABC$.

Find the midpoint D of side $\overline{AB}$ with endpoints $A(1, 10)$ and $B(5, 0)$.

$D\left(\frac{1+5}{2}, \frac{10+0}{2}\right) = D(3, 5)$

Graph point D. Notice that $\overline{DC}$ is a horizontal line. The distance from $D(3, 5)$ to $C(9, 5)$ is $9 - 3$ or 6 units.



If P is the centroid of $\triangle ABC$, then $PC = \frac{2}{3}DC$. So the centroid is $\frac{2}{3}(6)$ or 4 units to the left of C. The coordinates of P are $(9 - 4, 5)$ or $(5, 5)$.

The performer should balance the triangle at the point (5, 5).

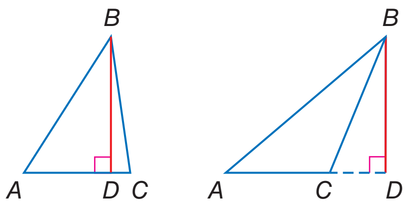
Check Use a different median to check your answer. The midpoint F of side $\overline{AC}$ is $F\left(\frac{1+9}{2}, \frac{10+5}{2}\right)$ or $F(5, 7.5)$. $\overline{BF}$ is a vertical line, so the distance from B to F is $7.5 - 0$ or 7.5. $\overline{PB} = \frac{2}{3}(7.5)$ or 5, so P is 5 units up from B. The coordinates of P are $(5, 0 + 5)$ or $(5, 5)$. ✓

GuidedPractice

3. A second triangle has vertices at (0, 4), (6, 11.5), and (12, 1). What are the coordinates of the point where the artist should support the triangle so that it will balance? Explain your reasoning.

2 Altitudes

An **altitude** of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side. An altitude can lie in the interior, exterior, or on the side of a triangle.



$\overline{BD}$ is an altitude from B to $\overline{AC}$.

ReadingMath

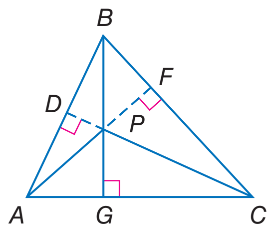
Height of a Triangle The length of an altitude is known as the *height* of the triangle. The height of a triangle is used to calculate the triangle's area.

Every triangle has three altitudes. If extended, the altitudes of a triangle intersect in a common point.

KeyConcept Orthocenter

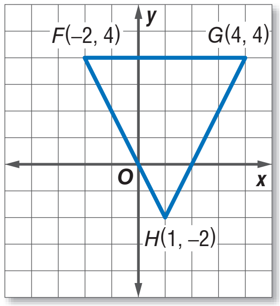
The lines containing the altitudes of a triangle are concurrent, intersecting at a point called the **orthocenter**.

Example The lines containing altitudes $\overline{AF}$, $\overline{CD}$, and $\overline{BG}$ intersect at P, the orthocenter of $\triangle ABC$.



Example 4 Find the Orthocenter on a Coordinate Plane

COORDINATE GEOMETRY **The vertices of $\triangle FGH$ are $F(-2, 4)$, $G(4, 4)$, and $H(1, -2)$. Find the coordinates of the orthocenter of $\triangle FGH$.**



Step 1 Graph $\triangle FGH$. To find the orthocenter, find the point where two of the three altitudes intersect.

Step 2 Find an equation of the altitude from F to $\overline{GH}$. The slope of $\overline{GH}$ is $\frac{4-(-2)}{4-1}$ or 2, so the slope of the altitude, which is perpendicular to $\overline{GH}$, is $-\frac{1}{2}$.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = -\frac{1}{2}[x - (-2)]$	$m = -\frac{1}{2}$ and $(x_1, y_1) = F(-2, 4)$.
$y - 4 = -\frac{1}{2}(x + 2)$	Simplify.
$y - 4 = -\frac{1}{2}x - 1$	Distributive Property
$y = -\frac{1}{2}x + 3$	Add 4 to each side.

Find an equation of the altitude from G to $\overline{FH}$. The slope of $\overline{FH}$ is $\frac{-2-4}{1-(-2)}$ or -2, so the slope of the altitude is $\frac{1}{2}$.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = \frac{1}{2}(x - 4)$	$m = \frac{1}{2}$ and $(x_1, y_1) = G(4, 4)$
$y - 4 = \frac{1}{2}x - 2$	Distributive Property
$y = \frac{1}{2}x + 2$	Add 4 to each side.

Step 3 Solve the resulting system of equations $\begin{cases} y = -\frac{1}{2}x + 3 \\ y = \frac{1}{2}x + 2 \end{cases}$ to find the point of intersection of the altitudes.

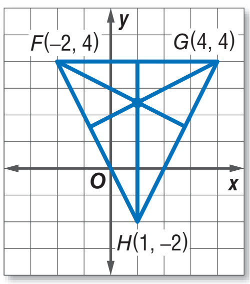
Adding the two equations to eliminate x results in $2y = 5$ or $y = \frac{5}{2}$.

$y = \frac{1}{2}x + 2$	Equation of altitude from G
$\frac{5}{2} = \frac{1}{2}x + 2$	$y = \frac{5}{2}$
$\frac{1}{2} = \frac{1}{2}x$	Subtract $\frac{4}{2}$ or 2 from each side.
$1 = x$	Multiply each side by 2.

The coordinates of the orthocenter of $\triangle JKL$ are $\left(1, \frac{5}{2}\right)$ or $\left(1, 2\frac{1}{2}\right)$.

StudyTip

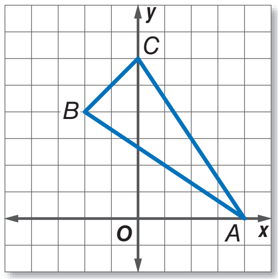
Check for Reasonableness Use the corner of a sheet of paper to draw the altitudes of each side of the triangle.



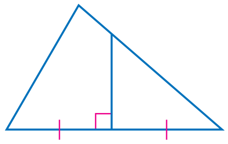
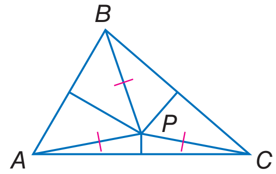
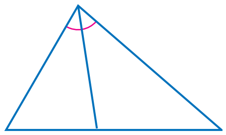
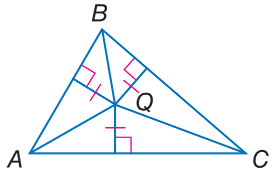
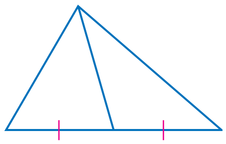
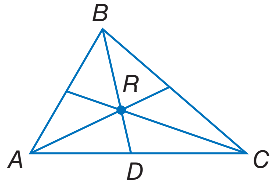
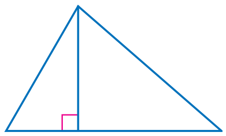
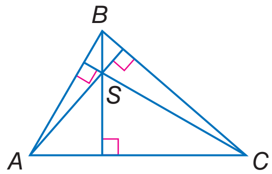
The intersection is located at approximately $\left(1, 2\frac{1}{2}\right)$, so the answer is reasonable.

GuidedPractice

4. Find the coordinates of the orthocenter of $\triangle ABC$ graphed at the right.



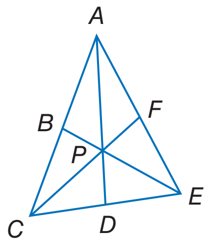
ConceptSummary Special Segments and Points in Triangles

Name	Example	Point of Concurrency	Special Property	Example
perpendicular bisector		circumcenter	The circumcenter *P* of $\triangle ABC$ is equidistant from each vertex.	
angle bisector		incenter	The incenter *Q* of $\triangle ABC$ is equidistant from each side of the triangle.	
median		centroid	The centroid *R* of $\triangle ABC$ is two thirds of the distance from each vertex to the midpoint of the opposite side.	
altitude		orthocenter	The lines containing the altitudes of $\triangle ABC$ are concurrent at the orthocenter *S*.	

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

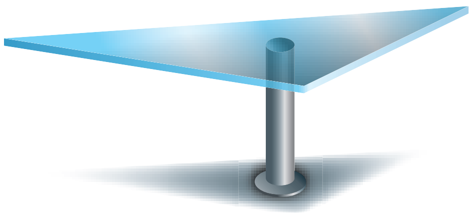
Examples 1–2 **In $\triangle ACE$, *P* is the centroid, $PF = 6$, and $AD = 15$. Find each measure.**



1. *PC*
2. *AP*

Example 3

3. **INTERIOR DESIGN** An interior designer is creating a custom coffee table for a client. The top of the table is a glass triangle that needs to balance on a single support. If the coordinates of the vertices of the triangle are at (3, 6), (5, 2), and (7, 10), at what point should the support be placed?



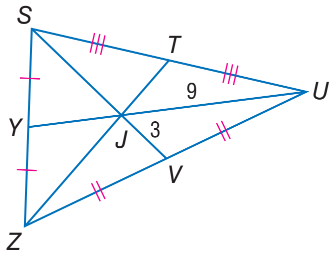
Example 4

4. **COORDINATE GEOMETRY** Find the coordinates of the orthocenter of $\triangle ABC$ with vertices $A(-3, 3)$, $B(-1, 7)$, and $C(3, 3)$.

Practice and Problem Solving

Extra Practice is on page R5.

Examples 1–2 **In $\triangle SZU$, $UJ = 9$, $VJ = 3$, and $ZT = 18$. Find each length.**



5. YJ

6. SJ

7. YU

8. SV

9. JT

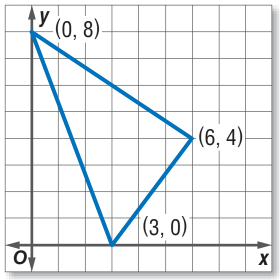
10. ZJ

Example 3 **COORDINATE GEOMETRY** **Find the coordinates of the centroid of each triangle with the given vertices.**

11. $A(-1, 11)$, $B(3, 1)$, $C(7, 6)$

12. $X(5, 7)$, $Y(9, -3)$, $Z(13, 2)$

13 **INTERIOR DESIGN** Emilia made a collage with pictures of her friends. She wants to hang the collage from the ceiling in her room so that it is parallel to the ceiling. A diagram of the collage is shown in the graph at the right. At what point should she place the string?



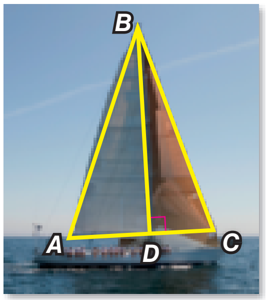
Example 4 **COORDINATE GEOMETRY** **Find the coordinates of the orthocenter of each triangle with the given vertices.**

14. $J(3, -2)$, $K(5, 6)$, $L(9, -2)$

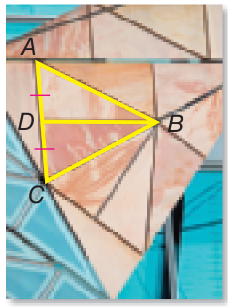
15. $R(-4, 8)$, $S(-1, 5)$, $T(5, 5)$

Identify each segment $\overline{BD}$ as a(n) altitude, median, or perpendicular bisector.

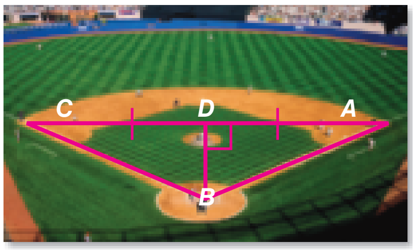
16.



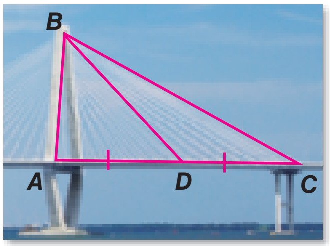
17.



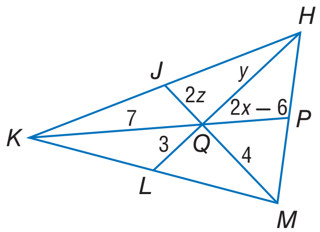
18.



19.



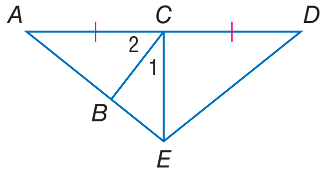
20. CCSS **SENSE-MAKING** In the figure at the right, if J, P, and L are the midpoints of $\overline{KH}$, $\overline{HM}$, and $\overline{MK}$, respectively, find x, y, and z.



Copy and complete each statement for $\triangle RST$ for medians $\overline{RM}$, $\overline{SL}$ and $\overline{TK}$, and centroid J.

21. $SL = x(JL)$ **22.** $JT = x(TK)$ **23.** $JM = x(RJ)$

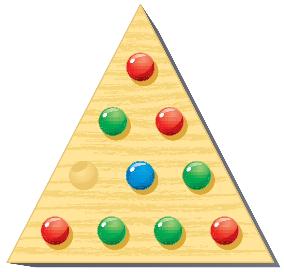
ALGEBRA **Use the figure at the right.**



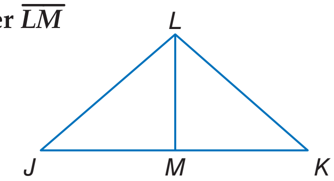
24. If $\overline{EC}$ is an altitude of $\triangle AED$, $m\angle 1 = 2x + 7$, and $m\angle 2 = 3x + 13$, find $m\angle 1$ and $m\angle 2$.

25 Find the value of x if $AC = 4x - 3$, $DC = 2x + 9$, $m\angle ECA = 15x + 2$, and $\overline{EC}$ is a median of $\triangle AED$. Is $\overline{EC}$ also an altitude of $\triangle AED$? Explain.

26. **GAMES** The game board shown is shaped like an equilateral triangle and has indentations for game pieces. The game's objective is to remove pegs by jumping over them until there is only one peg left. Copy the game board's outline and determine which of the points of concurrency the blue peg represents: *circumcenter, incenter, centroid,* or *orthocenter*. Explain your reasoning.



CCSS ARGUMENTS **Use the given information to determine whether $\overline{LM}$ is a *perpendicular bisector, median,* and/or an *altitude* of $\triangle JKL$.**



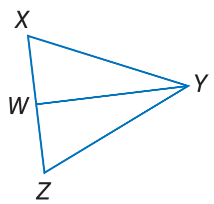
27. $\overline{LM} \perp \overline{JK}$ **28.** $\triangle JLM \cong \triangle KLM$

29. $\overline{JM} \cong \overline{KM}$ **30.** $\overline{LM} \perp \overline{JK}$ and $\overline{JL} \cong \overline{KL}$

31. **PROOF** Write a paragraph proof.

Given: $\triangle XYZ$ is isosceles. $\overline{WY}$ bisects $\angle Y$.

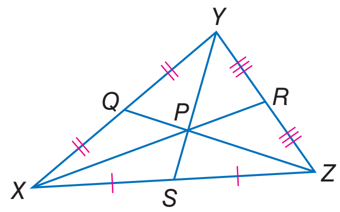
Prove: $\overline{WY}$ is a median.



32. **PROOF** Write an algebraic proof.

Given: $\triangle XYZ$ with medians $\overline{XR}$, $\overline{YS}$, $\overline{ZQ}$

Prove: $\frac{XP}{PR} = 2$



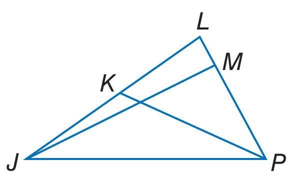
33. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the location of the points of concurrency for any equilateral triangle.

a. **Concrete** Construct three different equilateral triangles on tracing paper and cut them out. Fold each triangle to locate the circumcenter, incenter, centroid, and orthocenter.

b. **Verbal** Make a conjecture about the relationships among the four points of concurrency of any equilateral triangle.

c. **Graphical** Position an equilateral triangle and its circumcenter, incenter, centroid, and orthocenter on the coordinate plane using variable coordinates. Determine the coordinates of each point of concurrency.

ALGEBRA **In $\triangle JLP$, $m\angle JMP = 3x - 6$, $JK = 3y - 2$, and $LK = 5y - 8$.**



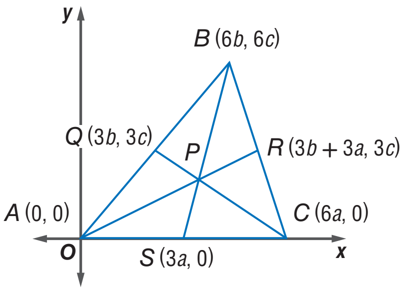
34. If $\overline{JM}$ is an altitude of $\triangle JLP$, find x.

35. Find LK if $\overline{PK}$ is a median.

36. **PROOF** **Write a coordinate proof to prove the Centroid Theorem.**

Given: $\triangle ABC$, medians $\overline{AR}$, $\overline{BS}$, and $\overline{CQ}$

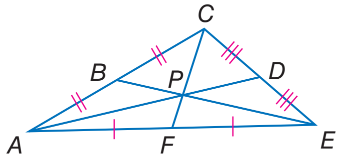
Prove: The medians intersect at point P and P is two thirds of the distance from each vertex to the midpoint of the opposite side.



(*Hint*: First, find the equations of the lines containing the medians. Then find the coordinates of point P and show that all three medians intersect at point P. Next, use the Distance Formula and multiplication to show $AP = \frac{2}{3}AR$, $BP = \frac{2}{3}BS$, and $CP = \frac{2}{3}CQ$.)

H.O.T. Problems Use Higher-Order Thinking Skills

37. **ERROR ANALYSIS** Based on the figure at the right, Luke says that $\frac{2}{3}AP = AD$. Kareem disagrees. Is either of them correct? Explain your reasoning.



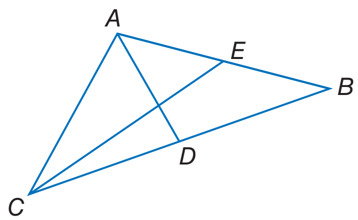
38. **CCSS ARGUMENTS** Determine whether the following statement is *true* or *false*. If true, explain your reasoning. If false, provide a counterexample.

The orthocenter of a right triangle is always located at the vertex of the right angle.

39. **CHALLENGE** $\triangle ABC$ has vertices $A(-3, 3)$, $B(2, 5)$, and $C(4, -3)$. What are the coordinates of the centroid of $\triangle ABC$? Explain the process you used to reach your conclusion.

40. **WRITING IN MATH** Compare and contrast the perpendicular bisectors, medians, and altitudes of a triangle.

41. **CHALLENGE** In the figure at the right, segments $\overline{AD}$ and $\overline{CE}$ are medians of $\triangle ACB$, $\overline{AD} \perp \overline{CE}$, $AB = 10$, and $CE = 9$. Find CA.



42. **OPEN ENDED** In this problem, you will investigate the relationships among three points of concurrency in a triangle.

a. Draw an acute triangle and find the circumcenter, centroid, and orthocenter.

b. Draw an obtuse triangle and find the circumcenter, centroid, and orthocenter.

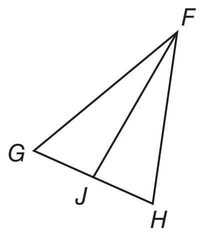
c. Draw a right triangle and find the circumcenter, centroid, and orthocenter.

d. Make a conjecture about the relationships among the circumcenter, centroid, and orthocenter.

43. **WRITING IN MATH** Use area to explain why the centroid of a triangle is its center of gravity. Then use this explanation to describe the location for the balancing point for a rectangle.

Standardized Test Practice

44. In the figure below, $\overline{GJ} \cong \overline{HJ}$. Which must be true?



A $\overline{FJ}$ is an altitude of $\triangle FGH$.

B $\overline{FJ}$ is an angle bisector of $\triangle FGH$.

C $\overline{FJ}$ is a median of $\triangle FGH$.

D $\overline{FJ}$ is a perpendicular bisector of $\triangle FGH$.

45. GRIDDED RESPONSE What is the x-intercept of the graph of $4x - 6y = 12$?

46. ALGEBRA Four students have volunteered to fold pamphlets for a local community action group. Which student is the fastest?

Student	Folding Speed
Neiva	1 page every 3 seconds
Sarah	2 pages every 10 seconds
Quinn	30 pages per minute
Deron	45 pages in 2 minutes

F Deron **H** Quinn
G Neiva **J** Sarah

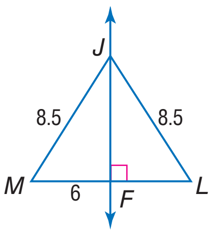
47. SAT/ACT 80 percent of 42 is what percent of 16?

A 240 **D** 50
B 210 **E** 30
C 150

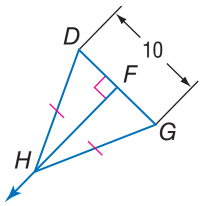
Spiral Review

Find each measure. (Lesson 5-1)

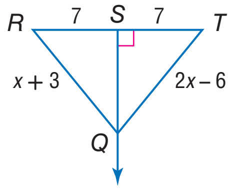
48. LM



49. DF



50. TQ



Position and label each triangle on the coordinate plane. (Lesson 4-8)

51. right $\triangle XYZ$ with hypotenuse $\overline{XZ}$, ZY is twice XY, and $\overline{XY}$ is b units long

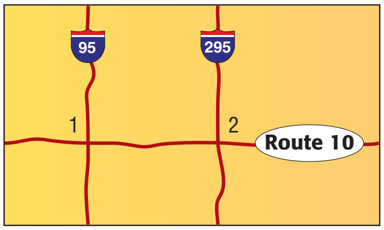
52. isosceles $\triangle QRT$ with base $\overline{QR}$ that is b units long

Determine whether $\overleftrightarrow{RS}$ and $\overleftrightarrow{JK}$ are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer. (Lesson 3-3)

53. $R(5, -4)$, $S(10, 0)$, $J(9, -8)$, $K(5, -13)$

54. $R(1, 1)$, $S(9, 8)$, $J(-6, 1)$, $K(2, 8)$

55. HIGHWAYS Near the city of Hopewell, Virginia, Route 10 runs perpendicular to Interstate 95 and Interstate 295. Show that the angles at the intersections of Route 10 with Interstate 95 and Interstate 295 are congruent. (Lesson 2-8)

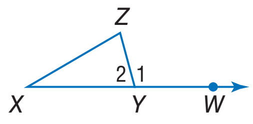


Skills Review

PROOF **Write a flow proof of the Exterior Angle Theorem.**

56. Given: $\triangle XYZ$

Prove: $m\angle X + m\angle Z = m\angle 1$



LESSON 5-3 Inequalities in One Triangle

Then

- You found the relationship between the angle measures of a triangle.

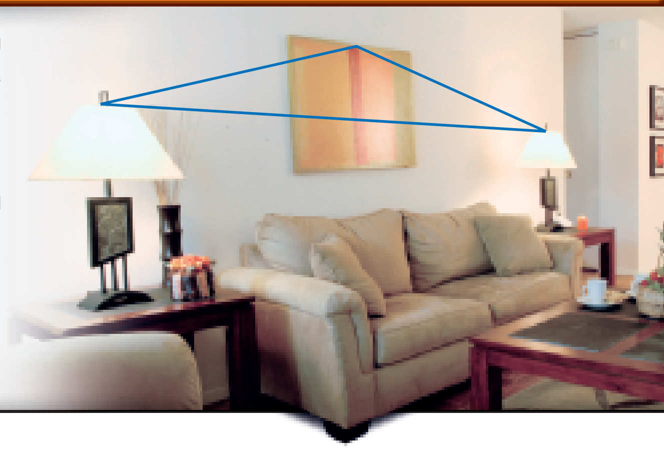
Now

1. Recognize and apply properties of inequalities to the measures of the angles of a triangle.
2. Recognize and apply properties of inequalities to the relationships between the angles and sides of a triangle.

Why?

- To create the appearance of depth in a room, interior designers use a technique called *triangulation*. A basic example of this technique is the placement of an end table on each side of a sofa with a painting over the sofa.

The measures of the base angles of the triangle should be less than the measure of the other angle.



Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

1 Angle Inequalities

In algebra, you learned about the inequality relationship between two real numbers. This relationship is often used in proofs.

KeyConcept Definition of Inequality

Words For any real numbers *a* and *b*, $a > b$ if and only if there is a positive number *c* such that $a = b + c$.

Example If $5 = 2 + 3$, then $5 > 2$ and $5 > 3$.

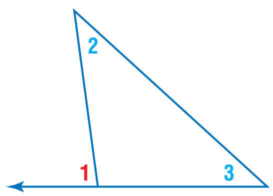
The table below lists some of the properties of inequalities you studied in algebra.

KeyConcept Properties of Inequality for Real Numbers

The following properties are true for any real numbers *a*, *b*, and *c*.

Comparison Property of Inequality	$a < b$, $a = b$, or $a > b$
Transitive Property of Inequality	**1.** If $a < b$ and $b < c$, then $a < c$. **2.** If $a > b$ and $b > c$, then $a > c$.
Addition Property of Inequality	**1.** If $a > b$, then $a + c > b + c$. **2.** If $a < b$, then $a + c < b + c$.
Subtraction Property of Inequality	**1.** If $a > b$, then $a - c > b - c$. **2.** If $a < b$, then $a - c < b - c$.

The definition of inequality and the properties of inequalities can be applied to the measures of angles and segments, since these are real numbers. Consider $\angle 1$, $\angle 2$, and $\angle 3$ in the figure shown.



By the Exterior Angle Theorem, you know that $m\angle 1 = m\angle 2 + m\angle 3$.

Since the angle measures are positive numbers, we can also say that

$$m\angle 1 > m\angle 2 \quad \text{and} \quad m\angle 1 > m\angle 3$$

by the definition of inequality. This result suggests the following theorem.

age fotostock/SuperStock

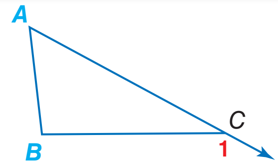
ReviewVocabulary

Remote Interior Angle Each exterior angle of a triangle has two *remote interior angles* that are not adjacent to the exterior angle.

Theorem 5.8 Exterior Angle Inequality

The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles.

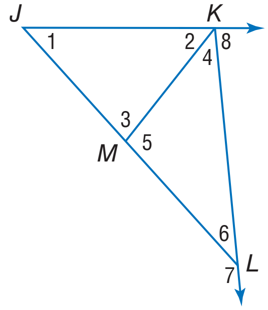
Example: $m\angle 1 > m\angle A$
$m\angle 1 > m\angle B$



The proof of Theorem 5.8 is in Lesson 5-4.

Example 1 Use the Exterior Angle Inequality Theorem

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



a. measures less than $m\angle 7$

$\angle 7$ is an exterior angle to $\triangle KML$, with $\angle 4$ and $\angle 5$ as corresponding remote interior angles. By the Exterior Angle Inequality Theorem, $m\angle 7 > m\angle 4$ and $m\angle 7 > m\angle 5$.

$\angle 7$ is also an exterior angle to $\triangle JKL$, with $\angle 1$ and $\angle JKL$ as corresponding remote interior angles. So, $m\angle 7 > m\angle 1$ and $m\angle 7 > m\angle JKL$. Since $m\angle JKL = m\angle 2 + m\angle 4$, by substitution $m\angle 7 > m\angle 2 + m\angle 4$. Therefore, $m\angle 7 > m\angle 2$.

So, the angles with measures less than $m\angle 7$ are $\angle 1$, $\angle 2$, $\angle 4$, $\angle 5$.

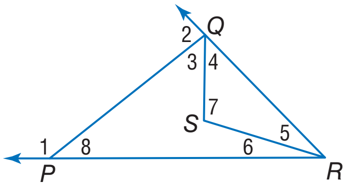
b. measures greater than $m\angle 6$

$\angle 3$ is an exterior angle to $\triangle KLM$. So by the Exterior Angle Inequality Theorem, $m\angle 3 > m\angle 6$. Because $\angle 8$ is an exterior angle to $\triangle JKL$, $m\angle 8 > m\angle 6$. Thus, the measures of $\angle 3$ and $\angle 8$ are greater than $m\angle 6$.

GuidedPractice

1A. measures less than $m\angle 1$

1B. measures greater than $m\angle 8$

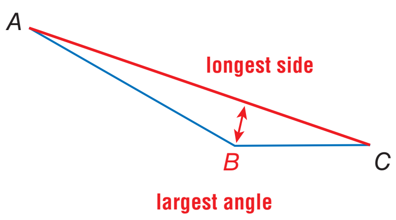


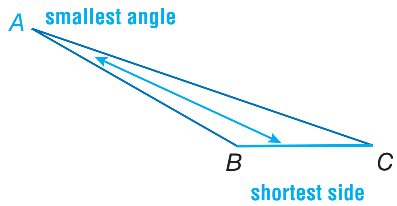
WatchOut!

Identifying Side Opposite Be careful to correctly identify the side opposite an angle. The sides that form the angle cannot be the sides opposite the angle.

2 Angle-Side Inequalities

In Lesson 4-6, you learned that if two sides of a triangle are congruent, or the triangle is isosceles, then the angles opposite those sides are congruent. What relationship exists if the sides are not congruent? Examine the longest and shortest sides and smallest and largest angles of a scalene obtuse triangle.





Notice that the longest side and largest angle of $\triangle ABC$ are opposite each other. Likewise, the shortest side and smallest angle are opposite each other.

WatchOut!

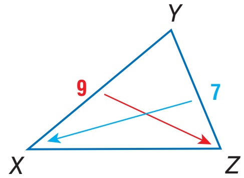
Symbols for Angles and Inequalities The symbol for angle ($\angle$) looks similar to the symbol for less than ($<$), especially when handwritten. Be careful to write the symbols correctly in situations where both are used.

The side-angle relationships in an obtuse scalene triangle are true for all triangles, and are stated using inequalities in the theorems below.

Theorems Angle-Side Relationships in Triangles

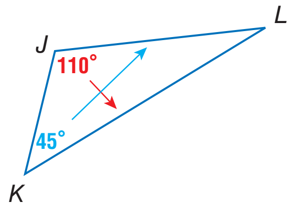
5.9 If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

Example: $XY > YZ$, so $m\angle Z > m\angle X$.



5.10 If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

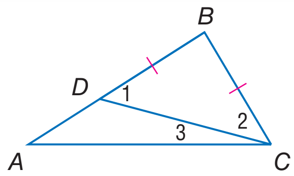
Example: $m\angle J > m\angle K$, so $KL > JL$.



Proof Theorem 5.9

Given: $\triangle ABC$, $AB > BC$

Prove: $m\angle BCA > m\angle A$



Proof:

Since $AB > BC$ in the given $\triangle ABC$, there exists a point D on $\overline{AB}$ such that $BD = BC$. Draw $\overline{CD}$ to form isosceles $\triangle BCD$. By the Isosceles Triangle Theorem, $\angle 1 \cong \angle 2$, so $m\angle 1 = m\angle 2$ by the definition of congruent angles.

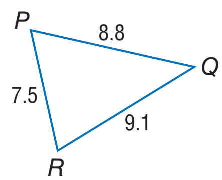
By the Angle Addition Postulate, $m\angle BCA = m\angle 2 + m\angle 3$, so $m\angle BCA > m\angle 2$ by the definition of inequality. By substitution, $m\angle BCA > m\angle 1$.

By the Exterior Angle Inequality Theorem, $m\angle 1 > m\angle A$. Therefore, because $m\angle BCA > m\angle 1$ and $m\angle 1 > m\angle A$, by the Transitive Property of Inequality, $m\angle BCA > m\angle A$.

You will prove Theorem 5.10 in Lesson 5-4, Exercise 31.

Example 2 Order Triangle Angle Measures

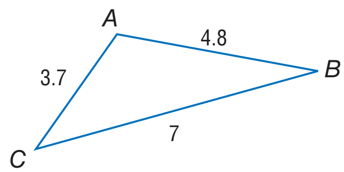
List the angles of $\triangle PQR$ in order from smallest to largest.



The sides from shortest to longest are $\overline{PR}$, $\overline{PQ}$, $\overline{QR}$. The angles opposite these sides are $\angle Q$, $\angle R$, and $\angle P$, respectively. So the angles from smallest to largest are $\angle Q$, $\angle R$, and $\angle P$.

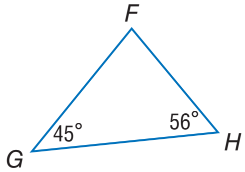
GuidedPractice

2. List the angles and sides of $\triangle ABC$ in order from smallest to largest.



Example 3 Order Triangle Side Lengths

List the sides of $\triangle FGH$ in order from shortest to longest.



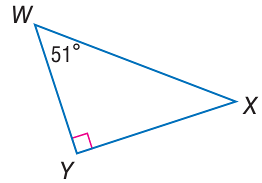
First find the missing angle measure using the Triangle Angle Sum Theorem.

$m\angle F = 180 - (45 + 56)$ or 79

So, the angles from smallest to largest are $\angle G$, $\angle H$, and $\angle F$. The sides opposite these angles are $\overline{FH}$, $\overline{FG}$, and $\overline{GH}$, respectively. So, the sides from shortest to longest are $\overline{FH}$, $\overline{FG}$, $\overline{GH}$.

GuidedPractice

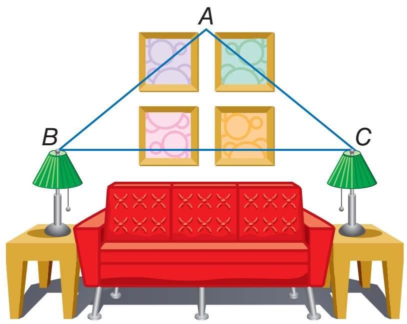
3. List the angles and sides of $\triangle WXY$ in order from smallest to largest.



You can use angle-side relationships in triangles to solve real-world problems.

Real-World Example 4 Angle-Side Relationships

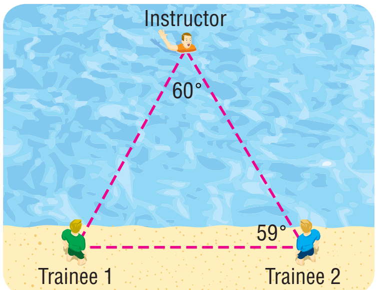
INTERIOR DESIGN An interior designer uses triangulation to create depth in a client's living room. If $m\angle B$ is to be less than $m\angle A$, which distance should be longer—the distance between the two lamps or the distance from the lamp at B to the midpoint of the top of the artwork? Explain.

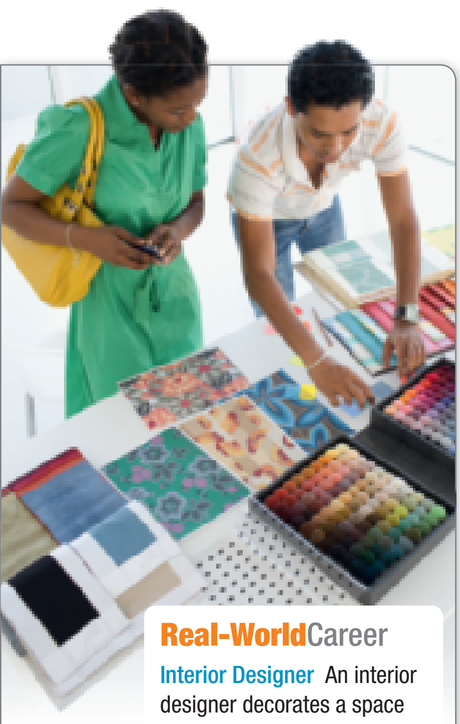


According to Theorem 5.10, in order for $m\angle B < m\angle A$, the length of the side opposite $\angle B$ must be less than the length of the side opposite $\angle A$. Since $\overline{AC}$ is opposite $\angle B$, and $\overline{BC}$ is opposite $\angle A$, then $AC < BC$ and $BC > AC$. So BC, the distance between the lamps, must be greater than the distance from the lamp at B to the midpoint of the top of the artwork.

GuidedPractice

4. LIFEGUARDING During lifeguard training, an instructor simulates a person in distress so that trainees can practice their rescue skills. If the instructor, Trainee 1, and Trainee 2 are located in the positions shown on the diagram, which of the two trainees is closest to the instructor?





Stuart O'Sullivan/Getty Images

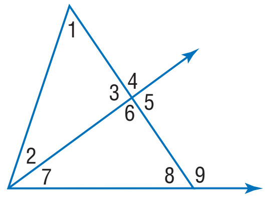
Real-WorldCareer

Interior Designer An interior designer decorates a space so that it is visually pleasing and comfortable for people to live or work in. Designers must know color and paint theory, lighting design, and space planning. A bachelor's degree is recommended for entry-level positions. Graduates usually enter a 1- to 3-year apprenticeship before taking a licensing exam.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.**



1. measures less than $m\angle 4$

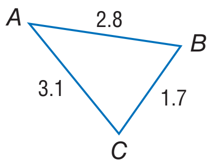
2. measures greater than $m\angle 7$

3. measures greater than $m\angle 2$

4. measures less than $m\angle 9$

Examples 2–3 **List the angles and sides of each triangle in order from smallest to largest.**

5.



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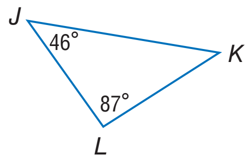
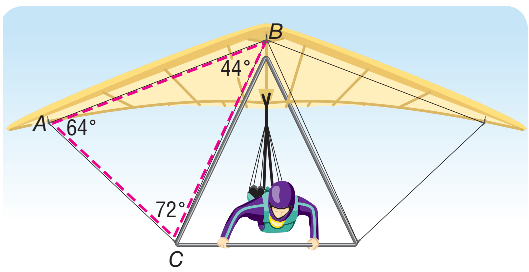


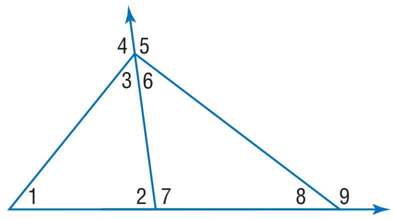
Example 4 **7.** **HANG GLIDING** The supports on a hang glider form triangles like the one shown. Which is longer—the support represented by $\overline{AC}$ or the support represented by $\overline{BC}$? Explain your reasoning.



Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **CCSS SENSE-MAKING** **Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.**



8. measures greater than $m\angle 2$

9. measures less than $m\angle 4$

10. measures less than $m\angle 5$

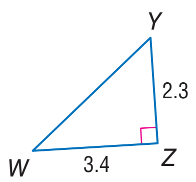
11. measures less than $m\angle 9$

12. measures greater than $m\angle 8$

13. measures greater than $m\angle 7$

Examples 2–3 **List the angles and sides of each triangle in order from smallest to largest.**

14.



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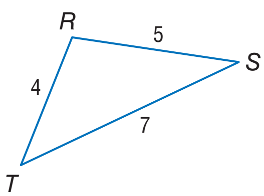

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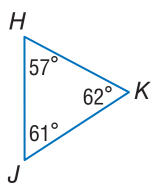
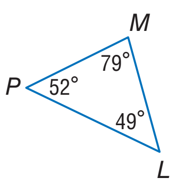


17.



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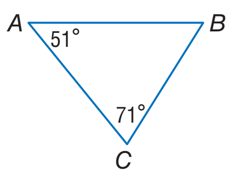

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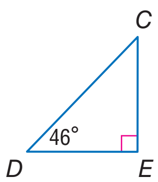
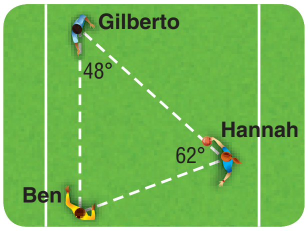
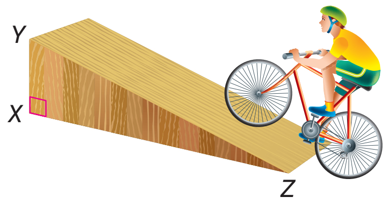


Example 4

20. SPORTS Ben, Gilberto, and Hannah are playing Ultimate. Hannah is trying to decide if she should pass to Ben or Gilberto. Which player should she choose in order to have the shorter passing distance? Explain your reasoning.

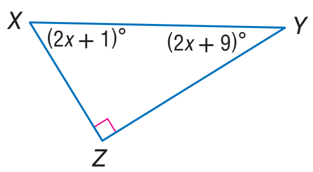


21 RAMPS The wedge below represents a bike ramp. Which is longer, the length of the ramp $\overline{XZ}$ or the length of the top surface of the ramp $\overline{YZ}$? Explain your reasoning using Theorem 5.9.

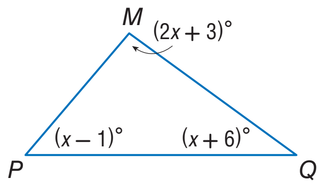


List the angles and sides of each triangle in order from smallest to largest.

22.



23.

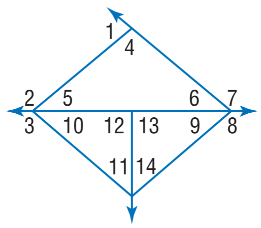


Use the figure at the right to determine which angle has the greatest measure.

24. ∠1, ∠5, ∠6 **25.** ∠2, ∠4, ∠6

26. ∠7, ∠4, ∠5 **27.** ∠3, ∠11, ∠12

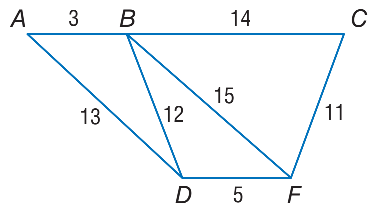
28. ∠3, ∠9, ∠14 **29.** ∠8, ∠10, ∠11



CCSS SENSE-MAKING Use the figure at the right to determine the relationship between the measures of the given angles.

30. ∠ABD, ∠BDA **31.** ∠BCF, ∠CFB

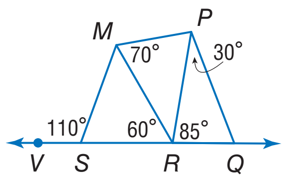
32. ∠BFD, ∠BDF **33.** ∠DBF, ∠BFD



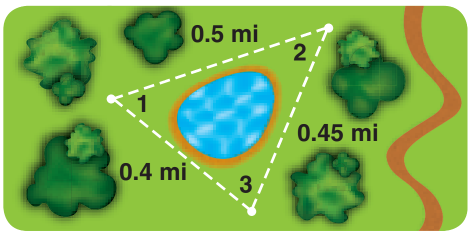
Use the figure at the right to determine the relationship between the given lengths.

34. SM, MR **35.** RP, MP

36. RQ, PQ **37.** RM, RQ



38. HIKING Justin and his family are hiking around a lake as shown in the diagram at the right. Order the angles of the triangle formed by their path from largest to smallest.

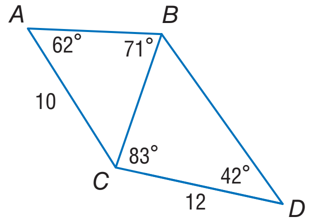


COORDINATE GEOMETRY **List the angles of each triangle with the given vertices in order from smallest to largest. Justify your answer.**

39. $A(-4, 6)$, $B(-2, 1)$, $C(5, 6)$

40. $X(-3, -2)$, $Y(3, 2)$, $Z(-3, -6)$

41. List the side lengths of the triangles in the figure from shortest to longest. Explain your reasoning.



42. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the sides of a triangle.

a. Geometric Draw three triangles, including one acute, one obtuse, and one right angle. Label the vertices of each triangle *A*, *B*, and *C*.

b. Tabular Measure the length of each side of the three triangles. Then copy and complete the table.

Triangle	*AB*	*BC*	*AB* + *BC*	*CA*
Acute				
Obtuse				
Right				

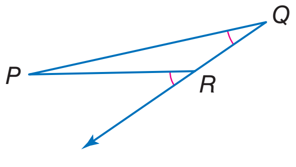
c. Tabular Create two additional tables like the one above, finding the sum of *BC* and *CA* in one table and the sum of *AB* and *CA* in the other.

d. Algebraic Write an inequality for each of the tables you created relating the measure of the sum of two of the sides to the measure of the third side of a triangle.

e. Verbal Make a conjecture about the relationship between the measure of the sum of two sides of a triangle and the measure of the third side.

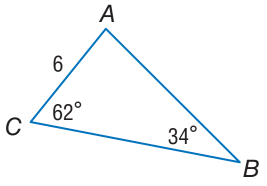
H.O.T. Problems Use Higher-Order Thinking Skills

43. **WRITING IN MATH** Analyze the information given in the diagram and explain why the markings must be incorrect.



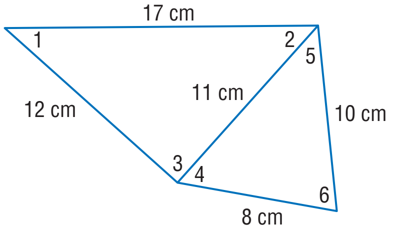
44. **CHALLENGE** Using only a ruler, draw $\triangle ABC$ such that $m\angle A > m\angle B > m\angle C$. Justify your drawing.

45. **OPEN ENDED** Give a possible measure for $\overline{AB}$ in $\triangle ABC$ shown. Explain your reasoning.



46. **CCSS ARGUMENTS** Is the base of an isosceles triangle *always*, *sometimes*, or *never* the longest side of the triangle? Explain.

47. **CHALLENGE** Use the side lengths in the figure to list the numbered angles in order from smallest to largest given that $m\angle 2 = m\angle 5$. Explain your reasoning.



48. **WRITING IN MATH** Why is the hypotenuse always the longest side of a triangle?

Standardized Test Practice

49. STATISTICS The chart shows the number and types of DVDs sold at three stores.

DVD Type	Store 1	Store 2	Store 3
Comedy	75	80	92
Action	54	37	65
Horror	30	48	62
Science Fiction	21	81	36
Total	180	246	255

According to the information in the chart, which of these statements is true?

A The mean number of DVDs sold per store was 56.

B Store 1 sold twice as many action and horror films as store 3 sold of science fiction.

C Store 2 sold fewer comedy and science fiction than store 3 sold.

D The mean number of science fiction DVDs sold per store was 46.

50. Two angles of a triangle have measures 45° and 92°. What type of triangle is it?

F obtuse scalene **H** acute scalene

G obtuse isosceles **J** acute isosceles

51. EXTENDED RESPONSE At a five-star restaurant, a waiter earns a total of t dollars for working h hours in which he receives $198 in tips and makes $2.50 per hour.

a. Write an equation to represent the total amount of money the waiter earns.

b. If the waiter earned a total of $213, how many hours did he work?

c. If the waiter earned $150 in tips and worked for 12 hours, what is the total amount of money he earned?

52. SAT/ACT Which expression has the *least* value?

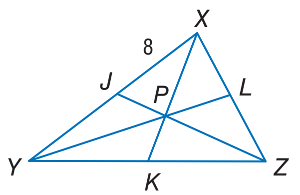
A $|-99|$ **D** $|-28|$

B $|45|$ **E** $|15|$

C $|-39|$

Spiral Review

In $\triangle XYZ$, P is the centroid, $KP = 3$, and $XJ = 8$. Find each length. (Lesson 5-2)



53. XK

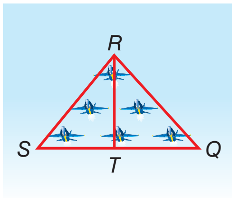
54. YJ

COORDINATE GEOMETRY **Write an equation in slope-intercept form for the perpendicular bisector of the segment with the given endpoints. Justify your answer.** (Lesson 5-1)

55. $D(-2, 4)$ and $E(3, 5)$

56. $D(-2, -4)$ and $E(2, 1)$

57. JETS The United States Navy Flight Demonstration Squadron, the Blue Angels, flies in a formation that can be viewed as two triangles with a common side. Write a two-column proof to prove that $\triangle SRT \cong \triangle QRT$ if T is the midpoint of $\overline{SQ}$ and $\overline{SR} \cong \overline{QR}$. (Lesson 4-4)



58. POOLS A rectangular pool is 20 feet by 30 feet. The depth of the pool is 60 inches, but the depth of the water is $\frac{3}{4}$ of the depth of the pool. Find each measure to the nearest tenth. (Lesson 1-7)

a. the surface area of the pool

b. the volume of water in the pool

Skills Review

Determine whether each statement is true or false if $x = 8$, $y = 2$, and $z = 3$.

59. $z(x - y) = 13$

60. $2x = 3yz$

61. $x + y > z + y$

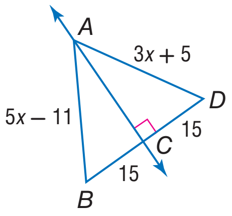
CHAPTER 5

Mid-Chapter Quiz

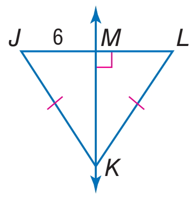
Lessons 5-1 through 5-3

Find each measure. (Lesson 5-1)

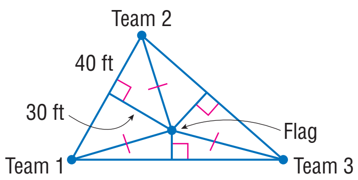
1. AB



2. JL

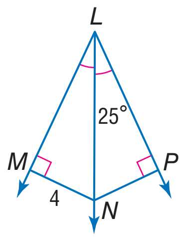


3. CAMP Camp Onawatchi ends with a game of capture the flag. If the starting locations of three teams are shown in the diagram below, with the flag at a point equidistant from each team's base, how far from each base is the flag in feet? (Lesson 5-1)

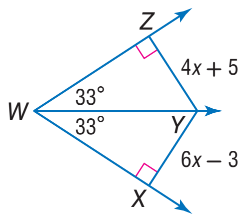


Find each measure. (Lesson 5-1)

4. $\angle MNP$



5. XY

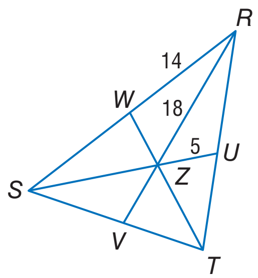


In $\triangle RST$, Z is the centroid and $RZ = 18$. Find each length. (Lesson 5-2)

6. ZV

7. SZ

8. SR



COORDINATE GEOMETRY **Find the coordinates of the centroid of each triangle with the given vertices.** (Lesson 5-2)

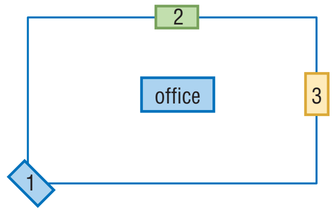
9. $A(1, 7)$, $B(4, 2)$, $C(7, 7)$

10. $X(-11, 0)$, $Y(-11, -8)$, $Z(-1, -4)$

11. $R(-6, 4)$, $S(-2, -2)$, $T(2, 4)$

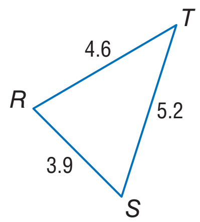
12. $J(-5, 5)$, $K(-5, -1)$, $L(1, 2)$

13. ARCHITECTURE An architect is designing a high school building. Describe how to position the central office so that it is at the intersection of each hallway connected to the three entrances to the school. (Lesson 5-2)

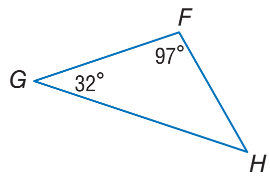


List the angles and sides of each triangle in order from smallest to largest. (Lesson 5-3)

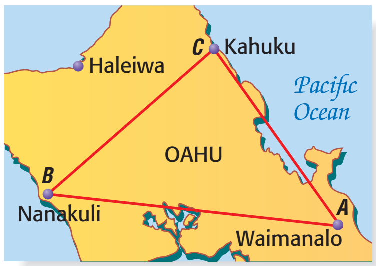
14.



15.



16. VACATION Kailey plans to fly over the route marked on the map of Hawaii below. (Lesson 5-3)



a. If $m\angle A = 2 + m\angle B$ and $m\angle C = 2(m\angle B) - 14$, what are the measures of the three angles?

b. What are the lengths of Kailey's trip in order of least to greatest?

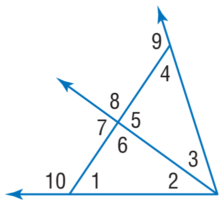
c. The length of the entire trip is about 68 miles. The middle leg is 11 miles greater than one-half the length of the shortest leg. The longest leg is 12 miles greater than three-fourths of the shortest leg. What are the lengths of the legs of the trip?

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition. (Lesson 5-3)

17. measures less than $m\angle 8$

18. measures greater than $m\angle 3$

19. measures less than $m\angle 10$



EXPLORE

5-4 Geometry Lab
Matrix Logic

Matrix logic uses a rectangular array in which you record what you have learned from clues in order to solve a logic or reasoning problem. Once all the rows and columns are filled, you can deduce the answer.

FOOD **Matt, Abby, Javier, Corey, and Keisha go to an Italian restaurant. Each orders their favorite dish: ravioli, pizza, lasagna, manicotti, or spaghetti. Javier loves ravioli, but Matt does not like pasta dishes. Abby does not like lasagna or manicotti. Corey's favorite dish does not end in the letter i. What does each person order?**

Step 1 Create an appropriate matrix.

Use a 5 × 5 matrix that includes each person's name as the header for each row and their possible favorite foods as the header for each column.

Name	ravioli	pizza	lasagna	manicotti	spaghetti
Matt	×	✓	×	×	×
Abby	×	×			
Javier	✓	×	×	×	×
Corey	×	×			
Keisha	×	×			

Step 2 Use each clue and logical reasoning to fill in the matrix.

- Since Javier loves ravioli, place a ✓ in Javier's row under ravioli and an × in every other cell in his row. Since only one person likes each dish, you can place an × in every other cell in the ravioli column.

- Since Matt does not like pasta, you know that Matt cannot like manicotti, ravioli, lasagna, or spaghetti, which are all pasta dishes. Therefore, Matt must like pizza. Place a ✓ in Matt's row under pizza. Place an × in every other cell in Matt's row and in every other cell in the pizza column.

Name	ravioli	pizza	lasagna	manicotti	spaghetti
Matt	×	✓	×	×	×
Abby	×	×	×	×	✓
Javier	✓	×	×	×	×
Corey	×	×			×
Keisha	×	×			×

- Since Abby does not like lasagna or manicotti, place an × in Abby's row under lasagna and manicotti. This leaves only spaghetti without an × for Abby's row. Therefore, you can conclude that Abby must like spaghetti. Place a ✓ in that cell and an × in every other cell in the spaghetti column.

- From the matrix, you can see that Corey's favorite dish must be either lasagna or manicotti. However, since Corey's favorite dish does not end in the letter i, you can conclude that Corey must like lasagna. In Corey's row, place a ✓ under lasagna and an × under manicotti.

- This leaves only one empty cell in Keisha's row, so you can conclude that her favorite dish is manicotti.

Name	ravioli	pizza	lasagna	manicotti	spaghetti
Matt	×	✓	×	×	×
Abby	×	×	×	×	✓
Javier	✓	×	×	×	×
Corey	×	×	✓	×	×
Keisha	×	×	×	✓	×

Step 3 Use your matrix to state the answer to the problem.

From the matrix you can state that Matt orders pizza, Abby orders spaghetti, Javier orders ravioli, Corey orders lasagna, and Keisha orders manicotti.

Geometry Lab

Matrix Logic *Continued*

Exercises

Use a matrix to solve each problem.

1. **SPORTS** Trey, Nathan, Parker, and Chen attend the same school. Each participates in a different school sport: basketball, football, track, or tennis. Use the following clues to determine in which sport each student participates.

- Nathan does not like track or basketball.
- Trey does not participate in football or tennis.
- Parker prefers an indoor winter sport.
- Chen scored four touchdowns in the final game of the season.

2. **FAMILY** The Martin family has five children. Use the following clues to determine in what order the children were born.

- Grace is older than Hannah.
- Thomas is younger than Sarah.
- Hannah is older than Thomas and Samuel.
- Samuel is older than Thomas.
- Sarah is older than Grace.

3. **PETS** Alejandra, Tamika, and Emily went to a pet store. Each girl chose a different pet to adopt: a dog, a rabbit, or a cat. Each girl named her pet Sweet Pea, Zuzu, or Roscoe. Use the following clues and the matrix shown to determine the animal each girl adopted and what name she gave her pet.

- The girl who adopted a dog did not name it Sweet Pea.
- Tamika's pet, who she named Zuzu, is not the type of animal that hops.
- Roscoe, who is not a cat, was adopted by Emily.
- The rabbit was not adopted by Alejandra.

		Pet			Pet Name		
		dog	rabbit	cat	Sweet Pea	Zuzu	Roscoe
Names	Alejandra						
	Tamika						
	Emily						
Pet Name	Sweet Pea						
	Zuzu						
	Roscoe						

4. **GEOMETRY** Kasa, Marcus, and Jason each drew a triangle, no two of which share the same side or angle classification. Use the following clues to determine what type of triangle each person has drawn.

- Kasa did not draw an equilateral triangle.
- Marcus' triangle has one angle that measures 25 and another that measures 65.
- Jason drew a triangle with at least one pair of congruent sides.
- The obtuse triangle has two congruent angles.

LESSON 5-4 Indirect Proof

Then

- You wrote paragraph, two-column, and flow proofs.

Now

1. Write indirect algebraic proofs.
2. Write indirect geometric proofs.

Why?

- **Matthew:** "I'm almost positive Friday is not a teacher work day, but I can't prove it."
- **Kim:** "Let's assume that Friday *is* a teacher work day. What day is our next Geometry test?"
- **Ana:** "Hmmm . . . according to the syllabus, it's this Friday. But we don't have tests on teacher work days—we're not in school."
- **Jamal:** "Exactly—so that proves it! This Friday can't be a teacher work day."

NewVocabulary
indirect reasoning
indirect proof
proof by contradiction

Common Core State Standards

Content Standards
G.CO.10 Prove theorems about triangles.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
2 Reason abstractly and quantitatively.

1 Indirect Algebraic Proof

The proofs you have written have been *direct proofs*—you started with a true hypothesis and proved that the conclusion was true. In the example above, the students used **indirect reasoning**, by assuming that a conclusion was false and then showing that this assumption led to a contradiction.

In an **indirect proof** or **proof by contradiction**, you temporarily assume that what you are trying to prove is false. By showing this assumption to be logically impossible, you prove your assumption false and the original conclusion true. Sometimes this is called *proof by negation*.

KeyConcept How to Write an Indirect Proof

Step 1 Identify the conclusion you are asked to prove. Make the assumption that this conclusion is false by assuming that the opposite is true.

Step 2 Use logical reasoning to show that this assumption leads to a contradiction of the hypothesis, or some other fact, such as a definition, postulate, theorem, or corollary.

Step 3 Point out that since the assumption leads to a contradiction, the original conclusion, what you were asked to prove, must be true.

Example 1 State the Assumption for Starting an Indirect Proof

State the assumption necessary to start an indirect proof of each statement.

a. If 6 is a factor of n, then 2 is a factor of n.

The conclusion of the conditional statement is *2 is a factor of n*. The negation of the conclusion is *2 is not a factor of n*.

b. $\angle 3$ is an obtuse angle.

If *∠3 is an obtuse angle* is false, then *∠3 is not an obtuse angle* must be true.

GuidedPractice

1A. $x > 5$

1B. $\triangle XYZ$ is an equilateral triangle.

Digital Vision/Punchstock

Indirect proofs can be used to prove algebraic concepts.

Example 2 Write an Indirect Algebraic Proof

Write an indirect proof to show that if $-3x + 4 > 16$, then $x < -4$.

Given: $-3x + 4 > 16$

Prove: $x < -4$

Step 1 **Indirect Proof:**
The negation of $x < -4$ is $x \geq -4$. So, assume that $x > -4$ or $x = -4$ is true.

Step 2 Make a table with several possibilities for x assuming $x > -4$ or $x = -4$.

x	−4	−3	−2	−1	0
$-3x + 4$	16	13	10	7	4

When $x > -4$, $-3x + 4 < 16$ and when $x = -4$, $-3x + 4 = 16$.

Step 3 In both cases, the assumption leads to the contradiction of the given information that $-3x + 4 > 16$. Therefore, the assumption that $x \geq -4$ must be false, so the original conclusion that $x < -4$ must be true.

GuidedPractice

Write an indirect proof of each statement.

2A. If $7x > 56$, then $x > 8$.

2B. If $-c$ is positive, then c is negative.

ReadingMath

Contradiction
A contradiction is a principle of logic stating that an assumption cannot be both *A* and the opposite of *A* at the same time.

Indirect reasoning and proof can be used in everyday situations.

Real-World Example 3 Indirect Algebraic Proof

PROM COSTS **Javier asked his friend Christopher the cost of his meal and his date's meal when he went to dinner for prom. Christopher could not remember the individual costs, but he did remember that the total bill, not including tip, was over $60. Use indirect reasoning to show that at least one of the meals cost more than $30.**

Let the cost of one meal be x and the cost of the other meal be y.

Step 1 **Given:** $x + y > 60$

Prove: $x > 30$ or $y > 30$

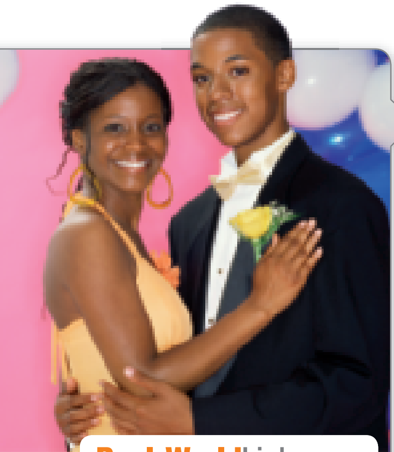
Indirect Proof:
Assume that $x \leq 30$ and $y \leq 30$.

Step 2 If $x \leq 30$ and $y \leq 30$, then $x + y \leq 30 + 30$ or $x + y \leq 60$. This is a contradiction because we know that $x + y > 60$.

Step 3 Since the assumption that $x \leq 30$ and $y \leq 30$ leads to a contradiction of a known fact, the assumption must be false. Therefore, the conclusion that $x > 30$ or $y > 30$ must be true. Thus, at least one of the meals had to cost more than $30.

GuidedPractice

3. TRAVEL Cleavon traveled over 360 miles on his trip, making just two stops. Use indirect reasoning to prove that he traveled more than 120 miles on one leg of his trip.



Real-WorldLink

$100–$300 the range in price of a girl's prom dress

$75–$125 the range in cost for a tuxedo rental

around $150 the cost of a fancy dinner for two

$100–$200 the range in cost of prom tickets per couple

Source: PromSpot

Indirect proofs are often used to prove concepts in number theory. In such proofs, it is helpful to remember that you can represent an even number with the expression $2k$ and an odd number with the expression $2k + 1$ for any integer k.

Example 4 Indirect Proofs in Number Theory

Write an indirect proof to show that if $x + 2$ is an even integer, then x is an even integer.

Step 1 **Given:** $x + 2$ is an even integer.

Prove: x is an even integer.

Indirect Proof:
Assume that x is an odd integer. This means that $x = 2k + 1$ for some integer k.

Step 2 $x + 2 = (2k + 1) + 2$ — Substitution of assumption

$= (2k + 2) + 1$ — Commutative Property

$= 2(k + 1) + 1$ — Distributive Property

Now determine whether $2(k + 1) + 1$ is an even or odd integer. Since k is an integer, $k + 1$ is also an integer. Let m represent the integer $k + 1$.

$2(k + 1) + 1 = 2m + 1$ — Substitution

So, $x + 2$ can be represented by $2m + 1$, where m is an integer. But this representation means that $x + 2$ is an odd integer, which contradicts the given statement that $x + 2$ is an even integer.

Step 3 Since the assumption that x is an odd integer leads to a contradiction of the given statement, the original conclusion that x is an even integer must be true.

WatchOut!

CCSS Arguments Proof by contradiction and using a counterexample are not the same. A counterexample helps you disprove a conjecture. It cannot be used to prove a conjecture.

GuidedPractice

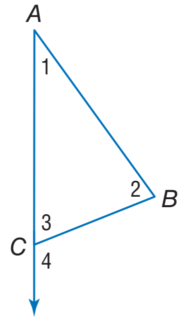
4. Write an indirect proof to show that if the square of an integer is odd, then the integer is odd.

2 Indirect Proof with Geometry

Indirect reasoning can be used to prove statements in geometry, such as the Exterior Angle Inequality Theorem.

Example 5 Geometry Proof

If an angle is an exterior angle of a triangle, prove that its measure is greater than the measure of either of its corresponding remote interior angles.



Step 1 Draw a diagram of this situation. Then identify what you are given and what you are asked to prove.

Given: $\angle 4$ is an exterior angle of $\triangle ABC$.

Prove: $m\angle 4 > m\angle 1$ and $m\angle 4 > m\angle 2$.

Indirect Proof:
Assume that $m\angle 4 \not> m\angle 1$ or $m\angle 4 \not> m\angle 2$.
In other words, $m\angle 4 \leq m\angle 1$ or $m\angle 4 \leq m\angle 2$.

(continued on the next page)

Step 2 You need only show that the assumption $m\angle 4 \leq m\angle 1$ leads to a contradiction. The argument for $m\angle 4 \leq m\angle 2$ follows the same reasoning.

$m\angle 4 \leq m\angle 1$ means that either $m\angle 4 = m\angle 1$ or $m\angle 4 < m\angle 1$.

Case 1 $m\angle 4 = m\angle 1$

$m\angle 4 = m\angle 1 + m\angle 2$ — Exterior Angle Theorem

$m\angle 4 = m\angle 4 + m\angle 2$ — Substitution

$0 = m\angle 2$ — Subtract $m\angle 4$ from each side.

This contradicts the fact that the measure of an angle is greater than 0, so $m\angle 4 \neq m\angle 1$.

Case 2 $m\angle 4 < m\angle 1$

By the Exterior Angle Theorem, $m\angle 4 = m\angle 1 + m\angle 2$. Since angle measures are positive, the definition of inequality implies that $m\angle 4 > m\angle 1$. This contradicts the assumption that $m\angle 4 < m\angle 1$.

Step 3 In both cases, the assumption leads to the contradiction of a theorem or definition. Therefore, the original conclusion that $m\angle 4 > m\angle 1$ and $m\angle 4 > m\angle 2$ must be true.

StudyTip

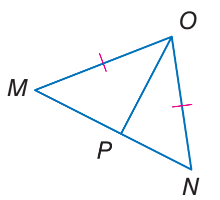
Recognizing Contradictions Remember that the contradiction in an indirect proof is not always of the given information or the assumption. It can be of a known fact or definition, such as in Case 1 of Example 5—the measure of an angle must be greater than 0.

GuidedPractice

5. Write an indirect proof.

Given: $\overline{MO} \cong \overline{ON}$, $\overline{MP} \ncong \overline{NP}$

Prove: $\angle MOP \ncong \angle NOP$



Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **State the assumption you would make to start an indirect proof of each statement.**

1. $\overline{AB} \cong \overline{CD}$

2. $\triangle XYZ$ is a scalene triangle.

3 If $4x < 24$, then $x < 6$.

4. $\angle A$ is not a right angle.

Example 2 **Write an indirect proof of each statement.**

5. If $2x + 3 < 7$, then $x < 2$.

6. If $3x - 4 > 8$, then $x > 4$.

Example 3 **7.** **LACROSSE** Christina scored 13 points for her high school lacrosse team during the last six games. Prove that her average points per game was less than 3.

Example 4 **8.** Write an indirect proof to show that if $5x - 2$ is an odd integer, then x is an odd integer.

Example 5 **Write an indirect proof of each statement.**

9. The hypotenuse of a right triangle is the longest side.

10. If two angles are supplementary, then they both cannot be obtuse angles.

Example 1 **State the assumption you would make to start an indirect proof of each statement.**

11. If $2x > 16$, then $x > 8$.

12. $\angle 1$ and $\angle 2$ are not supplementary angles.

13. If two lines have the same slope, the lines are parallel.

14. If the consecutive interior angles formed by two lines and a transversal are supplementary, the lines are parallel.

15. If a triangle is not equilateral, the triangle is not equiangular.

16. An odd number is not divisible by 2.

Example 2 **Write an indirect proof of each statement.**

17. If $2x - 7 > -11$, then $x > -2$.

18. If $5x + 12 < -33$, then $x < -9$.

19. If $-3x + 4 < 7$, then $x > -1$.

20. If $-2x - 6 > 12$, then $x < -9$.

Example 3

21. **COMPUTER GAMES** Kwan-Yong bought two computer games for just over \$80 before tax. A few weeks later, his friend asked how much each game cost. Kwan-Yong could not remember the individual prices. Use indirect reasoning to show that at least one of the games cost more than \$40.

22. **FUNDRAISING** Jamila's school is having a Fall Carnival to raise money for a local charity. The cost of an adult ticket to the carnival is \$6 and the cost of a child's ticket is \$2.50. If 375 total tickets were sold and the profit was more than \$1460, prove that at least 150 adult tickets were sold.

Examples 4–5 CCSS **ARGUMENTS** **Write an indirect proof of each statement.**

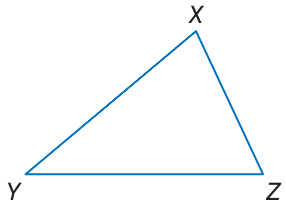
23. **Given:** xy is an odd integer.
Prove: x and y are both odd integers.

24. **Given:** n^2 is even.
Prove: n^2 is divisible by 4.

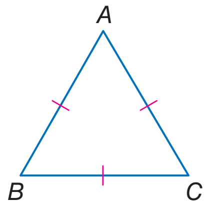
25. **Given:** x is an odd number.
Prove: x is not divisible by 4.

26. **Given:** xy is an even integer.
Prove: x or y is an even integer.

27. **Given:** $XZ > YZ$
Prove: $\angle X \not\cong \angle Y$



28. **Given:** $\triangle ABC$ is equilateral.
Prove: $\triangle ABC$ is equiangular.



29. In an isosceles triangle neither of the base angles can be a right angle.

30. A triangle can have only one right angle.

31. Write an indirect proof for Theorem 5.10.

32. Write an indirect proof to show that if $\frac{1}{b} < 0$, then b is negative.

33. **BASKETBALL** In basketball, there are three possible ways to score three points in a single possession. A player can make a basket from behind the three-point line, a player may be fouled while scoring a two-point shot and be allowed to shoot one free throw, or a player may be fouled behind the three-point line and be allowed to shoot three free throws. When Katsu left to get in the concession line, the score was 28 home team to 26 visiting team. When she returned, the score was 28 home team to 29 visiting team. Katsu concluded that a player on the visiting team had made a three-point basket. Prove or disprove her assumption using an indirect proof.

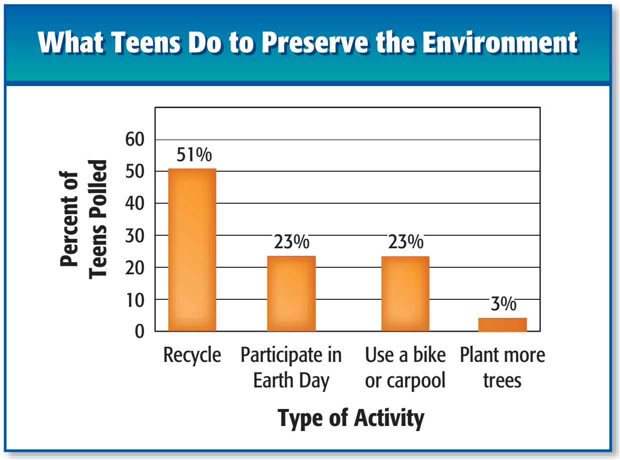
34. **GAMES** A computer game involves a knight on a quest for treasure. At the end of the journey, the knight approaches the two doors shown below.

A servant tells the knight that one of the signs is true and the other is false. Use indirect reasoning to determine which door the knight should choose. Explain your reasoning.

35 **SURVEYS** Luisa's local library conducted an online poll of teens to find out what activities teens participate in to preserve the environment. The results of the poll are shown in the graph.

a. Prove: *More than half of teens polled said that they recycle to preserve the environment.*

b. If 400 teens were polled, verify that 92 said that they participate in Earth Day.



36. CCSS **REASONING** James, Hector, and Mandy all have different color cars. Only one of the statements below is true. Use indirect reasoning to determine which statement is true. Explain.

(1) James has a red car.

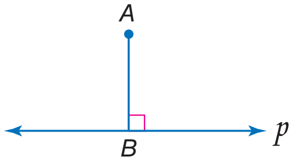
(2) Hector does not have a red car.

(3) Mandy does not have a blue car.

Determine whether each statement about the shortest distance between a point and a line or plane can be proved using a direct or indirect proof. Then write a proof of each statement.

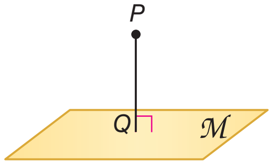
37. **Given:** $\overline{AB} \perp$ line p

Prove: $\overline{AB}$ is the shortest segment from A to line p.



38. **Given:** $\overline{PQ} \perp$ plane M

Prove: $\overline{PQ}$ is the shortest segment from P to plane M.



39. **NUMBER THEORY** In this problem, you will make and prove a conjecture about a number theory relationship.

a. Write an expression for *the sum of the cube of a number and three.*

b. Create a table that includes the value of the expression for 10 different values of n. Include both odd and even values of n.

c. Write a conjecture about n when the value of the expression is even.

d. Write an indirect proof of your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

40. **WRITING IN MATH** Explain the procedure for writing an indirect proof.

41. **OPEN ENDED** Write a statement that can be proven using indirect proof. Include the indirect proof of your statement.

42. **CHALLENGE** If x is a rational number, then it can be represented by the quotient $\frac{a}{b}$ for some integers a and b, if $b \neq 0$. An irrational number cannot be represented by the quotient of two integers. Write an indirect proof to show that the product of a nonzero rational number and an irrational number is an irrational number.

43. **CCSS CRITIQUE** Amber and Raquel are trying to verify the following statement using indirect proof. Is either of them correct? Explain your reasoning.

If the sum of two numbers is even, then the numbers are even.

Amber

The statement is true. If one of the numbers is even and the other number is zero, then the sum is even. Since the hypothesis is true even when the conclusion is false, the statement is true.

Raquel

The statement is true. If the two numbers are odd, then the sum is even. Since the hypothesis is true when the conclusion is false, the statement is true.

44. **WRITING IN MATH** Refer to Exercise 8. Write the contrapositive of the statement and write a direct proof of the contrapositive. How are the direct proof of the contrapositive of the statement and the indirect proof of the statement related?

Standardized Test Practice

45. SHORT RESPONSE Write an equation in slope-intercept form to describe the line that passes through the point (5, 3) and is parallel to the line represented by the equation $-2x + y = -4$.

46. Statement: If $\angle A \cong \angle B$ and $\angle A$ is supplementary to $\angle C$, then $\angle B$ is supplementary to $\angle C$.

Dia is proving the statement above by contradiction. She began by assuming that $\angle B$ is not supplementary to $\angle C$. Which of the following definitions will Dia use to reach a contradiction?

A definition of congruence
B definition of a linear pair
C definition of a right angle
D definition of supplementary angles

47. List the angles of $\triangle MNO$ in order from smallest to largest if $MN = 9$, $NO = 7.5$, and $OM = 12$.

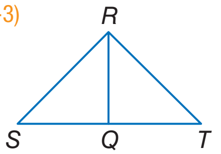
F $\angle N, \angle O, \angle M$
G $\angle O, \angle M, \angle N$
H $\angle O, \angle N, \angle M$
J $\angle M, \angle O, \angle N$

48. SAT/ACT If $b > a$, which of the following must be true?

A $-a > -b$
B $3a > b$
C $a^2 < b^2$
D $a^2 < ab$
E $-b > -a$

Spiral Review

49. PROOF Write a two-column proof. (Lesson 5-3)
Given: $\overline{RQ}$ bisects $\angle SRT$.
Prove: $m\angle SQR > m\angle SRQ$

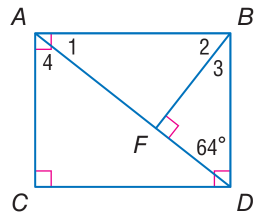


COORDINATE GEOMETRY **Find the coordinates of the circumcenter of each triangle with the given vertices.** (Lesson 5-1)

50. $D(-3, 3), E(3, 2), F(1, -4)$

51. $A(4, 0), B(-2, 4), C(0, 6)$

Find each measure. (Lesson 4-2)



52. $m\angle 1$

53. $m\angle 4$

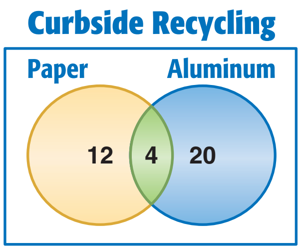
COORDINATE GEOMETRY **Find the distance between each pair of parallel lines with the given equations.** (Lesson 3-6)

54. $x + 3y = 6$
$x + 3y = -14$

55. $y = 2x + 2$
$y = 2x - 3$

56. RECYCLING Refer to the Venn diagram that represents the number of neighborhoods in a city with a curbside recycling program for paper or aluminum. (Lesson 2-2)

a. How many neighborhoods recycle aluminum?
b. How many neighborhoods recycle paper or aluminum or both?
c. How many neighborhoods recycle paper and aluminum?



Skills Review

Determine whether each inequality is *true* or *false*.

57. $23 - 11 > 9$

58. $41 - 19 < 21$

59. $57 + 68 < 115$

EXPLORE 5-5

Graphing Technology Lab
The Triangle Inequality

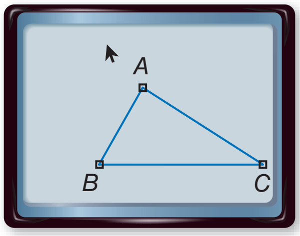
You can use the Cabri™ Jr. application on a TI-83/84 Plus graphing calculator to discover properties of triangles.

Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Activity 1

Construct a triangle. Observe the relationship between the sum of the lengths of two sides and the length of the other side.

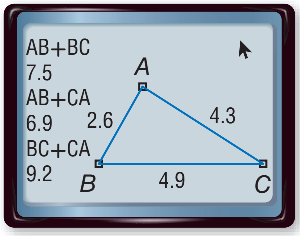
Step 1 Construct a triangle using the triangle tool on the **F2** menu. Then use the **Alph-Num** tool on the **F5** menu to label the vertices as A, B, and C.



Step 1

Step 2 Access the **distance & length** tool, shown as **D. & Length**, under **Measure** on the **F5** menu. Use the tool to measure each side of the triangle.

Step 3 Display $AB + BC$, $AB + CA$, and $BC + CA$ by using the **Calculate** tool on the **F5** menu. Label the measures.



Steps 2 and 3

Step 4 Click and drag the vertices to change the shape of the triangle.

Analyze the Results

1. Replace each ● with <, >, or = to make a true statement.
$AB + BC$ ● CA $AB + CA$ ● BC $BC + CA$ ● AB

2. Click and drag the vertices to change the shape of the triangle. Then review your answers to Exercise 1. What do you observe?

3. Click on point A and drag it to lie on line BC. What do you observe about AB, BC, and CA? Are A, B, and C the vertices of a triangle? Explain.

4. **Make a conjecture** about the sum of the lengths of two sides of a triangle and the length of the third side.

5. Do the measurements and observations you made in the Activity and in Exercises 1–3 constitute a proof of the conjecture you made in Exercise 4? Explain.

6. Replace each ● with <, >, or = to make a true statement.
$|AB - BC|$ ● CA $|AB - CA|$ ● BC $|BC - CA|$ ● AB

Then click and drag the vertices to change the shape of the triangle and review your answers. What do you observe?

7. How could you use your observations to determine the possible lengths of the third side of a triangle if you are given the lengths of the other two sides?

LESSON 5-5

The Triangle Inequality

Then

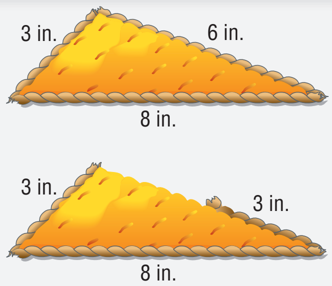
- You recognized and applied properties of inequalities to the relationships between the angles and sides of a triangle.

Now

- **1** Use the Triangle Inequality Theorem to identify possible triangles.
- **2** Prove triangle relationships using the Triangle Inequality Theorem.

Why?

- On a home improvement show, a designer wants to use scrap pieces of cording from another sewing project to decorate the triangular throw pillows that she and the homeowner have made. To minimize waste, she wants to use the scraps without cutting them. She selects three scraps at random and tries to form a triangle. Two such attempts are shown.



CCSS Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

1 The Triangle Inequality

The Triangle Inequality While a triangle is formed by three segments, a special relationship must exist among the lengths of the segments in order for them to form a triangle.

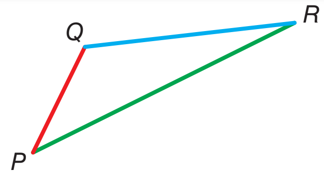
Theorem 5.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Examples $PQ + QR > PR$

$QR + PR > PQ$

$PR + PQ > QR$



You will prove Theorem 5.11 in Exercise 23.

To show that it is not possible to form a triangle with three side lengths, you need only show that one of the three triangle inequalities is not true.

Example 1 Identify Possible Triangles Given Side Lengths

PT

Is it possible to form a triangle with the given side lengths? If not, explain why not.

a. 8 in., 15 in., 17 in.

Check each inequality.

$8 + 15 \stackrel{?}{>} 17$ | $8 + 17 \stackrel{?}{>} 15$ | $15 + 17 \stackrel{?}{>} 8$

$23 > 17$ ✓ | $25 > 15$ ✓ | $32 > 8$ ✓

Since the sum of each pair of side lengths is greater than the third side length, sides with lengths 8, 15, and 17 inches will form a triangle.

b. 6 m, 8 m, 14 m

$6 + 8 \stackrel{?}{>} 14$

$14 \not> 14$ ✗

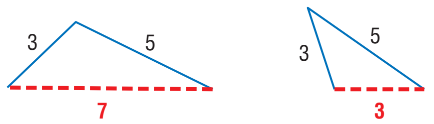
Since the sum of one pair of side lengths is not greater than the third side length, sides with lengths 6, 8, and 14 meters will not form a triangle.

Guided Practice

1A. 15 yd, 16 yd, 30 yd

1B. 2 ft, 8 ft, 11 ft

When the lengths of two sides of a triangle are known, the third side can be any length in a range of values. You can use the Triangle Inequality Theorem to determine the range of possible lengths for the third side.



> **Test-Taking Tip**
>
> **Testing Choices** If you are short on time, you can test each choice to find the correct answer and eliminate any remaining choices.

Standardized Test Example 2 Find Possible Side Lengths

If the measures of two sides of a triangle are 3 feet and 7 feet, which is the *least* possible whole number measure for the third side?

A 3 ft **B** 4 ft **C** 5 ft **D** 10 ft

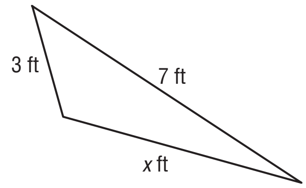
Read the Test Item

You need to determine which value is the least possible measure for the third side of a triangle with sides that measure 3 feet and 7 feet.

Solve the Test Item

To determine the least possible measure from the choices given, first determine the range of possible measures for the third side.

Draw a diagram and let x represent the length of the third side.



Next, set up and solve each of the three triangle inequalities.

$3 + 7 > x$	$3 + x > 7$	$x + 7 > 3$
$10 > x$ or $x < 10$	$x > 4$	$x > -4$

Notice that $x > -4$ is always true for any whole number measure for x. Combining the two remaining inequalities, the range of values that fit both inequalities is $x > 4$ and $x < 10$, which can be written as $4 < x < 10$.

> **Reading Math**
>
> **Multiple Inequality Symbols** The compound inequality $4 < x < 10$ is read *x is between 4 and 10.*

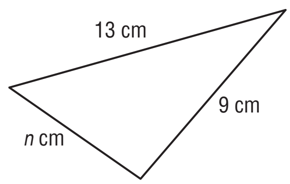
The least whole number value between 4 and 10 is 5. So the correct answer is choice C.

Guided Practice

2. Which of the following could *not* be the value of n?

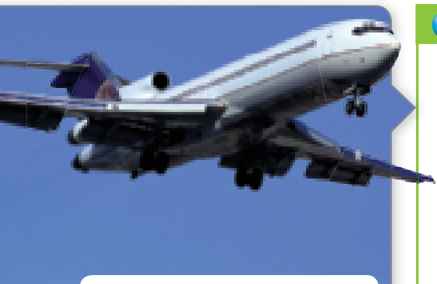
F 7 **H** 13

G 10 **J** 22



2 Proofs Using the Triangle Inequality Theorem

You can use the Triangle Inequality Theorem as a reason in proofs.

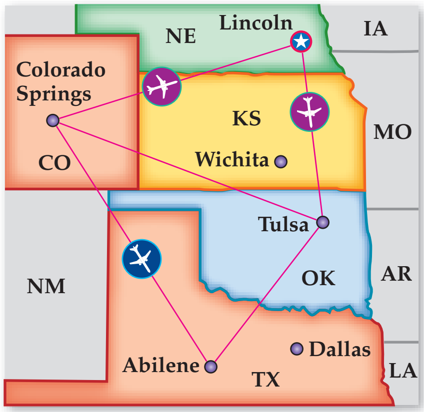


Real-World Link

A direct flight is not the same as a nonstop flight. For a direct flight, passengers do not change planes, but the plane may make one or more stops before continuing to its final destination.

Real-World Example 3 Proof Using Triangle Inequality Theorem

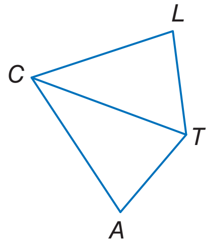
TRAVEL **The distance from Colorado Springs, Springs, Colorado, to Abilene, Texas, is the same as the distance from Colorado Springs to Tulsa, Oklahoma. Prove that a direct flight from Colorado Springs to Tulsa through Lincoln, Nebraska, is a greater distance than a nonstopflight from Colorado Springs to Abilene.**



Draw a simpler diagram of the situation and label the diagram. Draw in side $\overline{LT}$ to form $\triangle CTL$.

Given: $CA = CT$

Prove: $CL + LT > CA$



Proof:

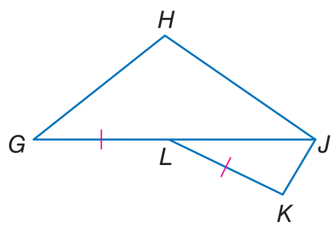
Statements	Reasons
1. $CA = CT$	**1.** Given
2. $CL + LT > CT$	**2.** Triangle Inequality Theorem
3. $CL + LT > CA$	**3.** Substitution

Guided Practice

3. Write a two-column proof.

Given: $GL = LK$

Prove: $JH + GH > JK$



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Is it possible to form a triangle with the given side lengths? If not, explain why not.**

1. 5 cm, 7 cm, 10 cm   **2.** 3 in., 4 in., 8 in.   **3.** 6 m, 14 m, 10 m

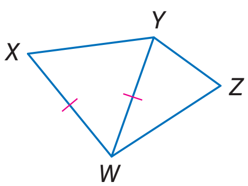
Example 2 **4.** **MULTIPLE CHOICE** If the measures of two sides of a triangle are 5 yards and 9 yards, what is the least possible measure of the third side if the measure is an integer?

A 4 yd   **B** 5 yd   **C** 6 yd   **D** 14 yd

Example 3 **5.** **PROOF** Write a two-column proof.

Given: $\overline{XW} \cong \overline{YW}$

Prove: $YZ + ZW > XW$



Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **Is it possible to form a triangle with the given side lengths? If not, explain why not.**

6. 4 ft, 9 ft, 15 ft

7. 11 mm, 21 mm, 16 mm

8. 9.9 cm, 1.1 cm, 8.2 cm

9. 2.1 in., 4.2 in., 7.9 in.

10. $2\frac{1}{2}$ m, $1\frac{3}{4}$ m, $5\frac{1}{8}$ m

11. $1\frac{1}{5}$ km, $4\frac{1}{2}$ km, $3\frac{3}{4}$ km

Example 2 **Find the range for the measure of the third side of a triangle given the measures of two sides.**

12. 4 ft, 8 ft

13. 5 m, 11 m

14. 2.7 cm, 4.2 cm

15. 3.8 in., 9.2 in.

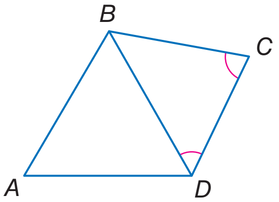
16. $\frac{1}{2}$ km, $3\frac{1}{4}$ km

17. $2\frac{1}{3}$ yd, $7\frac{2}{3}$ yd

Example 3 **PROOF** **Write a two-column proof.**

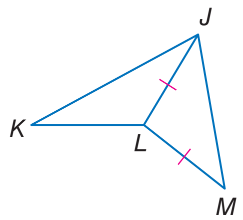
18. **Given:** $\angle BCD \cong \angle CDB$

Prove: $AB + AD > BC$



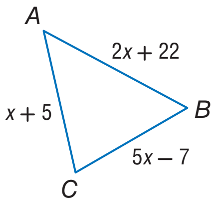
19. **Given:** $\overline{JL} \cong \overline{LM}$

Prove: $KJ + KL > LM$



CCSS SENSE-MAKING **Determine the possible values of x.**

20.



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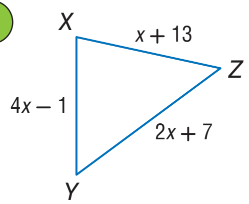
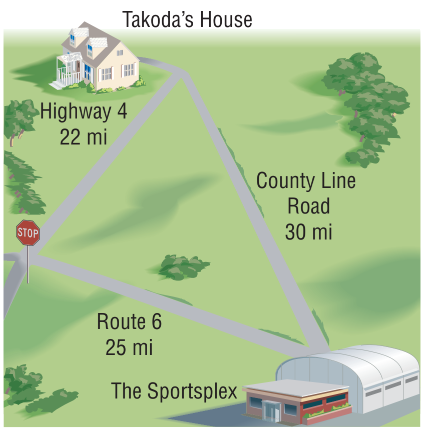


22. **DRIVING** Takoda wants to take the most efficient route from his house to a soccer tournament at The Sportsplex. He can take County Line Road or he can take Highway 4 and then Route 6 to the get to The Sportsplex.

a. Which of the two possible routes is the shortest? Explain your reasoning.

b. Suppose Takoda always drives below the speed limit. If the speed limit on County Line Road is 30 miles per hour and on both Highway 4 and Route 6 it is 55 miles per hour, which route will be faster? Explain.

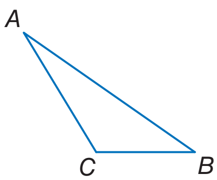


23. **PROOF** **Write a two-column proof.**

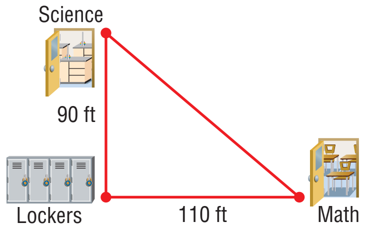
Given: $\triangle ABC$

Prove: $AC + BC > AB$ (Triangle Inequality Theorem)

(*Hint*: Draw auxiliary segment $\overline{CD}$, so that C is between B and D and $\overline{CD} \cong \overline{AC}$.)



24. **SCHOOL** When Toya goes from science class to math class, she usually stops at her locker. The distance from her science classroom to her locker is 90 feet, and the distance from her locker to her math classroom is 110 feet. What are the possible distances from science class to math class if she takes the hallway that goes directly between the two classrooms?



Find the range of possible measures of x if each set of expressions represents measures of the sides of a triangle.

25. $x, 4, 6$

26. $8, x, 12$

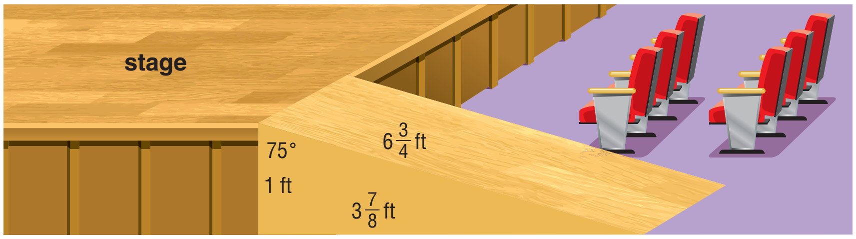
27. $x + 1, 5, 7$

28. $x - 2, 10, 12$

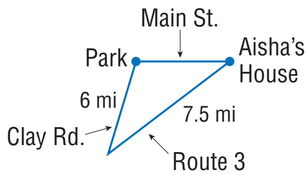
29. $x + 2, x + 4, x + 6$

30. $x, 2x + 1, x + 4$

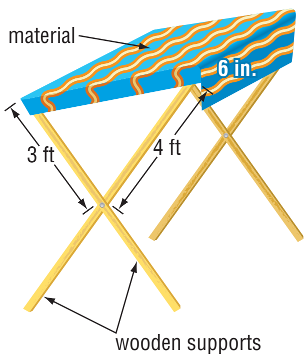
31. **DRAMA CLUB** Anthony and Catherine are working on a ramp up to the stage for the drama club's next production. Anthony's sketch of the ramp is shown below. Catherine is concerned about the measurements and thinks they should recheck the measures before they start cutting the wood. Is Catherine's concern valid? Explain your reasoning.



32. CCSS **SENSE-MAKING** Aisha is riding her bike to the park and can take one of two routes. The most direct route from her house is to take Main Street, but it is safer to take Route 3 and then turn right on Clay Road as shown. The additional distance she will travel if she takes Route 3 to Clay Road is between how many miles?



33. **DESIGN** Carlota designed an awning that she and her friends could take to the beach. Carlota decides to cover the top of the awning with material that will drape 6 inches over the front. What length of material should she buy to use with her design so that it covers the top of the awning, including the drape, when the supports are open as far as possible? Assume that the width of the material is sufficient to cover the awning.



ESTIMATION **Without using a calculator, determine if it is possible to form a triangle with the given side lengths. Explain.**

34. $\sqrt{8}$ ft, $\sqrt{2}$ ft, $\sqrt{35}$ ft

35. $\sqrt{99}$ yd, $\sqrt{48}$ yd, $\sqrt{65}$ yd

36. $\sqrt{3}$ m, $\sqrt{15}$ m, $\sqrt{24}$ m

37. $\sqrt{122}$ in., $\sqrt{5}$ in., $\sqrt{26}$ in.

CCSS REASONING **Determine whether the given coordinates are the vertices of a triangle. Explain.**

38. $X(1, -3), Y(6, 1), Z(2, 2)$

39. $F(-4, 3), G(3, -3), H(4, 6)$

40. $J(-7, -1), K(9, -5), L(21, -8)$

41. $Q(2, 6), R(6, 5), S(1, 2)$

42. **MULTIPLE REPRESENTATIONS** In this problem, you will use inequalities to make comparisons between the sides and angles of two triangles.

a. **Geometric** Draw three pairs of triangles that have two pairs of congruent sides and one pair of sides that is not congruent. Mark each pair of congruent sides. Label each triangle pair *ABC* and *DEF*, where $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

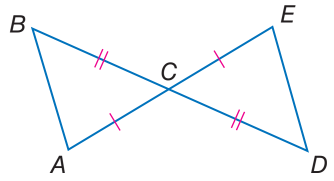
b. **Tabular** Copy the table below. Measure and record the values of *BC*, $m\angle A$, *EF*, and $m\angle D$ for each triangle pair.

Triangle Pair	*BC*	$m\angle A$	*EF*	$m\angle D$
1				
2				
3				

c. **Verbal** Make a conjecture about the relationship between the angles opposite the noncongruent sides of a pair of triangles that have two pairs of congruent legs.

H.O.T. Problems Use Higher-Order Thinking Skills

43. **CHALLENGE** What is the range of possible perimeters for figure *ABCDE* if $AC = 7$ and $DC = 9$? Explain your reasoning.



44. **REASONING** What is the range of lengths of each leg of an isosceles triangle if the measure of the base is 6 inches? Explain.

45. **WRITING IN MATH** What can you tell about a triangle when given three side lengths? Include at least two items.

46. **CHALLENGE** The sides of an isosceles triangle are whole numbers, and its perimeter is 30 units. What is the probability that the triangle is equilateral?

47. **OPEN ENDED** The length of one side of a triangle is 2 inches. Draw a triangle in which the 2-inch side is the shortest side and one in which the 2-inch side is the longest side. Include side and angle measures on your drawing.

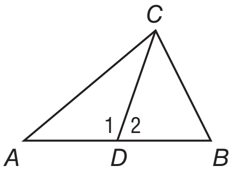
48. **WRITING IN MATH** Suppose your house is $\frac{3}{4}$ mile from a park and the park is 1.5 miles from a shopping center.

a. If your house, the park, and the shopping center are noncollinear, what do you know about the distance from your house to the shopping center? Explain your reasoning.

b. If the three locations are collinear, what do you know about the distance from your house to the shopping center? Explain your reasoning.

Standardized Test Practice

49. If $\overline{DC}$ is a median of $\triangle ABC$ and $m\angle 1 > m\angle 2$, which of the following statements is not true?



A $AD = BD$

B $m\angle ADC = m\angle BDC$

C $AC > BC$

D $m\angle 1 > m\angle B$

50. SHORT RESPONSE A high school soccer team has a goal of winning at least 75% of their 15 games this season. In the first three weeks, the team has won 5 games. How many more games must the team win to meet their goal?

51. Which of the following is a logical conclusion based on the statement and its converse below?

Statement: If a polygon is a rectangle, then it has four sides.

Converse: If a polygon has four sides, then it is a rectangle.

F The statement and its converse are both true.

G The statement is false; the converse is false.

H The statement is true; the converse is false.

J The statement is false; the converse is true.

52. SAT/ACT When 7 is subtracted from $14w$, the result is z. Which of the following equations represents this statement?

A $7 - 14w = z$

B $z = 14w + 7$

C $7 - z = 14w$

D $z = 14w - 7$

E $7 + 14w = 7z$

Spiral Review

State the assumption you would make to start an indirect proof of each statement. (Lesson 5-4)

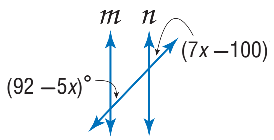
53. If $4y + 17 = 41$, then $y = 6$.

54. If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the two lines are parallel.

55. GEOGRAPHY The distance between San Jose, California, and Las Vegas, Nevada, is about 375 miles. The distance from Las Vegas to Carlsbad, California, is about 243 miles. Use the Triangle Inequality Theorem to find the possible distance between San Jose and Carlsbad. (Lesson 5-3)

Find x so that $m \parallel n$. Identify the postulate or theorem you used. (Lesson 3-5)

56.



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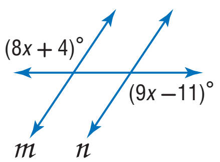
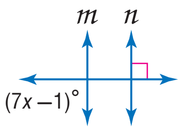


58.



ALGEBRA Find x and JK if J is between K and L. (Lesson 1-2)

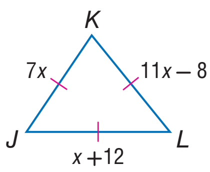
59. $KJ = 3x$, $JL = 6x$, and $KL = 12$

60. $KJ = 3x - 6$, $JL = x + 6$, and $KL = 24$

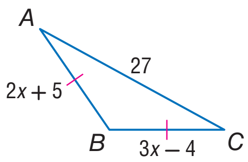
Skills Review

Find x and the measures of the unknown sides of each triangle.

61.



62.



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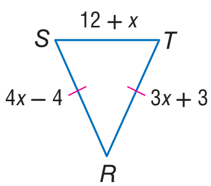


LESSON 5-6

Inequalities in Two Triangles

Then

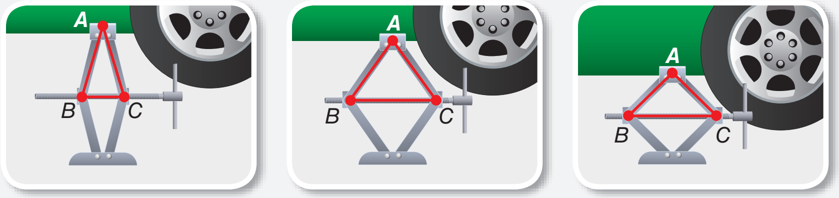
- You used inequalities to make comparisons in one triangle.

Now

1. Apply the Hinge Theorem or its converse to make comparisons in two triangles.
2. Prove triangle relationships using the Hinge Theorem or its converse.

Why?

- A car jack is used to lift a car. The jack shown below is one of the simplest still in use today. Notice that as the jack is lowered, the legs of isosceles $\triangle ABC$ remain congruent, but the included angle A widens and $\overline{BC}$, the side opposite $\angle A$, lengthens.



Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

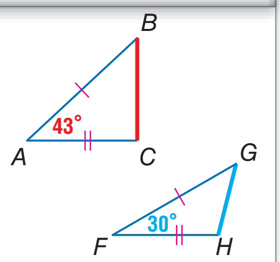
1 Make sense of problems and persevere in solving them.

1 Hinge Theorem

The observation in the example above is true of any type of triangle and illustrates the following theorems.

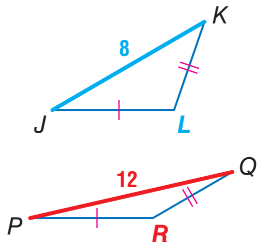
Theorems Inequalities in Two Triangles

5.13 Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.



Example: If $\overline{AB} \cong \overline{FG}$, $\overline{AC} \cong \overline{FH}$, and $m\angle A > m\angle F$, then $BC > GH$.

5.14 Converse of the Hinge Theorem If two sides of a triangle are congruent to two sides of another triangle, and the third side in the first is longer than the third side in the second triangle, then the included angle measure of the first triangle is greater than the included angle measure in the second triangle.



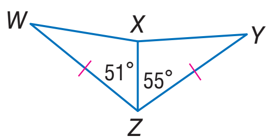
Example: If $\overline{JL} \cong \overline{PR}$, $\overline{KL} \cong \overline{QR}$, and $PQ > JK$, then $m\angle R > m\angle L$.

The proof of Theorem 5.13 is on p. 372. You will prove Theorem 5.14 in Exercise 28.

Example 1 Use the Hinge Theorem and its Converse

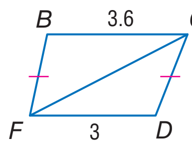
Compare the given measures.

a. WX and XY



In $\triangle WXZ$ and $\triangle YXZ$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ} \cong \overline{XZ}$, and $\angle YZX > \angle WZX$. By the Hinge Theorem, $m\angle WZX < m\angle YZX$, so $WX < XY$.

b. $m\angle FCD$ and $m\angle BFC$

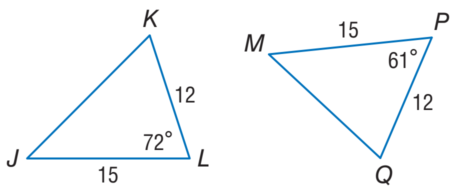


In $\triangle BCF$ and $\triangle DFC$, $\overline{BF} \cong \overline{DC}$, $\overline{FC} \cong \overline{CF}$, and $BC > FD$. By the Converse of the Hinge Theorem, $\angle BFC > \angle DCF$.

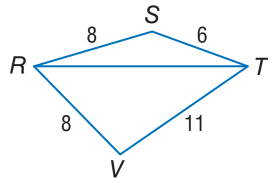
GuidedPractice

Compare the given measures.

1A. JK and MQ



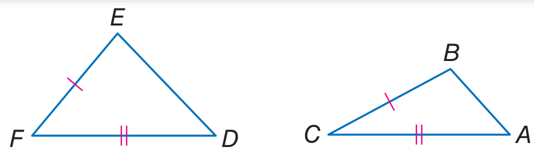
1B. $m\angle SRT$ and $m\angle VRT$



Proof Hinge Theorem

Given: $\triangle ABC$ and $\triangle DEF$,
$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
$m\angle F > m\angle C$

Prove: $DE > AB$



Proof:

We are given that $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$. We also know that $m\angle F > m\angle C$.

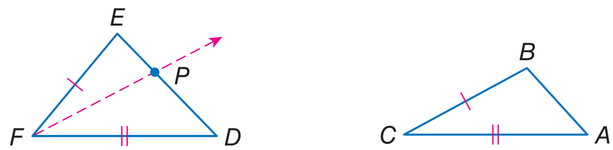
Draw auxiliary ray FP such that $m\angle DFP = m\angle C$ and that $\overline{PF} \cong \overline{BC}$. This leads to two cases.

StudyTip

SAS and SSS Inequality Theorem The Hinge Theorem is also called the SAS Inequality Theorem. The Converse of the Hinge Theorem is also called the SSS Inequality Theorem.

Case 1 P lies on $\overline{DE}$.

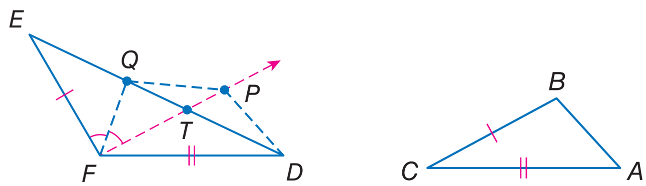
Then $\triangle FPD \cong \triangle CBA$ by SAS. Thus, $PD = BA$ by CPCTC and the definition of congruent segments.



By the Segment Addition Postulate, $DE = EP + PD$. Also, $DE > PD$ by the definition of inequality. Therefore, $DE > AB$ by substitution.

Case 2 P does not lie on $\overline{DE}$.

Then let the intersection of $\overline{FP}$ and $\overline{ED}$ be point T, and draw another auxiliary segment $\overline{FQ}$ such that Q is on $\overline{DE}$ and $\angle EFQ \cong \angle QFP$. Then draw auxiliary segments $\overline{PD}$ and $\overline{PQ}$.



Since $\overline{FP} \cong \overline{BC}$ and $\overline{BC} \cong \overline{EF}$, we have $\overline{FP} \cong \overline{EF}$ by the Transitive Property. Also $\overline{QF}$ is congruent to itself by the Reflexive Property. Thus, $\triangle EFQ \cong \triangle PFQ$ by SAS. By CPCTC, $\overline{EQ} \cong \overline{PQ}$ or $EQ = PQ$. Also, $\triangle FPD \cong \triangle CBA$ by SAS. So, $\overline{PD} \cong \overline{BA}$ by CPCTC and $PD = BA$.

In $\triangle QPD$, $QD + PQ > PD$ by the Triangle Inequality Theorem. By substitution, $QD + EQ >$ PD. Since $ED = QD + EQ$ by the Segment Addition Postulate, $ED > PD$. Using substitution, $ED > BA$ or $DE > AB$.

You can use the Hinge Theorem to solve real-world problems.

Real-WorldLink

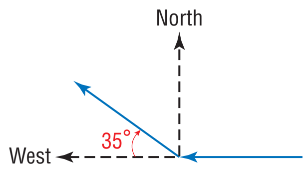
There are over 225,000 miles of groomed and marked snowmobile trails in North America.

Source: International Snowmobile Manufacturers Association

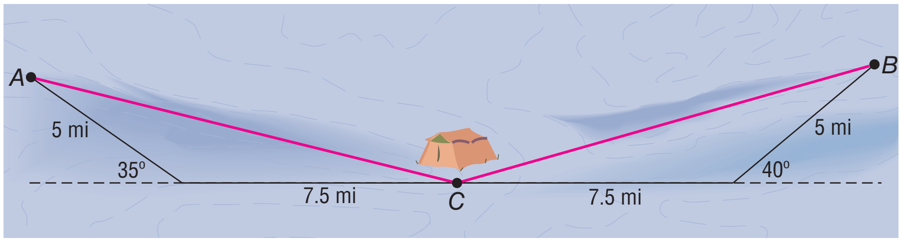
Real-World Example 2 Use the Hinge Theorem

SNOWMOBILING Two groups of snowmobilers leave from the same base camp. Group A goes 7.5 miles due west and then turns 35° north of west and goes 5 miles. Group B goes 7.5 miles due east and then turns 40° north of east and goes 5 miles. At this point, which group is farther from the base camp? Explain your reasoning.

Understand Using the sets of directions given in the problem, you need to determine which snowmobile group is farther from the base camp. A turn of 35° north of west is correctly interpreted as shown.



Plan Draw a diagram of the situation.



The paths taken by each group and the straight-line distance back to the camp form two triangles. Each group goes 7.5 miles and then turns and goes 5 miles.

Use linear pairs to find the measures of the included angles. Then apply the Hinge Theorem to compare the distance each group is from base camp.

Solve The included angle for the path made by Group A measures 180 − 35 or 145. The included angle for the path made by Group B is 180 − 40 or 140.

Since $145 > 140$, $AC > BC$ by the Hinge Theorem. So Group A is farther from the base camp.

Check Group B turned 5° more than Group A did back toward base camp, so they should be closer to base camp than Group A. Thus, Group A should be farther from the base camp. ✔

Problem-SolvingTip

Draw a Diagram Draw a diagram to help you see and correctly interpret a problem that has been described in words.

GuidedPractice

2A. SKIING Two groups of skiers leave from the same lodge. Group A goes 4 miles due east and then turns 70° north of east and goes 3 miles. Group B goes 4 miles due west and then turns 75° north of west and goes 3 miles. At this point, which group is *farther* from the lodge? Explain your reasoning.

2B. SKIING In problem 2A, suppose Group A instead went 4 miles west and then turned 45° north of west and traveled 3 miles. Which group would be *closer* to the lodge? Explain your reasoning.

When the included angle of one triangle is greater than the included angle in a second triangle, the Converse of the Hinge Theorem is used.

IndexStock/SuperStock

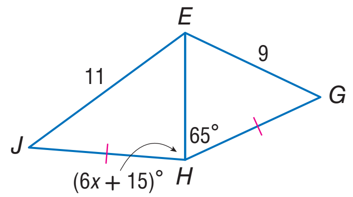
Study Tip

Using Additional Facts
When finding a range for the possible values for *x*, you may need to use one of the following facts.

- The measure of any angle is always greater than 0 and less than 180.
- The measure of any segment is always greater than 0.

Example 3 Apply Algebra to the Relationships in Triangles

ALGEBRA Find the range of possible values for x.



Step 1 From the diagram, we know that $\overline{JH} \cong \overline{GH}$, $\overline{EH} \cong \overline{EH}$, and $JE > EG$.

$m\angle JHE > m\angle EHG$	Converse of the Hinge Theorem
$6x + 15 > 65$	Substitution
$x > 8\frac{1}{3}$	Solve for x.

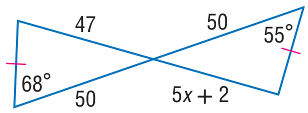
Step 2 Use the fact that the measure of any angle in a triangle is less than 180 to write a second inequality.

$m\angle JHE < 180$	
$6x + 15 < 180$	Substitution
$x < 27.5$	Solve for x.

Step 3 Write $x > 8\frac{1}{3}$ and $x < 27.5$ as the compound inequality $8\frac{1}{3} < x < 27.5$.

Guided Practice

3. Find the range of possible values for x.



2 Prove Relationships In Two Triangles

You can use the Hinge Theorem and its converse to prove relationships in two triangles.

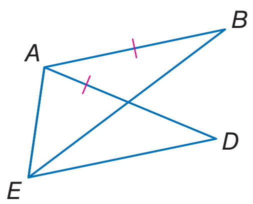
Example 4 Prove Triangle Relationships Using Hinge Theorem

Write a two-column proof.

Given: $\overline{AB} \cong \overline{AD}$

Prove: $EB > ED$

Proof:



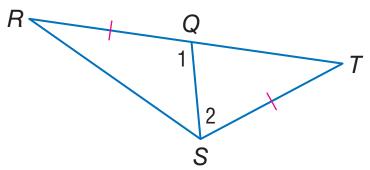
Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	**1.** Given
2. $\overline{AE} \cong \overline{AE}$	**2.** Reflexive Property
3. $m\angle EAB = m\angle EAD + m\angle DAB$	**3.** Angle Addition Postulate
4. $m\angle EAB > m\angle EAD$	**4.** Definition of Inequality
5. $EB > ED$	**5.** Hinge Theorem

Guided Practice

4. Write a two-column proof.

Given: $\overline{RQ} \cong \overline{ST}$

Prove: $RS > TQ$

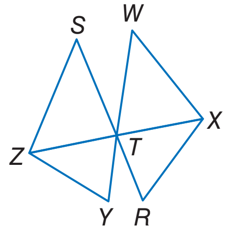


Example 5 Prove Relationships Using Converse of Hinge Theorem

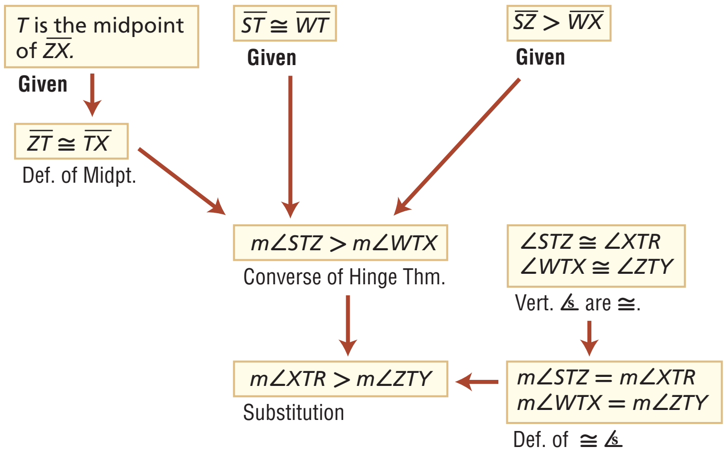
Write a flow proof.

Given: T is the midpoint of $\overline{ZX}$.
$\overline{ST} \cong \overline{WT}$
$SZ > WX$

Prove: $m\angle XTR > m\angle ZTY$



Flow Proof:

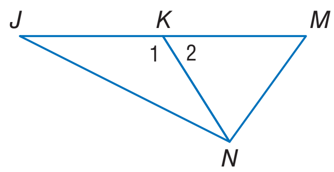


GuidedPractice

5. Write a two-column proof.

Given: $\overline{NK}$ is a median of $\triangle JMN$.
$JN > NM$

Prove: $m\angle 1 > m\angle 2$

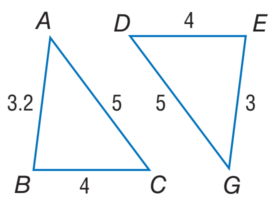


Check Your Understanding

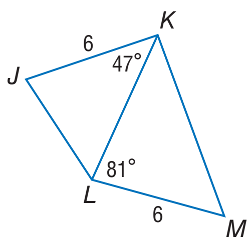
= Step-by-Step Solutions begin on page R14.

Example 1 **Compare the given measures.**

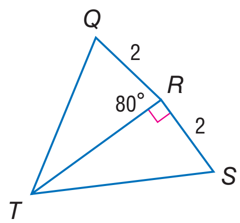
1. $m\angle ACB$ and $m\angle GDE$



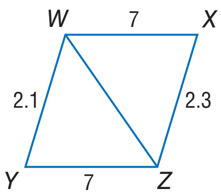
2. JL and KM



3. QT and ST



4. $m\angle XWZ$ and $m\angle YZW$

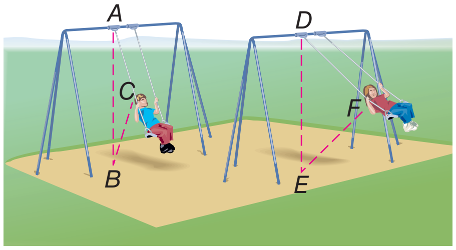


Example 2

5. **SWINGS** The position of the swing changes based on how hard the swing is pushed.

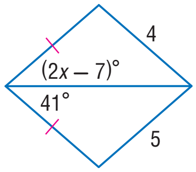
a. Which pairs of segments are congruent?

b. Is the measure of $\angle A$ or the measure of $\angle D$ greater? Explain.

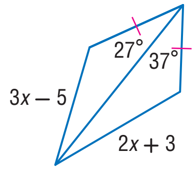


Example 3 **Find the range of possible values for x.**

6.

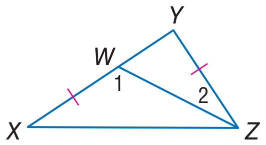


7.

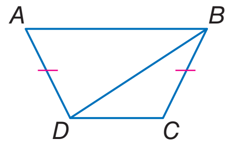


Examples 4–5 CCSS **ARGUMENTS** **Write a two-column proof.**

8. Given: $\triangle YZX$
$\overline{YZ} \cong \overline{XW}$
Prove: $ZX > YW$



9. Given: $\overline{AD} \cong \overline{CB}$
$DC < AB$
Prove: $m\angle CBD < m\angle ADB$

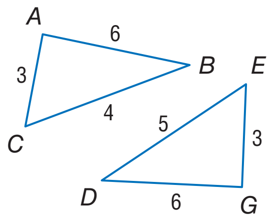


Practice and Problem Solving

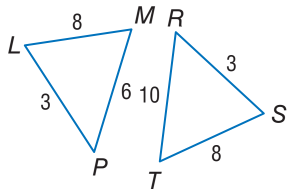
Extra Practice is on page R5.

Example 1 **Compare the given measures.**

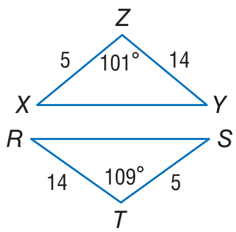
10. $m\angle BAC$ and $m\angle DGE$



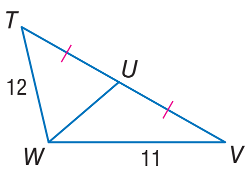
11. $m\angle MLP$ and $m\angle TSR$



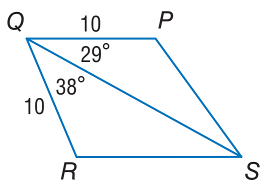
12. SR and XY



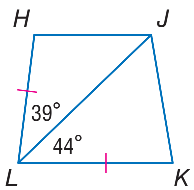
13. $m\angle TUW$ and $m\angle VUW$



14. PS and SR



15. JK and HJ



Example 2

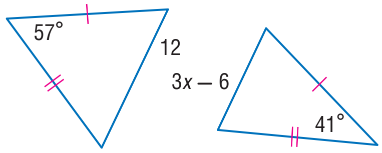
16. **CAMPING** Pedro and Joel are camping in a national park. One morning, Pedro decides to hike to the waterfall. He leaves camp and goes 5 miles east then turns 15° south of east and goes 2 more miles. Joel leaves the camp and travels 5 miles west, then turns 35° north of west and goes 2 miles to the lake for a swim.

a. When they reach their destinations, who is closer to the camp? Explain your reasoning. Include a diagram.

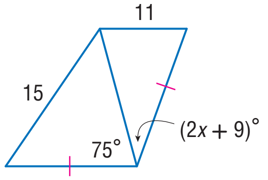
b. Suppose instead of turning 35° north of west, Joel turned 10° south of west. Who would then be farther from the camp? Explain your reasoning. Include a diagram.

Example 3 **Find the range of possible values for *x*.**

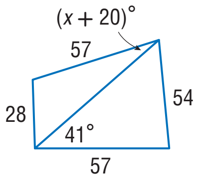
17.



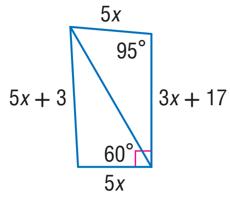
18.



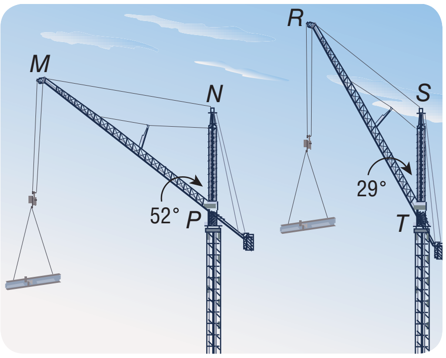
19



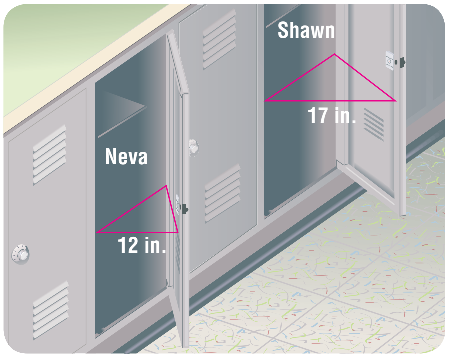
20.



21. CRANES In the diagram, a crane is shown lifting an object to two different heights. The length of the crane's arm is fixed, and $\overline{MP} \cong \overline{RT}$. Is $\overline{MN}$ or $\overline{RS}$ shorter? Explain your reasoning.



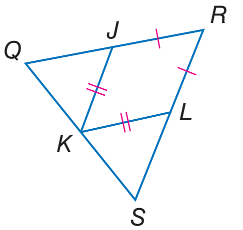
22. LOCKERS Neva and Shawn both have their lockers open as shown in the diagram. Whose locker forms a larger angle? Explain your reasoning.



Examples 4–5 **CCSS ARGUMENTS** **Write a two-column proof.**

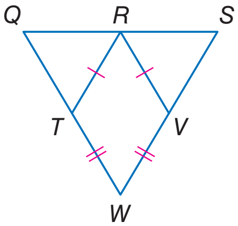
23. Given: $\overline{LK} \cong \overline{JK}$, $\overline{RL} \cong \overline{RJ}$
K is the midpoint of $\overline{QS}$.
$m\angle SKL > m\angle QKJ$

Prove: $RS > QR$



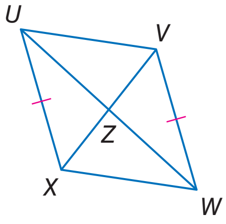
24. Given: $\overline{VR} \cong \overline{RT}$, $\overline{WV} \cong \overline{WT}$
$m\angle SRV > m\angle QRT$
R is the midpoint of $\overline{SQ}$.

Prove: $WS > WQ$



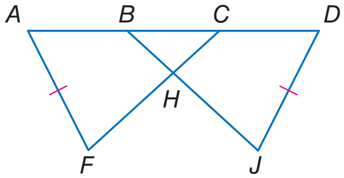
25. Given: $\overline{XU} \cong \overline{VW}$, $VW > XW$
$\overline{XU} \parallel \overline{VW}$

Prove: $m\angle XZU > m\angle UZV$

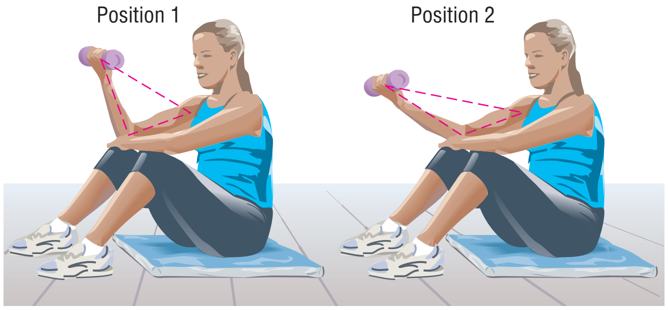


26. Given: $\overline{AF} \cong \overline{DJ}$, $\overline{FC} \cong \overline{JB}$
$AB > DC$

Prove: $m\angle AFC > m\angle DJB$



27 **EXERCISE** Anica is doing knee-supported bicep curls as part of her strength training.



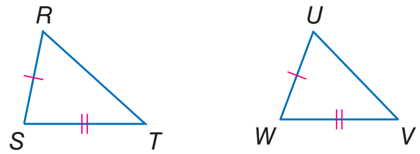
a. Is the distance from Anica's fist to her shoulder greater in Position 1 or Position 2? Justify your answer using measurement.

b. Is the measure of the angle formed by Anica's elbow greater in Position 1 or Position 2? Explain your reasoning.

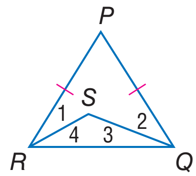
28. **PROOF** Use an indirect proof to prove the SSS Inequality Theorem (Theorem 5.14).

Given: $\overline{RS} \cong \overline{UW}$
$\overline{ST} \cong \overline{WV}$
$RT > UV$

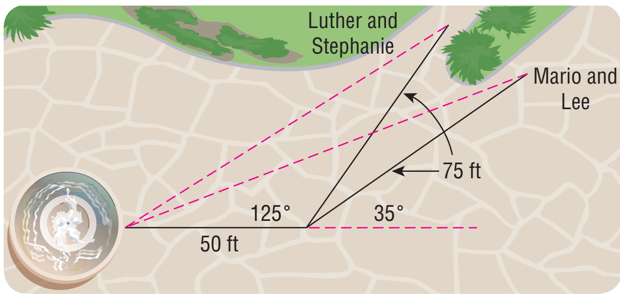
Prove: $m\angle S > m\angle W$



29. **PROOF** If $\overline{PR} \cong \overline{PQ}$ and $SQ > SR$, write a two-column proof to prove $m\angle 1 < m\angle 2$.



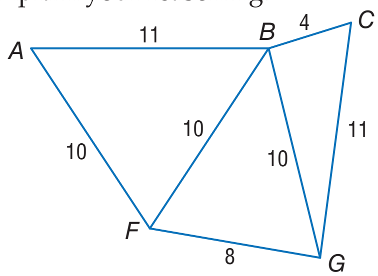
30. **SCAVENGER HUNT** Stephanie, Mario, Lee, and Luther are participating in a scavenger hunt as part of a geography lesson. Their map shows that the next clue is 50 feet due east and then 75 feet 35° east of north starting from the fountain in the school courtyard. When they get ready to turn and go 75 feet 35° east of north, they disagree about which way to go, so they split up and take the paths shown in the diagram below.



a. Which pair chose the correct path? Explain your reasoning.

b. Which pair is closest to the fountain when they stop? Explain your reasoning.

CCSS SENSE-MAKING **Use the figure at the right to write an inequality relating the given pair of angle or segment measures.**

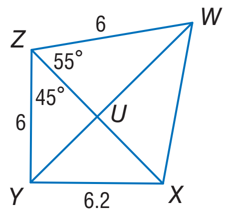


31. CB and AB

32. $m\angle FBG$ and $m\angle BFA$

33. $m\angle BGC$ and $m\angle FBA$

Use the figure at the right to write an inequality relating the given pair of angles or segment measures.



34. $m\angle ZUY$ and $m\angle ZUW$

35. WU and YU

36. WX and XY

37. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate properties of polygons.

a. Geometric Draw a three-sided, a four-sided, and a five-sided polygon. Label the 3-sided polygon *ABC*, the four-sided polygon *FGHJ*, and the five-sided polygon *PQRST*. Use a protractor to measure and label each angle.

b. Tabular Copy and complete the table below.

Number of sides	Angle Measures				Sum of Angles
3	$m\angle A$		$m\angle C$		
	$m\angle B$				
4	$m\angle F$		$m\angle H$		
	$m\angle G$		$m\angle J$		
5	$m\angle P$		$m\angle S$		
	$m\angle Q$		$m\angle T$		
	$m\angle R$				

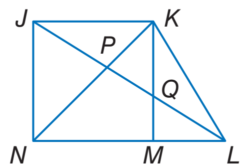
c. Verbal Make a conjecture about the relationship between the number of sides of a polygon and the sum of the measures of the angles of the polygon.

d. Logical What type of reasoning did you use in part **c**? Explain.

e. Algebraic Write an algebraic expression for the sum of the measures of the angles for a polygon with *n* sides.

H.O.T. Problems Use Higher-Order Thinking Skills

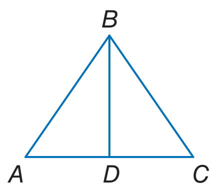
38. **CHALLENGE** If $m\angle LJN > m\angle KJL$, $KJ \cong JN$, and $JN \perp NL$, which is greater, $m\angle LKN$ or $m\angle LNK$? Explain your reasoning.



39. **OPEN ENDED** Give a real-world example of an object that uses a hinge. Draw two sketches in which the hinge on your object is adjusted to two different positions. Use your sketches to explain why Theorem 5.13 is called the Hinge Theorem.

40. **CHALLENGE** Given $\triangle RST$ with median $\overline{RQ}$, if RT is greater than or equal to RS, what are the possible classifications of $\triangle RQT$? Explain your reasoning.

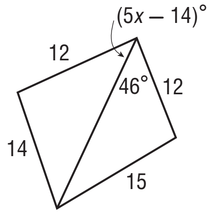
41. **CCSS PRECISION** If $\overline{BD}$ is a median and $AB < BC$, then $\angle BDC$ is *always*, *sometimes*, or *never* an acute angle. Explain.



42. **WRITING IN MATH** Compare and contrast the Hinge Theorem to the SAS Postulate for triangle congruence.

Standardized Test Practice

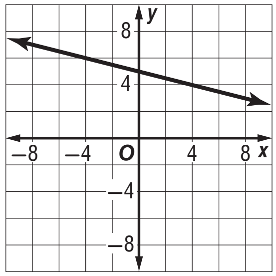
43. SHORT RESPONSE Write an inequality to describe the possible range of values for x.



44. Which of the following is the inverse of the statement *If it is snowing, then Steve wears his snow boots?*

A If Steve wears his snow boots, then it is snowing.
B If it is not snowing, then Steve does not wear his snow boots.
C If it is not snowing, then Steve wears his snow boots.
D If it never snows, then Steve does not own snow boots.

45. ALGEBRA Which linear function best describes the graph shown?



F $y = -\frac{1}{4}x + 5$

G $y = -\frac{1}{4}x - 5$

H $y = \frac{1}{4}x + 5$

J $y = \frac{1}{4}x - 5$

46. SAT/ACT If the side of a square is $x + 3$, then the diagonal of the square is

A $x^2 + 1$
B $x\sqrt{2} + 3\sqrt{2}$
C $2x + 6$
D $x^2\sqrt{2} + 6$
E $x^2 + 9$

Spiral Review

Find the range for the measure of the third side of a triangle given the measures of two sides. (Lesson 5-5)

47. 3.2 cm, 4.4 cm **48.** 5 ft, 10 ft **49.** 3 m, 9 m

50. CRUISES Ally asked Tavia the cost of a cruise she and her best friend went on after graduation. Tavia could not remember how much it cost per person, but she did remember that the total cost was over \$500. Use indirect reasoning to show that the cost for one person was more than \$250. (Lesson 5-4)

Draw and label a figure to represent the congruent triangles. Then find x. (Lesson 4-3)

51. $\triangle QRS \cong \triangle GHJ$, $RS = 12$, $QR = 10$, $QS = 6$, and $HJ = 2x - 4$.

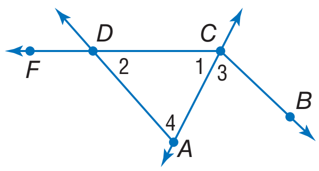
52. $\triangle ABC \cong \triangle XYZ$, $AB = 13$, $AC = 19$, $BC = 21$, and $XY = 3x + 7$.

Use the figure at the right. (Lesson 1-4)

53. Name the vertex of $\angle 4$.

54. What is another name for $\angle 2$?

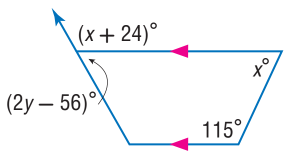
55. What is another name for $\angle BCA$?



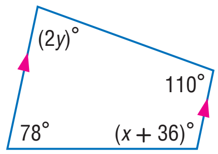
Skills Review

Find the value of the variable(s) in each figure. Explain your reasoning.

56.



57.



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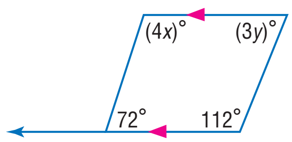


CHAPTER 5 Study Guide and Review

Study Guide

KeyConcepts

Special Segments in Triangles (Lessons 5-1 and 5-2)

- The special segments of triangles are perpendicular bisectors, angle bisectors, medians, and altitudes.
- The intersection points of each of the special segments of a triangle are called the points of concurrency.
- The points of concurrency for a triangle are the circumcenter, incenter, centroid, and orthocenter.

Indirect Proof (Lesson 5-4)

- Writing an Indirect Proof:

  1. Assume that the conclusion is false.

  2. Show that this assumption leads to a contradiction.

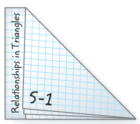
  3. Since the false conclusion leads to an incorrect statement, the original conclusion must be true.

Triangle Inequalities (Lessons 5-3, 5-5, and 5-6)

- The largest angle in a triangle is opposite the longest side, and the smallest angle is opposite the shortest side.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- **SAS Inequality** (Hinge Theorem): In two triangles, if two sides are congruent, then the measure of the included angle determines which triangle has the longer third side.
- **SSS Inequality**: In two triangles, if two corresponding sides of each triangle are congruent, then the length of the third side determines which triangle has the included angle with the greater measure.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

altitude (p. 337)

centroid (p. 335)

circumcenter (p. 325)

concurrent lines (p. 325)

incenter (p. 328)

indirect proof (p. 355)

indirect reasoning (p. 355)

median (p. 335)

orthocenter (p. 337)

perpendicular bisector (p. 324)

point of concurrency (p. 325)

proof by contradiction (p. 355)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The altitudes of a triangle intersect at the centroid.

2. The point of concurrency of the medians of a triangle is called the incenter.

3. The point of concurrency is the point at which three or more lines intersect.

4. The circumcenter of a triangle is equidistant from the vertices of the triangle.

5. To find the centroid of a triangle, first construct the angle bisectors.

6. The perpendicular bisectors of a triangle are concurrent lines.

7. To start a proof by contradiction, first assume that what you are trying to prove is true.

8. A proof by contradiction uses indirect reasoning.

9. A median of a triangle connects the midpoint of one side of the triangle to the midpoint of another side of the triangle.

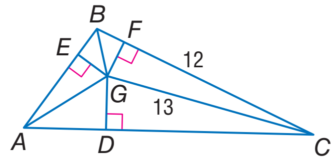
10. The incenter is the point at which the angle bisectors of a triangle intersect.

CHAPTER 5 Study Guide and Review *Continued*

Lesson-by-Lesson Review

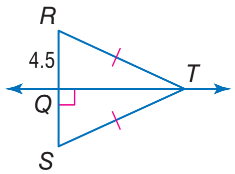
5-1 Bisectors of Triangles

11. Find EG if G is the incenter of $\triangle ABC$.

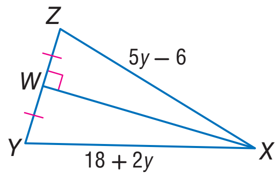


Find each measure.

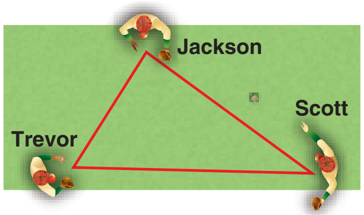
12. RS



13. XZ

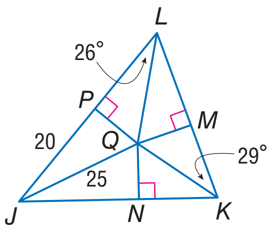


14. BASEBALL Jackson, Trevor, and Scott are warming up before a baseball game. One of their warm-up drills requires three players to form a triangle, with one player in the middle. Where should the fourth player stand so that he is the same distance from the other three players?



Example 1

Find each measure if Q is the incenter of $\triangle JKL$.



a. $\angle QJK$

$m\angle KLP + m\angle MKN + m\angle NJP = 180$ — $\triangle$ Sum Theorem

$2(26) + 2(29) + m\angle NJP = 180$ — Substitution

$110 + m\angle NJP = 180$ — Simplify.

$m\angle NJP = 70$ — Subtract.

Since $\overrightarrow{JQ}$ bisects $\angle NJP$, $2m\angle QJK = m\angle NJP$.
So, $m\angle QJK = \frac{1}{2}m\angle NJP$, so $m\angle QJK = \frac{1}{2}(70)$ or 35.

b. QP

$a^2 + b^2 = c^2$ — Pythagorean Theorem

$(QP)^2 + 20^2 = 25^2$ — Substitution

$(QP)^2 + 400 = 625$ — $20^2 = 400$ and $25^2 = 625$

$(QP)^2 = 225$ — Subtract.

$QP = 15$ — Simplify.

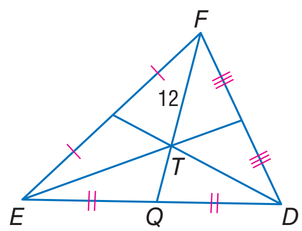
5-2 Medians and Altitudes of Triangles

15. The vertices of $\triangle DEF$ are $D(0, 0)$, $E(0, 7)$, and $F(6, 3)$. Find the coordinates of the orthocenter of $\triangle DEF$.

16. PROM Georgia is on the prom committee. She wants to hang a dozen congruent triangles from the ceiling so that they are parallel to the floor. She sketched out one triangle on a coordinate plane with coordinates (0, 4), (3, 8), and (6, 0). If each triangle is to be hung by one chain, what are the coordinates of the point where the chain should attach to the triangle?

Example 2

In $\triangle EDF$, T is the centroid and $FT = 12$. Find TQ.



$FT = \frac{2}{3}FQ$

$FT = \frac{2}{3}(FT + TQ)$

$12 = \frac{2}{3}(12 + TQ)$ — $FT = 12$

$12 = 8 + \frac{2}{3}TQ$ — Distributive Property

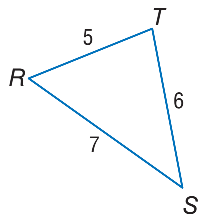
$4 = \frac{2}{3}TQ$ — Subtract.

$6 = TQ$ — Multiply.

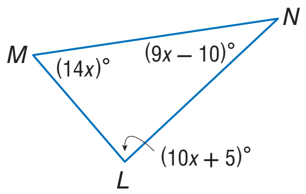
5-3 Inequalities in One Triangle

List the angles and sides of each triangle in order from smallest to largest.

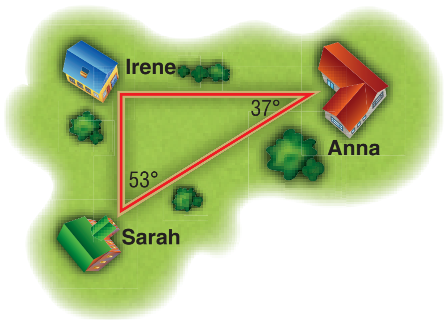
17.



18.

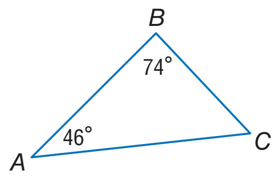


19. **NEIGHBORHOODS** Anna, Sarah, and Irene live at the intersections of the three roads that make the triangle shown. If the girls want to spend the afternoon together, is it a shorter path for Anna to stop and get Sarah and go onto Irene's house, or for Sarah to stop and get Irene and then go on to Anna's house?



Example 3

List the angles and sides of $\triangle ABC$ in order from smallest to largest.



a. First, find the missing angle measure using the Triangle Sum Theorem.

$m\angle C = 180 - (46 + 74)$ or 60

So, the angles from smallest to largest are $\angle A$, $\angle C$, and $\angle B$.

b. The sides from shortest to longest are $\overline{BC}$, $\overline{AB}$, and $\overline{AC}$.

5-4 Indirect Proof

State the assumption you would make to start an indirect proof of each statement.

20. $m\angle A \geq m\angle B$

21. $\triangle FGH \cong \triangle MNO$

22. $\triangle KLM$ is a right triangle.

23. If $3y < 12$, then $y < 4$.

24. Write an indirect proof to show that if two angles are complementary, neither angle is a right angle.

25. **MOVIES** Isaac bought two DVD's and spent over \$50. Use indirect reasoning to show that at least one of the DVD's he purchased was over \$25.

Example 4

State the assumption necessary to start an indirect proof of each statement.

a. $\overline{XY} \not\cong \overline{JK}$

$\overline{XY} \cong \overline{JK}$

b. **If $3x < 18$, then $x < 6$.**

The conclusion of the conditional statement is $x < 6$. The negation of the conclusion is $x \geq 6$.

c. **$\angle 2$ is an acute angle.**

If $\angle 2$ *is an acute angle* is false, then $\angle 2$ *is not an acute angle* must be true. This means that $\angle 2$ *is an obtuse or right angle* must be true.

5-5 The Triangle Inequality

Is it possible to form a triangle with the given lengths? If not, explain why not.

26. 5, 6, 9

27. 3, 4, 8

Find the range for the measure of the third side of a triangle given the measure of two sides.

28. 5 ft, 7 ft

29. 10.5 cm, 4 cm

30. **BIKES** Leonard rides his bike to visit Josh. Since High Street is closed, he has to travel 2 miles down Main Street and turn to travel 3 miles farther on 5th Street. If the three streets form a triangle with Leonard and Josh's house as two of the vertices, find the range of the possible distance between Leonard and Josh's houses when traveling straight down High Street.

Example 5

Is it possible to form a triangle with the lengths 7, 10, and 9 feet? If not, explain why not.

Check each inequality.

$7 + 10 > 9$ $7 + 9 > 10$ $10 + 9 > 7$

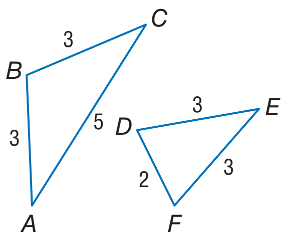
$17 > 9$ ✓ $16 > 10$ ✓ $19 > 7$ ✓

Since the sum of each pair of side lengths is greater than the third side length, sides with lengths 7, 10, and 9 feet will form a triangle.

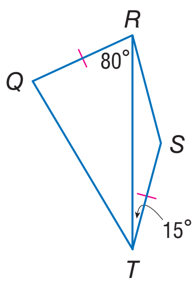
5-6 Inequalities in Two Triangles

Compare the given measures.

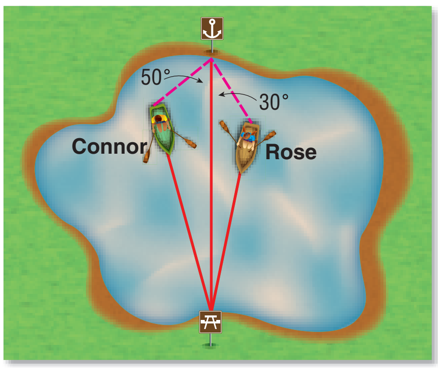
31. $m\angle ABC$, $m\angle DEF$



32. QT and RS

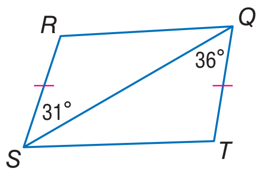


33. **BOATING** Rose and Connor each row across a pond heading to the same point. Neither of them has rowed a boat before, so they both go off course as shown in the diagram. After two minutes, they have each traveled 50 yards. Who is closer to their destination?



Example 6

Compare the given measures.

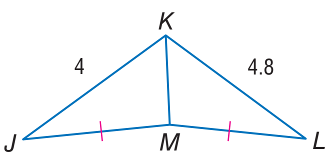


a. RQ and ST

In $\triangle QRS$ and $\triangle STQ$, $\overline{RS} \cong \overline{TQ}$, $\overline{QS} \cong \overline{QS}$, and $\angle SQT > \angle RSQ$. By the Hinge Theorem, $m\angle SQT < m\angle RSQ$, so $RQ < ST$.

b. $m\angle JKM$ and $m\angle LKM$

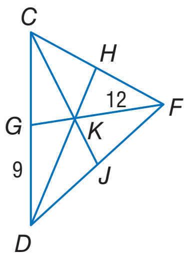
In $\triangle JKM$ and $\triangle LKM$, $\overline{JM} \cong \overline{LM}$, $\overline{KM} \cong \overline{KM}$, and $LK > JK$. By the Converse of the Hinge Theorem, $\angle LKM > \angle JKM$.



CHAPTER 5 Practice Test

1. **GARDENS** Maggie wants to plant a circular flower bed within a triangular area set off by three pathways. Which point of concurrency related to triangles would she use for the center of the largest circle that would fit inside the triangle?

In $\triangle CDF$, K is the centroid and $DK = 16$. Find each length.



2. KH
3. CD
4. FG

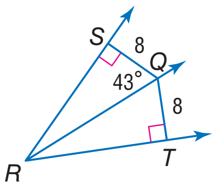
5. **PROOF** Write an indirect proof.

Given: $5x + 7 \geq 52$

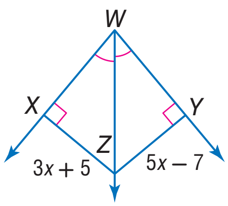
Prove: $x \geq 9$

Find each measure.

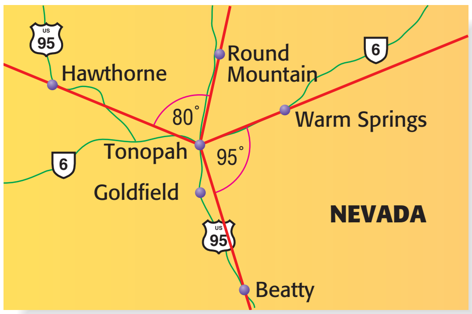
6. $m\angle TQR$



7. XZ



8. **GEOGRAPHY** The distance from Tonopah to Round Mountain is equal to the distance from Tonopah to Warm Springs. The distance from Tonopah to Hawthorne is the same as the distance from Tonopah to Beatty. Determine which distance is greater, Round Mountain to Hawthorne or Warm Springs to Beatty.



9. **MULTIPLE CHOICE** If the measures of two sides of a triangle are 3.1 feet and 4.6 feet, which is the *least* possible whole number measure for the third side?

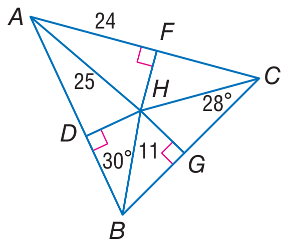
A 1.6 feet

B 2 feet

C 7.5 feet

D 8 feet

Point H is the incenter of $\triangle ABC$. Find each measure.



10. DH
11. BD
12. $m\angle HAC$
13. $m\angle DHG$

14. **MULTIPLE CHOICE** If the lengths of two sides of a triangle are 5 and 11, what is the range of possible lengths for the third side?

F $6 < x < 10$

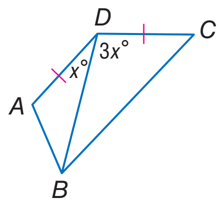
G $5 < x < 11$

H $6 < x < 16$

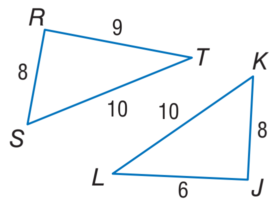
J $x < 5$ or $x > 11$

Compare the given measures.

15. AB and BC



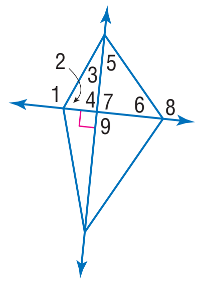
16. $\angle RST$ and $\angle JKL$



State the assumption necessary to start an indirect proof of each statement.

17. If 8 is a factor of n, then 4 is a factor of n.
18. $m\angle M > m\angle N$
19. If $3a + 7 \leq 28$, then $a \leq 7$.

Use the figure to determine which angle has the greatest measure.

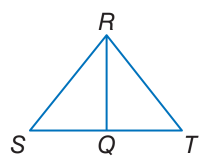


20. $\angle 1, \angle 5, \angle 6$
21. $\angle 9, \angle 8, \angle 3$
22. $\angle 4, \angle 3, \angle 2$

23. **PROOF** Write a two-column proof.

Given: $\overline{RQ}$ bisects $\angle SRT$.

Prove: $m\angle SQR > m\angle SRQ$



Find the range for the measure of the third side of a triangle given the measures of the two sides.

24. 10 ft, 16 ft
25. 23 m, 39 m

5 Preparing for Standardized Tests

Eliminate Unreasonable Answers

You can eliminate unreasonable answers to determine the correct answer when solving multiple choice test items.

Jack Hollingsworth/Photodisc/Getty Images

Strategies for Eliminating Unreasonable Answers

Step 1

Read the problem statement carefully to determine exactly what you are being asked to find.

- What am I being asked to solve?
- Is the correct answer a whole number, fraction, or decimal?
- Do I need to use a graph or table?
- What units (if any) will the correct answer have?

Step 2

Carefully look over each possible answer choice and evaluate for reasonableness. Do not write any digits or symbols outside the answer boxes.

- Identify any answer choices that are clearly incorrect and eliminate them.
- Eliminate any answer choices that are not in the proper format.
- Eliminate any answer choices that do not have the correct units.

Step 3

Solve the problem and choose the correct answer from those remaining. Check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

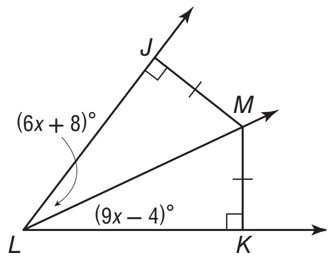
What is the measure of $\angle KLM$?

A 32

B 44

C 78

D 94



Read the problem and study the figure carefully. Triangle *KLM* is a right triangle. Since the sum of the interior angles of a triangle is 180°, $m\angle KLM + m\angle LMK$ must be equal to 90°. Otherwise, the sum would exceed 180°. Since answer choice D is an obtuse angle, it can be eliminated as unreasonable. The correct answer must be A, B, or C.

Solve the problem. According to the converse of the Angle Bisector Theorem, if a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle. Point *M* is equidistant from rays *LJ* and *LK*, so it lies on the angle bisector of $\angle JLK$. Therefore, $\angle JLM$ must be congruent to $\angle KLM$. Set up and solve an equation for *x*.

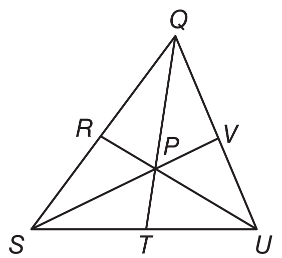
$$6x + 8 = 9x - 4$$
$$-3x = -12$$
$$x = 4$$

So, the measure of $\angle KLM$ is $[9(4) - 4]°$, or 32°. The correct answer is A.

Exercises

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Point *P* is the centroid of triangle *QUS*. If $QP = 14$ centimeters, what is the length of $\overline{QT}$?



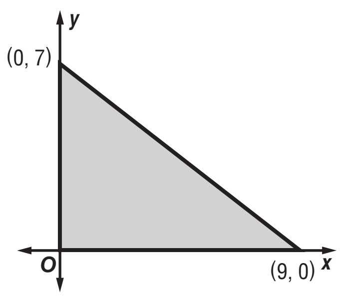
A 7 cm

B 12 cm

C 18 cm

D 21 cm

2. What is the area, in square units, of the triangle shown below?



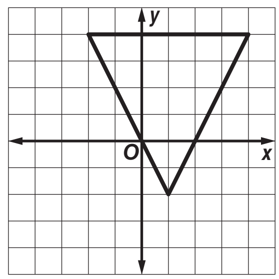
F 8

G 27.4

H 31.5

J 63

3. What are the coordinates of the orthocenter of the triangle below?



A $\left(-\frac{3}{4}, -1\right)$

B $\left(-\frac{4}{3}, 1\right)$

C $\left(1, \frac{5}{2}\right)$

D $\left(1, \frac{9}{4}\right)$

4. If $\triangle ABC$ is isosceles and $m\angle A = 94$, which of the following *must* be true?

F $m\angle B = 94$

G $m\angle B = 47$

H $AB = BC$

J $AB = AC$

5. Which of the following could *not* be the dimensions of a triangle?

A 1.9, 3.2, 4

B 1.6, 3, 4.6

C 3, 7.2, 7.5

D 2.6, 4.5, 6

CHAPTER 5

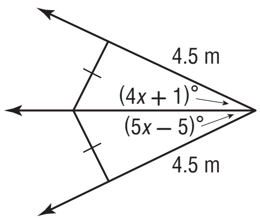
Standardized Test Practice

Cumulative, Chapters 1 through 5

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

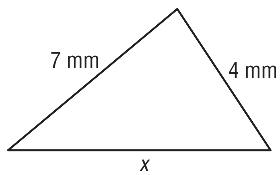
1. Solve for x.



A 3 **C** 5

B 4 **D** 6

2. Which of the following could not be the value of x?



F 8 mm **H** 10 mm

G 9 mm **J** 11 mm

3. Jesse claims that if you live in Lexington, then you live in Kentucky. Which assumption would you need to make to form an indirect proof of this claim?

A Suppose someone lives in Kentucky, but not in Lexington.

B Suppose someone lives in Kentucky and in Lexington.

C Suppose someone lives in Lexington and in Kentucky.

D Suppose someone lives in Lexington, but not in Kentucky.

4. Which of the following best describes the shortest distance from a vertex of a triangle to the opposite side?

F altitude **H** median

G diameter **J** segment

5. Lin started mowing lawns. Let x represent the number of weeks after he began mowing lawns, and y represent the number of customers. Use the points (3, 4) and (9, 6) to find the equation of a line that can be used to predict how many customers Lin has by the end of a certain week.

A $y = \frac{1}{3}x$ **C** $y = \frac{2}{3}x + 2$

B $y = \frac{1}{3}x + 3$ **D** $y = \frac{2}{3}x$

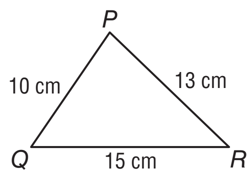
6. What is the correct relationship between the angle measures of $\triangle PQR$?

F $m\angle R < m\angle Q < m\angle P$

G $m\angle R < m\angle P < m\angle Q$

H $m\angle Q < m\angle P < m\angle R$

J $m\angle P < m\angle Q < m\angle R$



7. Which assumption would you need to make in order to start an indirect proof of the statement?

Angle S is not an obtuse angle.

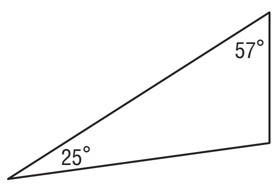
A $\angle S$ is a right angle.

B $\angle S$ is an obtuse angle.

C $\angle S$ is an acute angle.

D $\angle S$ is not an acute angle.

8. Classify the triangle below according to its angle measures.



F acute **H** obtuse

G equiangular **J** right

Test-Taking Tip

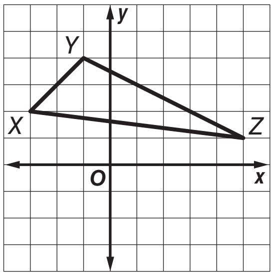
Question 2 The sum of any two sides of a triangle must be greater than the third side.

Short Response/Gridded Response

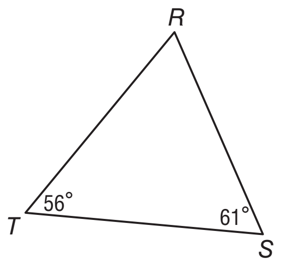
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. **GRIDDED RESPONSE** If the measures of two sides of a triangle are 9 centimeters and 15 centimeters, what is the least possible measure of the third side in centimeters if the measure is an integer?

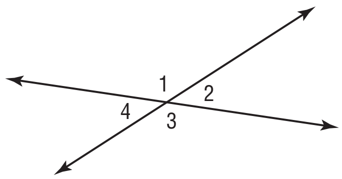
10. What are the coordinates of the orthocenter of the triangle below?



11. List the sides of the triangle below in order from shortest to longest.



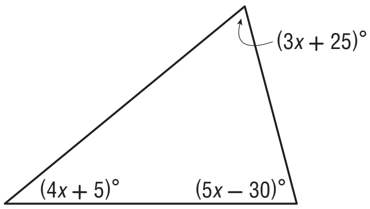
12. Suppose two lines intersect in a plane to form four angles.



What do you know about the pairs of adjacent angles formed? Explain.

13. Eric and Heather are each taking a group of campers hiking in the woods. Eric's group leaves camp and goes 2 miles east, then turns 20° south of east and goes 4 more miles. Heather's group leaves camp and travels 2 miles west, then turns 30° north of west and goes 4 more miles. How many degrees south of east would Eric have needed to turn in order for his group and Heather's group to be the same distance from camp after the two legs of the hike?

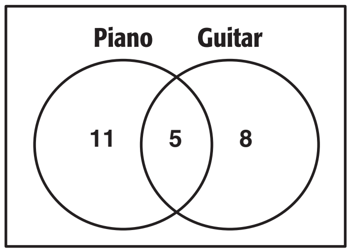
14. **GRIDDED RESPONSE** Solve for x in the triangle below.



Extended Response

Record your answers on a sheet of paper. Show your work.

15. Refer to the figure to answer each question.



a. How many students play the guitar?

b. How many students play the piano?

c. How many students play both piano and guitar?

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	5-1	5-5	5-4	5-2	3-4	5-3	5-4	4-1	5-5	5-2	5-3	1-5	5-6	4-2	2-2