CHAPTER 6

Quadrilaterals

Rubberball/Erik Isakson/Getty Images

Then

You classified polygons. You recognized and applied properties of polygons.

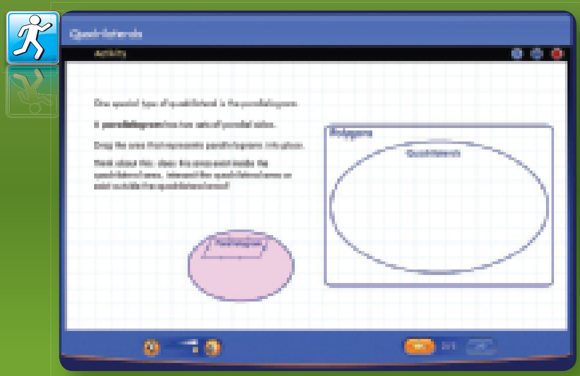
Now

In this chapter, you will:

- Find and use the sum of the measures of the interior and exterior angles of a polygon.
- Recognize and apply properties of quadrilaterals.
- Compare quadrilaterals.

Why? ▲

FUN AND GAMES The properties of quadrilaterals can be used to find various angle measures and side lengths such as the measures of angles in game equipment, playing fields, and game boards.



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Animation | Vocabulary | eGlossary | Personal Tutor | Virtual Manipulatives | Graphing Calculator | Audio | Foldables | Self-Check Practice | Worksheets

Get Ready for the Chapter

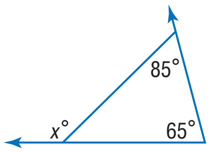
Diagnose Readiness | You have two options for checking Prerequisite Skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

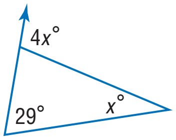
QuickCheck

Find x to the nearest tenth.

1.

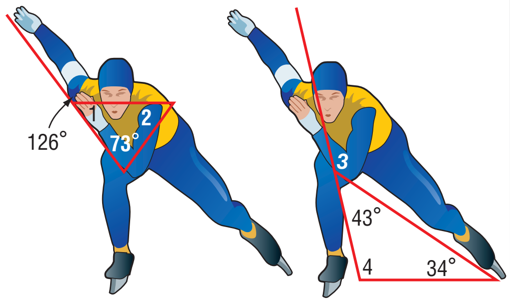


2.



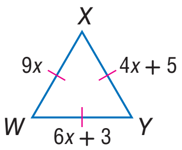
SPEED SKATING A speed skater forms at least two sets of triangles and exterior angles as she skates. Find each measure.

3. $m\angle 1$
4. $m\angle 2$
5. $m\angle 3$
6. $m\angle 4$

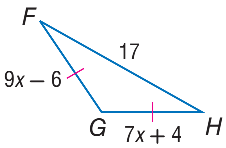


ALGEBRA Find x and the measures of the unknown sides of each triangle.

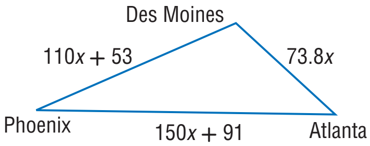
7.



8.



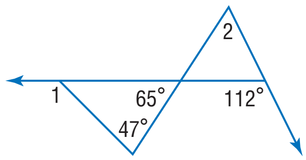
9. **TRAVEL** A plane travels from Des Moines to Phoenix, on to Atlanta, and back to Des Moines, as shown below. Find the distance in miles for each leg of the trip if the total trip was 3482 miles.



QuickReview

Example 1

Find the measure of each numbered angle.



a. $m\angle 1$

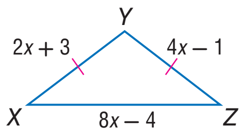
$m\angle 1 = 65 + 47$	Exterior ∠ Theorem
$m\angle 1 = 112$	Add.

b. $m\angle 2$

$180 = m\angle 2 + 68 + 65$	Triangle Sum Theorem
$180 = m\angle 2 + 133$	Simplify.
$m\angle 2 = 47$	Subtract.

Example 2

ALGEBRA Find the measures of the sides of isosceles $\triangle XYZ$.



$XY = YZ$	Given
$2x + 3 = 4x - 1$	Substitution
$-2x = -4$	Subtract.
$x = 2$	Simplify.
$XY = 2x + 3$	Given
$= 2(2) + 3$ or 7	$x = 2$
$YZ = XY$	Given
$= 7$	$XY = 7$
$XZ = 8x - 4$	Given
$= 8(2) - 4$ or 12	$x = 2$

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

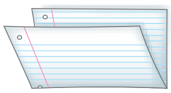
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 6. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

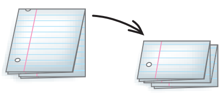
FOLDABLES® **Study**Organizer

Quadrilaterals Make this Foldable to help you organize your Chapter 6 notes about quadrilaterals. Begin with one sheet of notebook paper.

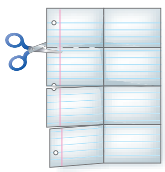
1 **Fold** lengthwise to the holes.



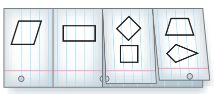
2 **Fold** along the width of the paper twice and unfold the paper.



3 **Cut** along the fold marks on the left side of the paper.



4 **Label** as shown.



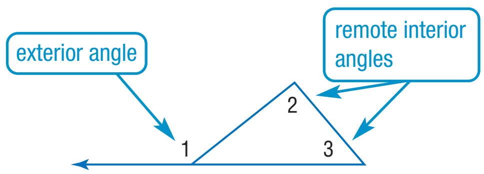
NewVocabulary

English		Español
diagonal	p. 393	diagonal
parallelogram	p. 403	paralelogramo
rectangle	p. 423	rectángulo
rhombus	p. 430	rombo
square	p. 431	cuadrado
trapezoid	p. 439	trapecio
base	p. 439	base
legs	p. 439	catetos
isosceles trapezoid	p. 439	trapecio isósceles
midsegment of a trapezoid	p. 441	segmento medio de un trapecio

ReviewVocabulary

exterior angle ángulo externo an angle formed by one side of a triangle and the extension of another side

remote interior angle ángulos internos no adyacentes the angles of a triangle that are not adjacent to a given exterior angle



slope pendiente for a (nonvertical) line containing two points (x_1, y_1) and (x_2, y_2), the number m given by the formula

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \text{ where } x_2 \neq x_1$$

LESSON 6-1 Angles of Polygons

Then

- You named and classified polygons.

Now

1. Find and use the sum of the measures of the interior angles of a polygon.
2. Find and use the sum of the measures of the exterior angles of a polygon.

Why?

- To create their honeycombs, young worker honeybees excrete flecks of wax that are carefully molded by other bees to form hexagonal cells. The cells are less than 0.1 millimeter thick, but they support almost 25 times their own weight. The cell walls all stand at exactly the same angle to one another. This angle is the measure of the interior angle of a regular hexagon.

NewVocabulary
diagonal

Common Core State Standards

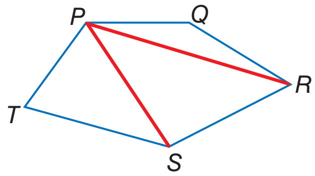
Content Standards
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★

Mathematical Practices
4 Model with mathematics.
3 Construct viable arguments and critique the reasoning of others.

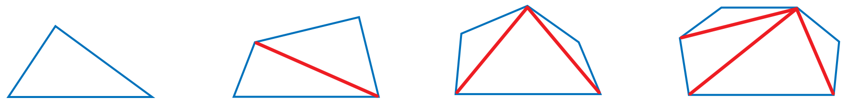
1 Polygon Interior Angles Sum

A **diagonal** of a polygon is a segment that connects any two nonconsecutive vertices.

The vertices of polygon *PQRST* that are not consecutive with vertex *P* are vertices *R* and *S*. Therefore, polygon *PQRST* has two diagonals from vertex *P*, $\overline{PR}$ and $\overline{PS}$. Notice that the diagonals from vertex *P* separate the polygon into three triangles.



The sum of the angle measures of a polygon is the sum of the angle measures of the triangles formed by drawing all the possible diagonals from one vertex.



Triangle **Quadrilateral** **Pentagon** **Hexagon**

Since the sum of the angle measures of a triangle is 180, we can make a table and look for a pattern to find the sum of the angle measures for any convex polygon.

Polygon	Number of Sides	Number of Triangles	Sum of Interior Angle Measures
Triangle	3	1	(1)180 or 180
Quadrilateral	4	2	(2)180 or 360
Pentagon	5	3	(3)180 or 540
Hexagon	6	4	(4)180 or 720
n-gon	*n*	$n - 2$	$(n - 2)180$

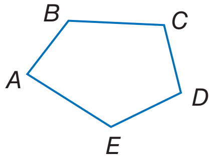
This leads to the following theorem.

Theorem 6.1 Polygon Interior Angles Sum

The sum of the interior angle measures of an *n*-sided convex polygon is $(n - 2) \cdot 180$.

Example $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = (5 - 2) \cdot 180$

$= 540$



You will prove Theorem 6.1 for octagons in Exercise 42.

You can use the Polygon Interior Angles Sum Theorem to find the sum of the interior angles of a polygon and to find missing measures in polygons.

Study Tip

Naming Polygons Remember, a polygon with n-sides is an *n-gon*, but several polygons have special names.

Number of Sides	Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
11	hendecagon
12	dodecagon
n	n-gon

Example 1 Find the Interior Angles Sum of a Polygon

a. Find the sum of the measures of the interior angles of a convex heptagon.

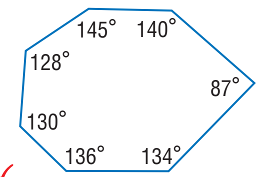
A heptagon has seven sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

$(n - 2) \cdot 180 = (7 - 2) \cdot 180$ $\quad n = 7$

$= 5 \cdot 180$ or 900 $\quad$ Simplify.

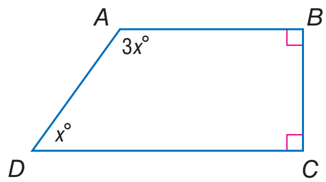
The sum of the measures is 900.

CHECK Draw a convex polygon with seven sides. Use a protractor to measure each angle to the nearest degree. Then find the sum of these measures.



$128 + 145 + 140 + 87 + 134 + 136 + 130 = 900$ ✓

b. ALGEBRA Find the measure of each interior angle of quadrilateral *ABCD*.



Step 1 Find x.

Since there are 4 angles, the sum of the interior angle measures is $(4 - 2) \cdot 180$ or 360.

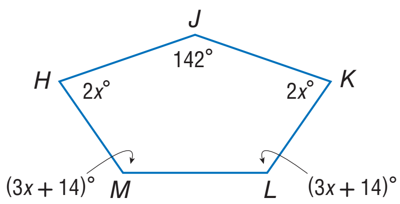
$360 = m\angle A + m\angle B + m\angle C + m\angle D$	Sum of interior angle measures
$360 = 3x + 90 + 90 + x$	Substitution
$360 = 4x + 180$	Combine like terms.
$180 = 4x$	Subtract 180 from each side.
$45 = x$	Divide each side by 4.

Step 2 Use the value of x to find the measure of each angle.

$m\angle A = 3x$ $\quad m\angle B = 90$ $\quad m\angle D = x$

$= 3(45)$ or 135 $\quad m\angle C = 90$ $\quad = 45$

Guided Practice



1A. Find the sum of the measures of the interior angles of a convex octagon.

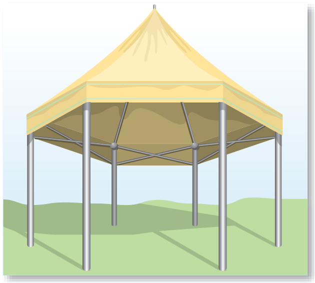
1B. Find the measure of each interior angle of pentagon *HJKLM* shown.

Recall from Lesson 1-6 that in a regular polygon, all of the interior angles are congruent. You can use this fact and the Polygon Interior Angle Sum Theorem to find the interior angle measure of any regular polygon.

Review Vocabulary

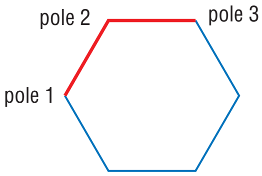
regular polygon
a convex polygon in which all of the sides are congruent and all of the angles are congruent

Real-World Example 2 Interior Angle Measure of Regular Polygon



TENTS The poles for a tent form the vertices of a regular hexagon. When the poles are properly positioned, what is the measure of the angle formed at a corner of the tent?

Understand Draw a diagram of the situation.



The measure of the angle formed at a corner of the tent is an interior angle of a regular hexagon.

Plan Use the Polygon Interior Angles Sum Theorem to find the sum of the measures of the angles. Since the angles of a regular polygon are congruent, divide this sum by the number of angles to find the measure of each interior angle.

Solve **Step 1** Find the sum of the interior angle measures.

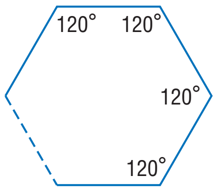
$$(n - 2) \cdot 180 = (6 - 2) \cdot 180 \qquad n = 6$$
$$= 4 \cdot 180 \text{ or } 720 \qquad \text{Simplify.}$$

Step 2 Find the measure of one interior angle.

$$\frac{\text{sum of interior angle measures}}{\text{number of congruent angles}} = \frac{720}{6} \qquad \text{Substitution}$$
$$= 120 \qquad \text{Divide.}$$

The angle at a corner of the tent measures 120.

Check To verify that this measure is correct, use a ruler and a protractor to draw a regular hexagon using 120 as the measure of each interior angle. The last side drawn should connect with the beginning point of the first segment drawn. ✓



Guided Practice

2A. COINS Find the measure of each interior angle of the regular hendecagon that appears on the face of a Susan B. Anthony one-dollar coin.

2B. HOT TUBS A certain company makes hot tubs in a variety of different shapes. Find the measure of each interior angle of the nonagon model.

PhotoSpin, Inc/Alamy

Real-World Link

Susan B. Anthony was a leader of the women's suffrage movement in the late 1800s, which eventually led to the Nineteenth Amendment giving women the right to vote. In 1979, the Susan B. Anthony one-dollar coin was first minted, making her the first woman to be depicted on U.S. currency.

Source: *Encyclopaedia Britannica*

Given the interior angle measure of a regular polygon, you can also use the Polygon Interior Angles Sum Theorem to find a polygon's number of sides.

Example 3 Find Number of Sides Given Interior Angle Measure

The measure of an interior angle of a regular polygon is 135. Find the number of sides in the polygon.

Let n = the number of sides in the polygon. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $135n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n - 2) \cdot 180$.

$135n = (n - 2) \cdot 180$	Write an equation.
$135n = 180n - 360$	Distributive Property
$-45n = -360$	Subtract $180n$ from each side.
$n = 8$	Divide each side by -45.

The polygon has 8 sides.

GuidedPractice

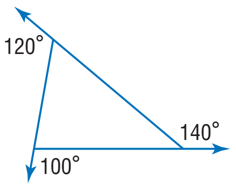
3. The measure of an interior angle of a regular polygon is 144. Find the number of sides in the polygon.

ReviewVocabulary

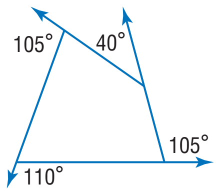
exterior angle an angle formed by one side of a polygon and the extension of another side

2 Polygon Exterior Angles Sum

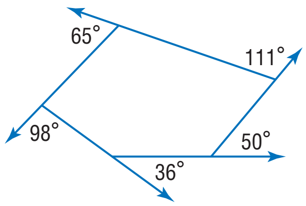
Does a relationship exist between the number of sides of a convex polygon and the sum of its exterior angle measures? Examine the polygons below in which an exterior angle has been measured at each vertex.



$120 + 100 + 140 = \mathbf{360}$



$105 + 110 + 105 + 40 = \mathbf{360}$



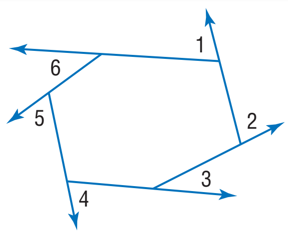
$65 + 98 + 36 + 50 + 111 = \mathbf{360}$

Notice that the sum of the exterior angle measures in each case is 360. This suggests the following theorem.

Theorem 6.2 Polygon Exterior Angles Sum

The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360.

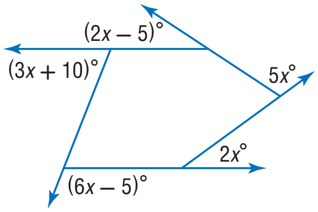
Example
$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360$



You will prove Theorem 6.2 in Exercise 43.

Example 4 Find Exterior Angle Measures of a Polygon

a. ALGEBRA Find the value of x in the diagram.



Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$(2x - 5) + 5x + 2x + (6x - 5) + (3x + 10) = 360$$
$$(2x + 5x + 2x + 6x + 3x) + [-5 + (-5) + 10] = 360$$
$$18x = 360$$
$$x = \frac{360}{18} \text{ or } 20$$

Study Tip

CCSS Perseverance To find the measure of each exterior angle of a regular polygon, you can find the measure of each interior angle and subtract this measure from 180, since an exterior angle and its corresponding interior angle are supplementary.

b. Find the measure of each exterior angle of a regular nonagon.

A regular nonagon has 9 congruent sides and 9 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n = the measure of each exterior angle and write and solve an equation.

$9n = 360$ Polygon Exterior Angles Sum Theorem

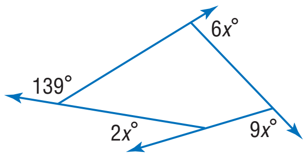
$n = 40$ Divide each side by 9.

The measure of each exterior angle of a regular nonagon is 40.

Guided Practice

4A. Find the value of x in the diagram.

4B. Find the measure of each exterior angle of a regular dodecagon.



Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

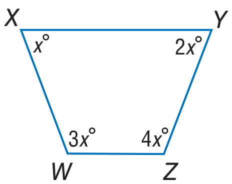
Example 1 **Find the sum of the measures of the interior angles of each convex polygon.**

1. decagon

2. pentagon

Find the measure of each interior angle.

3.



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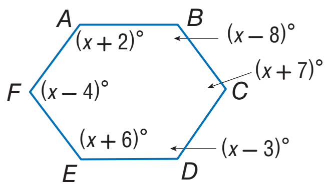
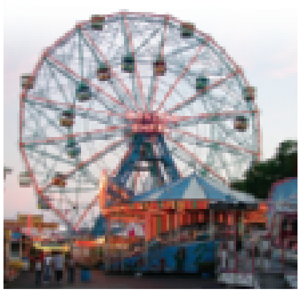


Example 2 **5. AMUSEMENT** The Wonder Wheel at Coney Island in Brooklyn, New York, is a regular polygon with 16 sides. What is the measure of each interior angle of the polygon?



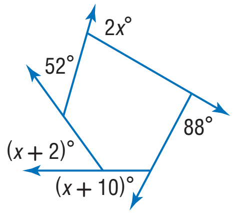
Example 3 **The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.**

6. 150

7. 170

Example 4 **Find the value of x in each diagram.**

8.



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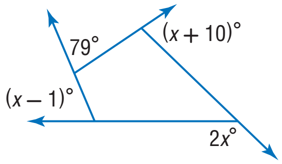


Find the measure of each exterior angle of each regular polygon.

10. quadrilateral

11. octagon

Practice and Problem Solving

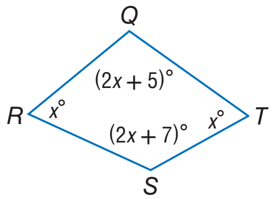
Extra Practice is on page R6.

Example 1 **Find the sum of the measures of the interior angles of each convex polygon.**

12. dodecagon **13.** 20-gon **14.** 29-gon **15.** 32-gon

Find the measure of each interior angle.

16.



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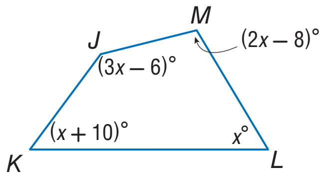
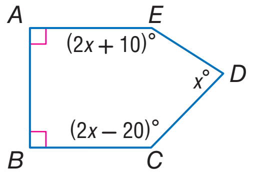
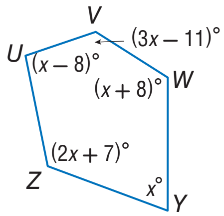


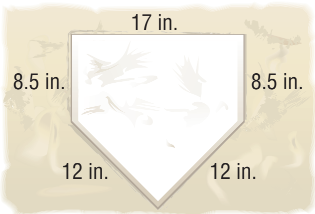
18.



19.



20. **BASEBALL** In baseball, home plate is a pentagon. The dimensions of home plate are shown. What is the sum of the measures of the interior angles of home plate?



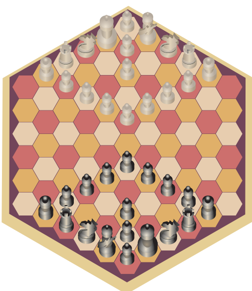
Example 2 **Find the measure of each interior angle of each regular polygon.**

21. dodecagon **22.** pentagon **23.** decagon **24.** nonagon

25. CCSS **MODELING** Hexagonal chess is played on a regular hexagonal board comprised of 92 small hexagons in three colors. The chess pieces are arranged so that a player can move any piece at the start of a game.

a. What is the sum of the measures of the interior angles of the chess board?

b. Does each interior angle have the same measure? If so, give the measure. Explain your reasoning.

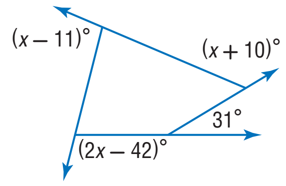


Example 3 **The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.**

26. 60 **27.** 90 **28.** 120 **29.** 156

Example 4 **Find the value of x in each diagram.**

30.



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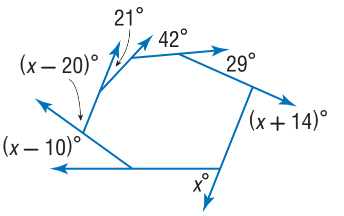
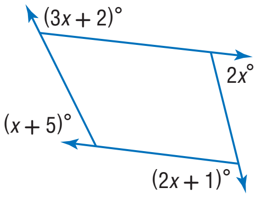
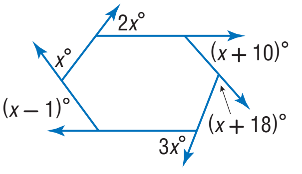


32.



33.



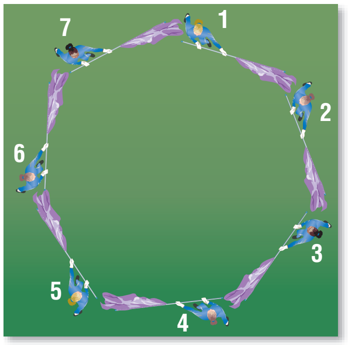
Find the measure of each exterior angle of each regular polygon.

34. decagon **35.** pentagon **36.** hexagon **37.** 15-gon

38. COLOR GUARD During the halftime performance for a football game, the color guard is planning a new formation in which seven members stand around a central point and stretch their flag to the person immediately to their left as shown.

a. What is the measure of each exterior angle of the formation?

b. If the perimeter of the formation is 38.5 feet, how long is each flag?



Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.

39. 7 **40.** 13 **41.** 14

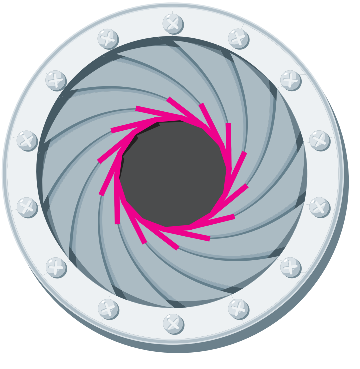
42. PROOF Write a paragraph proof to prove the Polygon Interior Angles Sum Theorem for octagons.

43. PROOF Use algebra to prove the Polygon Exterior Angles Sum Theorem.

44. CCSS MODELING The aperture on the camera lens shown is a regular 14-sided polygon.

a. What is the measure of each interior angle of the polygon?

b. What is the measure of each exterior angle of the polygon?

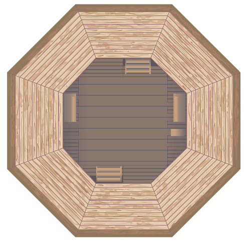


ALGEBRA **Find the measure of each interior angle.**

45. decagon, in which the measures of the interior angles are $x + 5$, $x + 10$, $x + 20$, $x + 30$, $x + 35$, $x + 40$, $x + 60$, $x + 70$, $x + 80$, and $x + 90$

46. polygon *ABCDE*, in which the measures of the interior angles are $6x$, $4x + 13$, $x + 9$, $2x - 8$, $4x - 1$

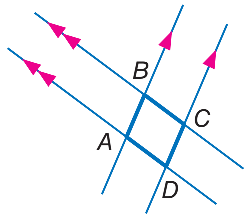
47 **THEATER** The drama club would like to build a theater in the round, so the audience can be seated on all sides of the stage, for its next production.



a. The stage is to be a regular octagon with a total perimeter of 60 feet. To what length should each board be cut to form the sides of the stage?

b. At what angle should each board be cut so that they will fit together as shown? Explain your reasoning.

48. **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle and side relationships in special quadrilaterals.



a. **Geometric** Draw two pairs of parallel lines that intersect like the ones shown. Label the quadrilateral formed by *ABCD*. Repeat these steps to form two additional quadrilaterals, *FGHJ* and *QRST*.

b. **Tabular** Copy and complete the table below.

Quadrilateral	Lengths and Measures							
ABCD	$m\angle A$		$m\angle B$		$m\angle C$		$m\angle D$	
	AB		*BC*		*CD*		*DA*	
FGHJ	$m\angle F$		$m\angle G$		$m\angle H$		$m\angle J$	
	FG		*GH*		*HJ*		*JF*	
QRST	$m\angle Q$		$m\angle R$		$m\angle S$		$m\angle T$	
	QR		*RS*		*ST*		*TQ*	

c. **Verbal** Make a conjecture about the relationship between the angles opposite each other in a quadrilateral formed by two pairs of parallel lines.

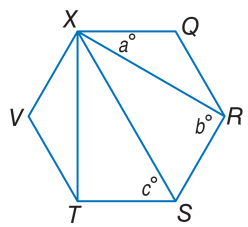
d. **Verbal** Make a conjecture about the relationship between two consecutive angles in a quadrilateral formed by two pairs of parallel lines.

e. **Verbal** Make a conjecture about the relationship between the sides opposite each other in a quadrilateral formed by two pairs of parallel lines.

H.O.T. Problems Use Higher-Order Thinking Skills

49. **ERROR ANALYSIS** Marcus says that the sum of the exterior angles of a decagon is greater than that of a heptagon because a decagon has more sides. Liam says that the sum of the exterior angles for both polygons is the same. Is either of them correct? Explain your reasoning.

50. **CHALLENGE** Find the values of *a*, *b*, and *c* if *QRSTVX* is a regular hexagon. Justify your answer.



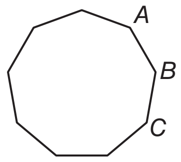
51. **CCSS** **ARGUMENTS** If two sides of a regular hexagon are extended to meet at a point in the exterior of the polygon, will the triangle formed *always*, *sometimes*, or *never* be equilateral? Justify your answer.

52. **OPEN ENDED** Sketch a polygon and find the sum of its interior angles. How many sides does a polygon with twice this interior angles sum have? Justify your answer.

53. **WRITING IN MATH** Explain how triangles are related to the Interior Angles Sum Theorem.

Standardized Test Practice

54. If the polygon shown is regular, what is $m\angle ABC$?



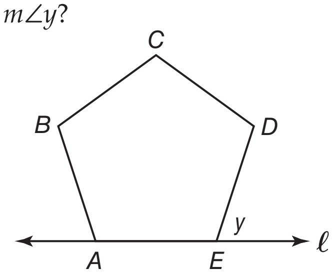
A 140

B 144

C 162

D 180

55. SHORT RESPONSE Figure *ABCDE* is a regular pentagon with line ℓ passing through side *AE*. What is $m\angle y$?



56. ALGEBRA $\dfrac{3^2 \cdot 4^5 \cdot 5^3}{5^3 \cdot 3^3 \cdot 4^6} =$

F $\frac{1}{60}$

G $\frac{1}{12}$

H $\frac{3}{4}$

J 12

57. SAT/ACT The sum of the measures of the interior angles of a polygon is twice the sum of the measures of its exterior angles. What type of polygon is it?

A square

B pentagon

C hexagon

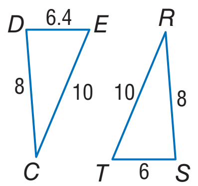
D octagon

E nonagon

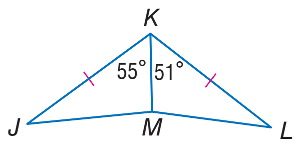
Spiral Review

Compare the given measures. (Lesson 5-6)

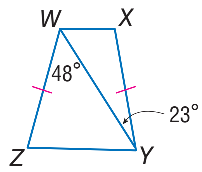
58. $m\angle DCE$ and $m\angle SRT$



59. *JM* and *ML*



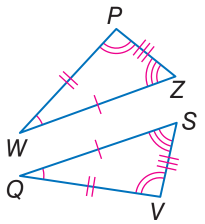
60. *WX* and *ZY*



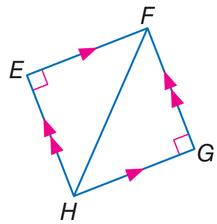
61. HISTORY The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. How many different triangles can be formed using the rope below? (Lesson 5-5)

Show that the triangles are congruent by identifying all congruent corresponding parts. Then write a congruence statement. (Lesson 4-3)

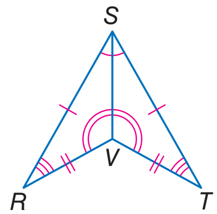
62.



63.

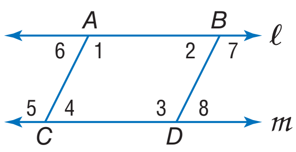


64.



Skills Review

In the figure, $\ell \parallel m$ and $\overline{AC} \parallel \overline{BD}$. Name all pairs of angles for each type indicated.



65. alternate interior angles

66. consecutive interior angles

EXTEND 6-1

Spreadsheet Lab
Angles of Polygons

It is possible to find the interior and exterior measurements along with the sum of the interior angles of any regular polygon with *n* number of sides by using a spreadsheet.

Activity

Design a spreadsheet using the following steps.

- Label the columns as shown in the spreadsheet below.
- Enter the digits 3–10 in the first column.
- The number of triangles in a polygon is 2 fewer than the number of sides. Write a formula for Cell B1 to subtract 2 from each number in Cell A1.
- Enter a formula for Cell C1 so the spreadsheet will calculate the sum of the measures of the interior angles. Remember that the formula is $S = (n - 2)180$.
- Continue to enter formulas so that the indicated computation is performed. Then, copy each formula through Row 9. The final spreadsheet will appear as below.

Polygons and Angles

	A	B	C	D	E	F
1	Number of Sides	Number of Triangles	Sum of Measures of Interior Angles	Measure of Each Interior Angle	Measure of Each Exterior Angle	Measures of Exterior Angles
2	3	1	180	60	120	360
3	4	2	360	90	90	360
4	5	3	540	108	72	360
5	6	4	720	120	60	360
6	7	5	900	128.57	51.43	360
7	8	6	1080	135	45	360
8	9	7	1260	140	40	360
9	10	8	1440	144	36	360

Sheet 1 | Sheet 2 | Sheet 3

Exercises

1. Write the formula to find the measure of each interior angle in the polygon.
2. Write the formula to find the sum of the measures of the exterior angles.
3. What is the measure of each interior angle if the number of sides is 1? 2?
4. Is it possible to have values of 1 and 2 for the number of sides? Explain.

For Exercises 5–8, use the spreadsheet.

5. How many triangles are in a polygon with 17 sides?
6. Find the measure of an exterior angle of a regular polygon with 16 sides.
7. Find the measure of an interior angle of a regular polygon with 115 sides.
8. If the measure of the exterior angles is 0, find the measure of the interior angles. Is this possible? Explain.

LESSON 6-2 Parallelograms

Then

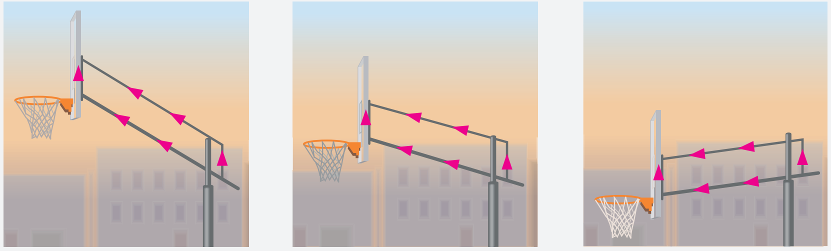
- You classified polygons with four sides as quadrilaterals.

Now

1. Recognize and apply properties of the sides and angles of parallelograms.
2. Recognize and apply properties of the diagonals of parallelograms.

Why?

- The arm of the basketball goal shown can be adjusted to a height of 10 feet or 5 feet. Notice that as the height is adjusted, each pair of opposite sides of the quadrilateral formed by the arms remains parallel.



NewVocabulary

parallelogram

Common Core State Standards

Content Standards

G.CO.11 Prove theorems about parallelograms.

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

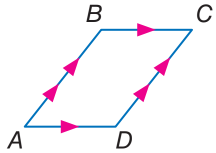
Mathematical Practices

4 Model with mathematics.

3 Construct viable arguments and critique the reasoning of others.

1 Sides and Angles of Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. To name a parallelogram, use the symbol ▱. In ▱$ABCD$, $\overline{BC} \parallel \overline{AD}$ and $\overline{AB} \parallel \overline{DC}$ by definition.



▱$ABCD$

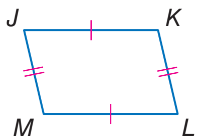
Other properties of parallelograms are given in the theorems below.

Theorem Properties of Parallelograms

6.3 If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Abbreviation *Opp. sides of a ▱ are ≅.*

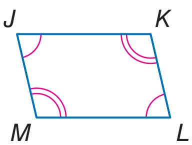
Example If $JKLM$ is a parallelogram, then $\overline{JK} \cong \overline{ML}$ and $\overline{JM} \cong \overline{KL}$.



6.4 If a quadrilateral is a parallelogram, then its opposite angles are congruent.

Abbreviation *Opp. ∠s of a ▱ are ≅.*

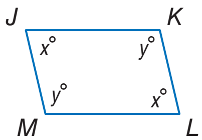
Example If $JKLM$ is a parallelogram, then $\angle J \cong \angle L$ and $\angle K \cong \angle M$.



6.5 If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

Abbreviation *Cons. ∠s in a ▱ are supplementary.*

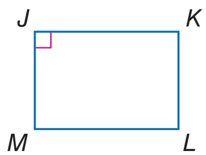
Example If $JKLM$ is a parallelogram, then $x + y = 180$.



6.6 If a parallelogram has one right angle, then it has four right angles.

Abbreviation *If a ▱ has 1 rt. ∠, it has 4 rt. ∠s.*

Example In ▱$JKLM$, if $\angle J$ is a right angle, then $\angle K$, $\angle L$, and $\angle M$ are also right angles.



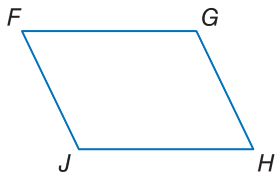
You will prove Theorems 6.3, 6.5, and 6.6 in Exercises 28, 26, and 7, respectively.

StudyTip

Including a Figure Theorems are presented in general terms. In a proof, you must include a drawing so that you can refer to segments and angles specifically.

Proof Theorem 6.4

Write a two-column proof of Theorem 6.4.



Given: ▱$FGHJ$

Prove: $\angle F \cong \angle H$, $\angle J \cong \angle G$

Proof:

Statements	Reasons
1. ▱$FGHJ$	1. Given
2. $\overline{FG} \parallel \overline{JH}$; $\overline{FJ} \parallel \overline{GH}$	2. Definition of parallelogram
3. $\angle F$ and $\angle J$ are supplementary. $\angle J$ and $\angle H$ are supplementary. $\angle H$ and $\angle G$ are supplementary.	3. If parallel lines are cut by a transversal, consecutive interior angles are supplementary.
4. $\angle F \cong \angle H$, $\angle J \cong \angle G$	4. Supplements of the same angles are congruent.

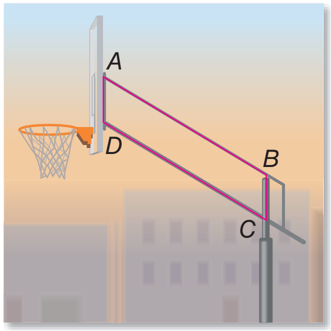
Real-World Career

Coach Coaches organize amateur and professional atheletes, teaching them the fundamentals of a sport. They manage teams during both practice sessions and competitions. Additional tasks may include selecting and issuing sports equiment, materials, and supplies. Head coaches at public secondary schools usually have a bachelor's degree.

PT

Real-World Example 1 Use Properties of Parallelograms

BASKETBALL **In ▱$ABCD$, suppose $m\angle A = 55$, $AB = 2.5$ feet, and $BC = 1$ foot. Find each measure.**



a. DC

$DC = AB$ — Opp. sides of a ▱ are ≅.

$= 2.5$ ft — Substitution

b. $m\angle B$

$m\angle B + m\angle A = 180$ — Cons. ∠s in a ▱ are supplementary.

$m\angle B + 55 = 180$ — Substitution

$m\angle B = 125$ — Subtract 55 from each side.

c. $m\angle C$

$m\angle C = m\angle A$ — Opp. ∠s of a ▱ are ≅.

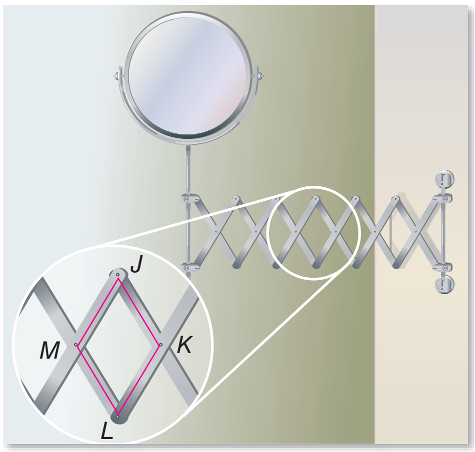
$= 55$ — Substitution

GuidedPractice

1. **MIRRORS** The wall-mounted mirror shown uses parallelograms that change shape as the arm is extended. In ▱$JKLM$, suppose $m\angle J = 47$. Find each measure.

A. $m\angle L$ **B.** $m\angle M$

C. Suppose the arm was extended further so that $m\angle J = 90$. What would be the measure of each of the other angles? Justify your answer.



2 Diagonals of Parallelograms

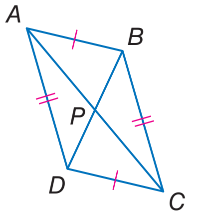
The diagonals of a parallelogram have special properties as well.

Theorem Diagonals of Parallelograms

6.7 If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Abbreviation *Diag. of a ▱ bisect each other.*

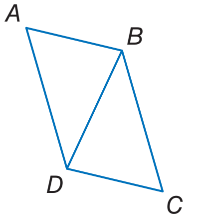
Example If $ABCD$ is a parallelogram, then $\overline{AP} \cong \overline{PC}$ and $\overline{DP} \cong \overline{PB}$.



6.8 If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two congruent triangles.

Abbreviation *Diag. separates a ▱ into 2 ≅ △s.*

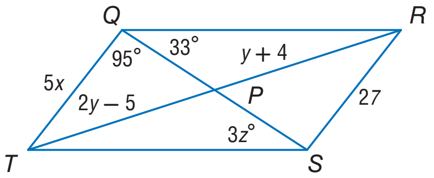
Example If $ABCD$ is a parallelogram, then $\triangle ABD \cong \triangle CDB$.



You will prove Theorems 6.7 and 6.8 in Exercises 29 and 27, respectively.

Example 2 Use Properties of Parallelograms and Algebra

ALGEBRA **If $QRST$ is a parallelogram, find the value of the indicated variable.**



a. x

$\overline{QT} \cong \overline{RS}$	Opp. sides of a ▱ are ≅.
$QT = RS$	Definition of congruence
$5x = 27$	Substitution
$x = 5.4$	Divide each side by 5.

b. y

$\overline{TP} \cong \overline{PR}$	Diag. of a ▱ bisect each other.
$TP = PR$	Definition of congruence
$2y - 5 = y + 4$	Substitution
$y = 9$	Subtract y and add 5 to each side.

c. z

$\triangle TQS \cong \triangle RSQ$	Diag. separates a ▱ into 2 ≅ △s.
$\angle QST \cong \angle SQR$	CPCTC
$m\angle QST = m\angle SQR$	Definition of congruence
$3z = 33$	Substitution
$z = 11$	Divide each side by 3.

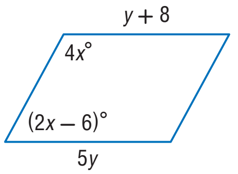
StudyTip

Congruent Triangles A parallelogram with two diagonals divides the figure into two pairs of congruent triangles.

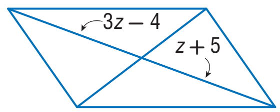
GuidedPractice

Find the value of each variable in the given parallelogram.

2A.



2B.



You can use Theorem 6.7 to determine the coordinates of the intersection of the diagonals of a parallelogram on a coordinate plane given the coordinates of the vertices.

Example 3 Parallelograms and Coordinate Geometry

COORDINATE GEOMETRY Determine the coordinates of the intersection of the diagonals of ▱*FGHJ* with vertices *F*(−2, 4), *G*(3, 5), *H*(2, −3), and *J*(−3, −4).

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of $\overline{FH}$ and $\overline{GJ}$. Find the midpoint of $\overline{FH}$ with endpoints (−2, 4) and (2, −3).

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 2}{2}, \frac{4 + (-3)}{2}\right) \quad \text{Midpoint Formula}$$

$$= (0, 0.5) \quad \text{Simplify.}$$

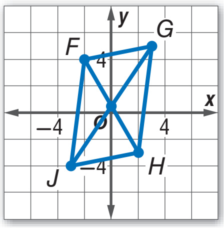
The coordinates of the intersection of the diagonals of ▱*FGHJ* are (0, 0.5).

CHECK Find the midpoint of $\overline{GJ}$ with endpoints (3, 5) and (−3, −4).

$$\left(\frac{3 + (-3)}{2}, \frac{5 + (-4)}{2}\right) = (0, 0.5) \checkmark$$

StudyTip

CCSS Regularity Graph the parallelogram in Example 3 and the point of intersection of the diagonals you found. Draw the diagonals. The point of intersection appears to be correct.



GuidedPractice

3. **COORDINATE GEOMETRY** Determine the coordinates of the intersection of the diagonals of *RSTU* with vertices *R*(−8, −2), *S*(−6, 7), *T*(6, 7), and *U*(4, −2).

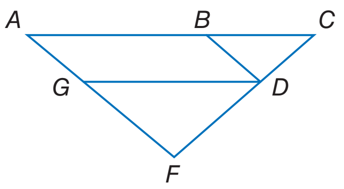
You can use the properties of parallelograms and their diagonals to write proofs.

Example 4 Proofs Using the Properties of Parallelograms

Write a paragraph proof.

Given: ▱*ABDG*, $\overline{AF} \cong \overline{CF}$

Prove: $\angle BDG \cong \angle C$



Proof:

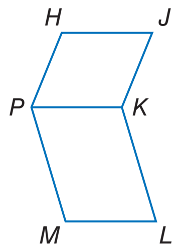
We are given *ABDG* is a parallelogram. Since opposite angles in a parallelogram are congruent, $\angle BDG \cong \angle A$. We are also given that $\overline{AF} \cong \overline{CF}$. By the Isosceles Triangle Theorem, $\angle A \cong \angle C$. So, by the Transitive Property of Congruence, $\angle BDG \cong \angle C$.

GuidedPractice

4. Write a two-column proof.

   Given: ▱*HJKP* and ▱*PKLM*

   Prove: $\overline{HJ} \cong \overline{ML}$

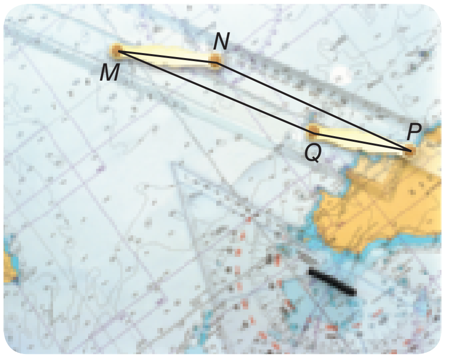


Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1

1. NAVIGATION To chart a course, sailors use a *parallel ruler.* One edge of the ruler is placed along the line representing the direction of the course to be taken. Then the other ruler is moved until its edge reaches the compass rose printed on the chart. Reading the compass determines which direction to travel. The rulers and the crossbars of the tool form $\square MNPQ$.



a. If $m\angle NMQ = 32$, find $m\angle MNP$.

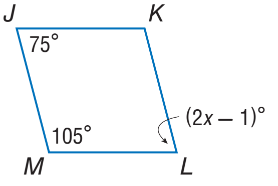
b. If $m\angle MQP = 125$, find $m\angle MNP$.

c. If $MQ = 4$, what is NP?

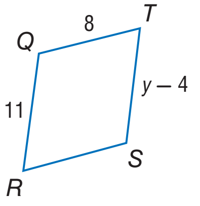
Example 2

ALGEBRA **Find the value of each variable in each parallelogram.**

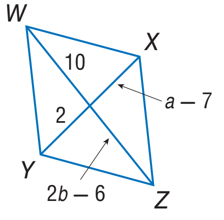
2.



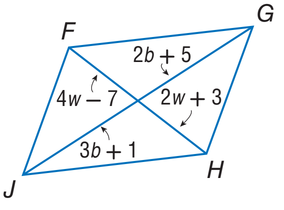
3.



4.



5.



Example 3

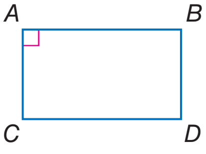
6. COORDINATE GEOMETRY Determine the coordinates of the intersection of the diagonals of $\square ABCD$ with vertices $A(-4, 6)$, $B(5, 6)$, $C(4, -2)$, and $D(-5, -2)$.

Example 4

CCSS **ARGUMENTS** **Write the indicated type of proof.**

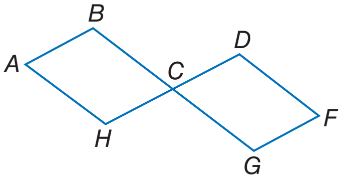
7. paragraph

Given: $\square ABCD$, $\angle A$ is a right angle.
Prove: $\angle B$, $\angle C$, and $\angle D$ are right angles. (Theorem 6.6)



8. two-column

Given: *ABCH* and *DCGF* are parallelograms.
Prove: $\angle A \cong \angle F$

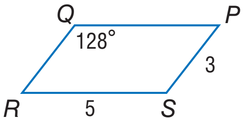


Practice and Problem Solving

Extra Practice is on page R6.

Example 1

Use $\square PQRS$ to find each measure.



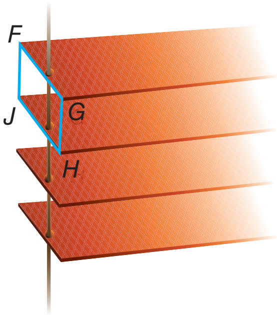
9 $m\angle R$

10. QR

11. QP

12. $m\angle S$

13 **HOME DECOR** The slats on Venetian blinds are designed to remain parallel in order to direct the path of light coming in a window. In ▱$FGHJ$, $FJ = \frac{3}{4}$ inch, $FG = 1$ inch, and $m\angle JHG = 62$. Find each measure.



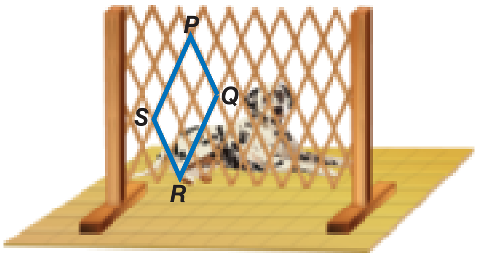
a. JH

b. GH

c. $m\angle JFG$

d. $m\angle FJH$

14. **CCSS MODELING** Wesley is a member of the kennel club in his area. His club uses accordion fencing like the section shown at the right to block out areas at dog shows.

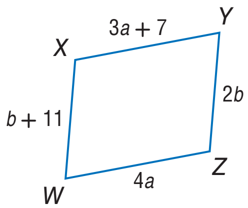


a. Identify two pairs of congruent segments.

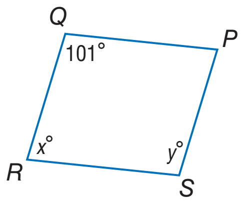
b. Identify two pairs of supplementary angles.

Example 2 **ALGEBRA** **Find the value of each variable in each parallelogram.**

15.



16.



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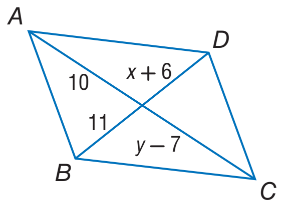

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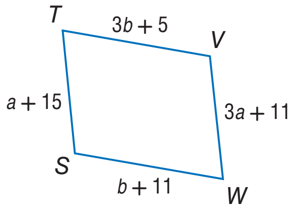
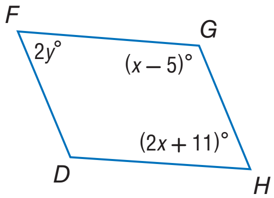
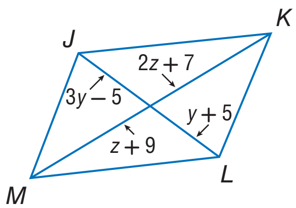


19.



20.



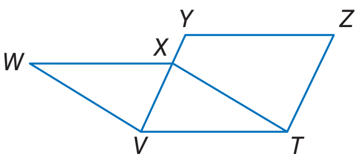
Example 3 **COORDINATE GEOMETRY** **Find the coordinates of the intersection of the diagonals of ▱$WXYZ$ with the given vertices.**

21. $W(-1, 7)$, $X(8, 7)$, $Y(6, -2)$, $Z(-3, -2)$

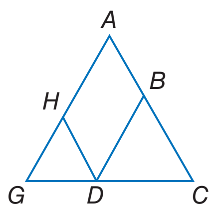
22. $W(-4, 5)$, $X(5, 7)$, $Y(4, -2)$, $Z(-5, -4)$

Example 4 **PROOF** **Write a two-column proof.**

23. Given: $WXTV$ and $ZYVT$ are parallelograms.
Prove: $\overline{WX} \cong \overline{ZY}$



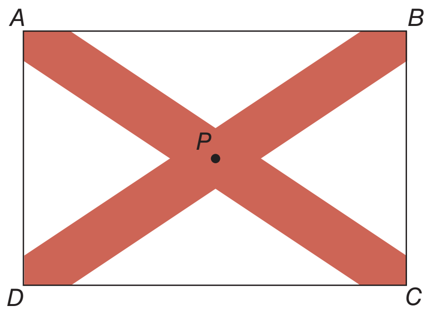
24. Given: ▱$BDHA$, $\overline{CA} \cong \overline{CG}$
Prove: $\angle BDH \cong \angle G$



25. FLAGS Refer to the Alabama state flag at the right.

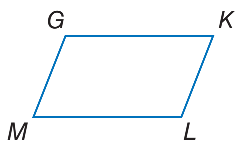
Given: $\triangle ACD \cong \triangle CAB$

Prove: $\overline{DP} \cong \overline{PB}$

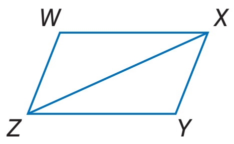


CCSS ARGUMENTS **Write the indicated type of proof.**

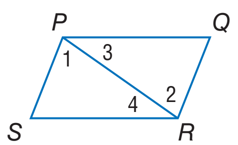
26. two-column
Given: $\square GKLM$
Prove: $\angle G$ and $\angle K$, $\angle K$ and $\angle L$, $\angle L$ and $\angle M$, and $\angle M$ and $\angle G$ are supplementary. (Theorem 6.5)



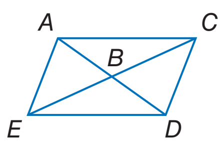
27. two-column
Given: $\square WXYZ$
Prove: $\triangle WXZ \cong \triangle YZX$ (Theorem 6.8)



28. two-column
Given: $\square PQRS$
Prove: $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{SP}$ (Theorem 6.3)



29. paragraph
Given: $\square ACDE$ is a parallelogram.
Prove: $\overline{EC}$ bisects $\overline{AD}$. (Theorem 6.7)

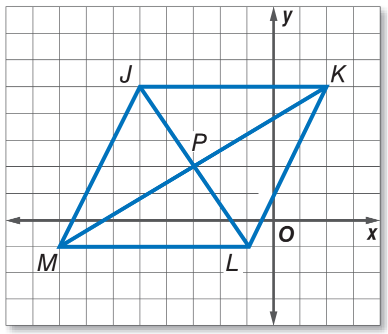


30. COORDINATE GEOMETRY Use the graph shown.

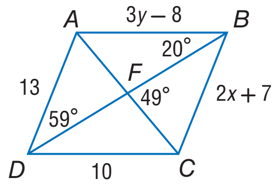
a. Use the Distance Formula to determine if the diagonals of *JKLM* bisect each other. Explain.

b. Determine whether the diagonals are congruent. Explain.

c. Use slopes to determine if the consecutive sides are perpendicular. Explain.



ALGEBRA **Use $\square ABCD$ to find each measure or value.**



31. x

32. y

33. $m\angle AFB$

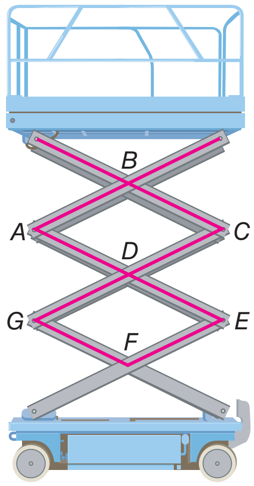
34. $m\angle DAC$

35. $m\angle ACD$

36. $m\angle DAB$

37. COORDINATE GEOMETRY $\square ABCD$ has vertices $A(-3, 5)$, $B(1, 2)$, and $C(3, -4)$. Determine the coordinates of vertex D if it is located in Quadrant III.

38. **MECHANICS** Scissor lifts are variable elevation work platforms. One is shown at the right. In the diagram, *ABCD* and *DEFG* are congruent parallelograms.



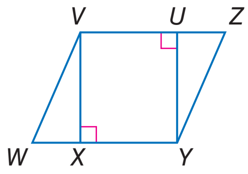
a. List the angle(s) congruent to $\angle A$. Explain your reasoning.

b. List the segment(s) congruent to $\overline{BC}$. Explain your reasoning.

c. List the angle(s) supplementary to $\angle C$. Explain your reasoning.

PROOF **Write a two-column proof.**

39. **Given:** $\square YWVZ$, $\overline{VX} \perp \overline{WY}$, $\overline{YU} \perp \overline{VZ}$
Prove: $\triangle YUZ \cong \triangle VXW$



40. **MULTIPLE REPRESENTATIONS** In this problem, you will explore tests for parallelograms.

a. **Geometric** Draw three pairs of segments that are both congruent and parallel and connect the endpoints to form quadrilaterals. Label one quadrilateral *ABCD*, one *MNOP*, and one *WXYZ*. Measure and label the sides and angles of the quadrilaterals.

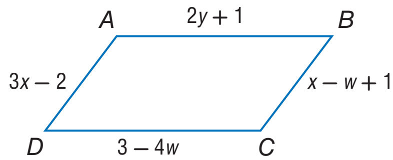
b. **Tabular** Copy and complete the table below.

Quadrilateral	Opposite Sides Congruent?	Opposite Angles Congruent?	Parallelogram
ABCD			
MNOP			
WXYZ			

c. **Verbal** Make a conjecture about quadrilaterals with one pair of segments that are both congruent and parallel.

H.O.T. Problems Use Higher-Order Thinking Skills

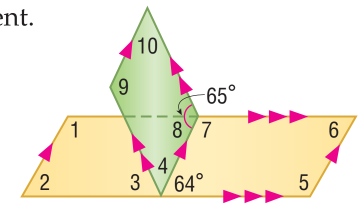
41. **CHALLENGE** *ABCD* is a parallelogram with side lengths as indicated in the figure at the right. The perimeter of *ABCD* is 22. Find *AB*.



42. **WRITING IN MATH** Explain why parallelograms are *always* quadrilaterals, but quadrilaterals are *sometimes* parallelograms.

43. **OPEN ENDED** Provide a counterexample to show that parallelograms are not always congruent if their corresponding sides are congruent.

44. **CCSS REASONING** Find $m\angle 1$ and $m\angle 10$ in the figure at the right. Explain.



45. **WRITING IN MATH** Summarize the properties of the sides, angles, and diagonals of a parallelogram.

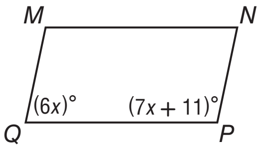
Standardized Test Practice

46. Two consecutive angles of a parallelogram measure $3x + 42$ and $9x - 18$. What are the measures of the angles?

A 13, 167 **C** 39, 141

B 58.5, 31.5 **D** 81, 99

47. GRIDDED RESPONSE Parallelogram *MNPQ* is shown. What is the value of x?



48. ALGEBRA In a history class with 32 students, the ratio of girls to boys is 5 to 3. How many more girls are there than boys?

F 2 **G** 8 **H** 12 **J** 15

49. SAT/ACT The table shows the heights of the tallest buildings in Kansas City, Missouri. To the nearest tenth, what is the positive difference between the median and the mean of the data?

Name	Height (m)
One Kansas City Place	193
Town Pavillion	180
Hyatt Regency	154
Power and Light Building	147
City Hall	135
1201 Walnut	130

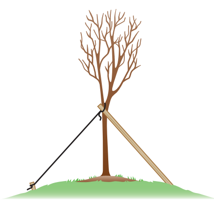
A 5
B 6
C 7
D 8
E 10

Spiral Review

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon. (Lesson 6-1)

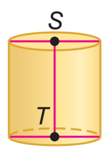
50. 108 **51.** 140 **52.** ≈ 147.3 **53.** 160 **54.** 135 **55.** 176.4

56. LANDSCAPING When landscapers plant new trees, they usually brace the tree using a stake tied to the trunk of the tree. Use the SAS or SSS Inequality to explain why this is an effective method for keeping a newly planted tree perpendicular to the ground. Assume that the tree does not lean forward or backward. (Lesson 5-6)

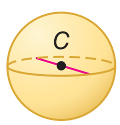


Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices. (Lesson 1-7)

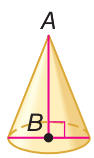
57.



58.



59.



Skills Review

The vertices of a quadrilateral are $W(3, -1)$, $X(4, 2)$, $Y(-2, 3)$ and $Z(-3, 0)$. Determine whether each segment is a side or diagonal of the quadrilateral, and find the slope of each segment.

60. $\overline{YZ}$ **61.** $\overline{YW}$ **62.** $\overline{ZW}$

EXPLORE 6-3

Graphing Technology Lab
Parallelograms

You can use the Cabri™ Jr. application on a TI-83/84 Plus graphing calculator to discover properties of parallelograms.

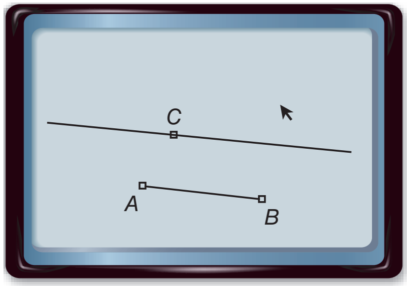
CCSS Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Activity

Construct a quadrilateral with one pair of sides that are both parallel and congruent.

Step 1 Construct a segment using the **Segment** tool on the **F2** menu. Label the segment $\overline{AB}$. This is one side of the quadrilateral.

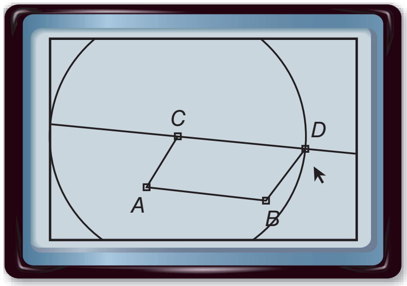
Step 2 Use the **Parallel** tool on the **F3** menu to construct a line parallel to the segment. Pressing ENTER will draw the line and a point on the line. Label the point C.



Steps 1 and 2

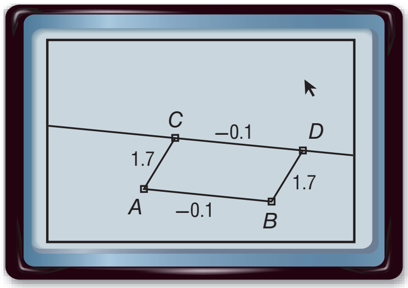
Step 3 Access the **Compass** tool on the **F3** menu. Set the compass to the length of $\overline{AB}$ by selecting one endpoint of the segment and then the other. Construct a circle centered at C.

Step 4 Use the **Point Intersection** tool on the **F2** menu to draw a point at the intersection of the line and the circle. Label the point D. Then use the **Segment** tool on the **F2** menu to draw $\overline{AC}$ and $\overline{BD}$.



Steps 3 and 4

Step 5 Use the **Hide/Show** tool on the **F5** menu to hide the circle. Then access **Slope** tool under **Measure** on the **F5** menu. Display the slopes of $\overline{AB}$, $\overline{BD}$, $\overline{CD}$, and $\overline{AC}$.



Step 5

Analyze the Results

1. What is the relationship between sides $\overline{AB}$ and $\overline{CD}$? Explain how you know.
2. What do you observe about the slopes of opposite sides of the quadrilateral? What type of quadrilateral is $ABDC$? Explain.
3. Click on point A and drag it to change the shape of $ABDC$. What do you observe?
4. Make a conjecture about a quadrilateral with a pair of opposite sides that are both congruent and parallel.
5. Use a graphing calculator to construct a quadrilateral with both pairs of opposite sides congruent. Then analyze the slopes of the sides of the quadrilateral. Make a conjecture based on your observations.

LESSON 6-3 Tests for Parallelograms

Then

- You recognized and applied properties of parallelograms.

Now

1. Recognize the conditions that ensure a quadrilateral is a parallelogram.
2. Prove that a set of points forms a parallelogram in the coordinate plane.

Why?

- Lexi and Rosalinda cut strips of bulletin board paper at an angle to form the hallway display shown. Their friends asked them how they cut the strips so that their sides were parallel without using a protractor.

Rosalinda explained that since the left and right sides of the paper were parallel, she only needed to make sure that the sides were cut to the same length to guarantee that a strip would form a parallelogram.

CCSS Common Core State Standards

Content Standards

G.CO.11 Prove theorems about parallelograms.

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

2 Reason abstractly and quantitatively.

1 Conditions for Parallelograms

If a quadrilateral has each pair of opposite sides parallel, it is a parallelogram by definition.

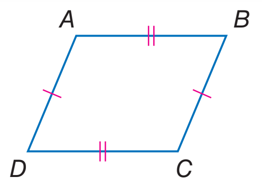
This is not the only test, however, that can be used to determine if a quadrilateral is a parallelogram.

Theorems Conditions for Parallelograms

6.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation *If both pairs of opp. sides are ≅, then quad. is a ▱.*

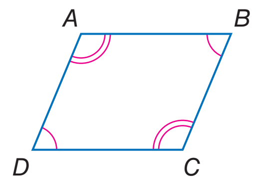
Example If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, then $ABCD$ is a parallelogram.



6.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation *If both pairs of opp. ∠s are ≅, then quad. is a ▱.*

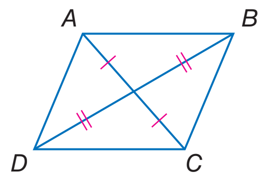
Example If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.



6.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Abbreviation *If diag. bisect each other, then quad. is a ▱.*

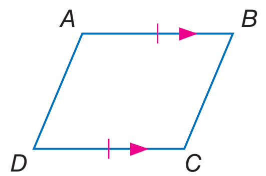
Example If $\overline{AC}$ and $\overline{DB}$ bisect each other, then $ABCD$ is a parallelogram.



6.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Abbreviation *If one pair of opp. sides is ≅ and ||, then the quad. is a ▱.*

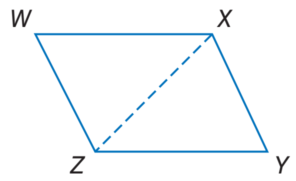
Example If $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$, then $ABCD$ is a parallelogram.



You will prove Theorems 6.10, 6.11, and 6.12 in Exercises 30, 32, and 33, respectively.

Proof Theorem 6.9

Write a paragraph proof of Theorem 6.9.



Given: $\overline{WX} \cong \overline{ZY}$, $\overline{WZ} \cong \overline{XY}$

Prove: *WXYZ* is a parallelogram.

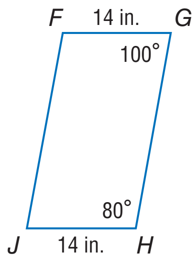
Paragraph Proof:

Two points determine a line, so we can draw auxiliary line $\overline{ZX}$ to form $\triangle ZWX$ and $\triangle XYZ$. We are given that $\overline{WX} \cong \overline{ZY}$ and $\overline{WZ} \cong \overline{XY}$. Also, $\overline{ZX} \cong \overline{XZ}$ by the Reflexive Property of Congruence. So $\triangle ZWX \cong \triangle XYZ$ by SSS. By CPCTC, $\angle WXZ \cong \angle YZX$ and $\angle WZX \cong \angle YXZ$. This means that $\overline{WX} \parallel \overline{ZY}$ and $\overline{WZ} \parallel \overline{XY}$ by the Alternate Interior Angles Converse. Opposite sides of *WXYZ* are parallel, so by definition *WXYZ* is a parallelogram.

PT

Example 1 Identify Parallelograms

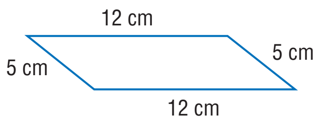
Determine whether the quadrilateral is a parallelogram. Justify your answer.



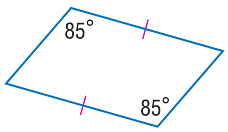
Opposite sides $\overline{FG}$ and $\overline{JH}$ are congruent because they have the same measure. Also, since $\angle FGH$ and $\angle GHJ$ are supplementary consecutive interior angles, $\overline{FG} \parallel \overline{JH}$. Therefore, by Theorem 6.12, *FGHJ* is a parallelogram.

GuidedPractice

1A.



1B.



You can use the conditions of parallelograms to prove relationships in real-world situations.

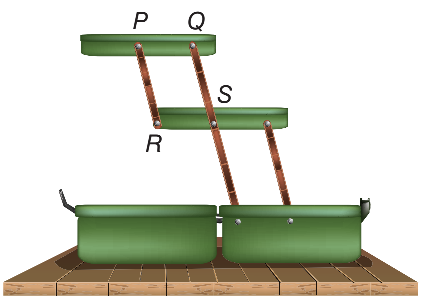
Real-WorldLink

A 2- or 3-cantilever tackle box is often used to organize lures and other fishing supplies. The trays lift up and away so that all items in the box are easily accessible.

PT

Real-World Example 2 Use Parallelograms to Prove Relationships

FISHING The diagram shows a side view of the tackle box at the left. In the diagram, $PQ = RS$ and $PR = QS$. Explain why the upper and middle trays remain parallel no matter to what height the trays are raised or lowered.



Since both pairs of opposite sides of quadrilateral *PQSR* are congruent, *PQRS* is a parallelogram by Theorem 6.9. By the definition of a parallelogram, opposite sides are parallel, so $\overline{PQ} \parallel \overline{RS}$. Therefore, no matter the vertical position of the trays, they will always remain parallel.

GuidedPractice

2. BANNERS In the example at the beginning of the lesson, explain why the cuts made by Lexi and Rosalinda are parallel.

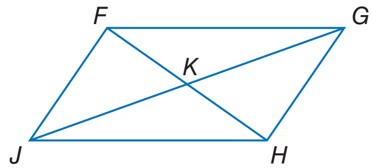
You can also use the conditions of parallelograms along with algebra to find missing values that make a quadrilateral a parallelogram.

WatchOut!

Parallelograms In Example 3, if x is 4, then y must be 2.5 in order for $FGHJ$ to be a parallelogram. In other words, if x is 4 and y is 1, then $FGHJ$ is not a parallelogram.

Example 3 Use Parallelograms and Algebra to Find Values

If $FK = 3x - 1$, $KG = 4y + 3$, $JK = 6y - 2$, and $KH = 2x + 3$, find x and y so that the quadrilateral is a parallelogram.



By Theorem 6.11, if the diagonals of a quadrilateral bisect each other, then it is a parallelogram. So find x such that $\overline{FK} \cong \overline{KH}$ and y such that $\overline{JK} \cong \overline{KG}$.

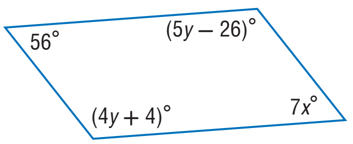
$FK = KH$	Definition of $\cong$
$3x - 1 = 2x + 3$	Substitution
$x - 1 = 3$	Subtract $2x$ from each side.
$x = 4$	Add 1 to each side.
$JK = KG$	Definition of $\cong$
$6y - 2 = 4y + 3$	Substitution
$2y - 2 = 3$	Subtract $4y$ from each side.
$2y = 5$	Add 2 to each side.
$y = 2.5$	Divide each side by 2.

So, when x is 4 and y is 2.5, quadrilateral $FGHJ$ is a parallelogram.

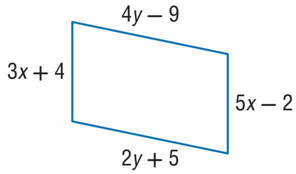
GuidedPractice

Find x and y so that each quadrilateral is a parallelogram.

3A.



3B.



You have learned the conditions of parallelograms. The following list summarizes how to use the conditions to prove a quadrilateral is a parallelogram.

Concept Summary

Prove that a Quadrilateral Is a Parallelogram

- Show that both pairs of opposite sides are parallel. (Definition)
- Show that both pairs of opposite sides are congruent. (Theorem 6.9)
- Show that both pairs of opposite angles are congruent. (Theorem 6.10)
- Show that the diagonals bisect each other. (Theorem 6.11)
- Show that a pair of opposite sides is both parallel and congruent. (Theorem 6.12)

StudyTip

Midpoint Formula To show that a quadrilateral is a parallelogram, you can also use the Midpoint Formula. If the midpoint of each diagonal is the same point, then the diagonals bisect each other.

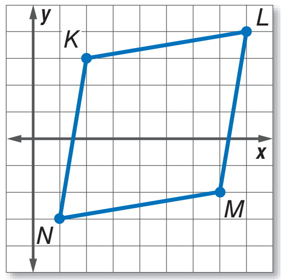
2 Parallelograms on the Coordinate Plane

We can use the Distance, Slope, and Midpoint Formulas to determine whether a quadrilateral in the coordinate plane is a parallelogram.

Example 4 Parallelograms and Coordinate Geometry

COORDINATE GEOMETRY Graph quadrilateral $KLMN$ with vertices $K(2, 3)$, $L(8, 4)$, $M(7, -2)$, and $N(1, -3)$. Determine whether the quadrilateral is a parallelogram. Justify your answer using the Slope Formula.

If the opposite sides of a quadrilateral are parallel, then it is a parallelogram.



slope of $\overline{KL} = \frac{4-3}{8-2}$ or $\frac{1}{6}$

slope of $\overline{NM} = \frac{-2-(-3)}{7-1}$ or $\frac{1}{6}$

slope of $\overline{KN} = \frac{-3-3}{1-2} = \frac{-6}{-1}$ or 6

slope of $\overline{LM} = \frac{-2-4}{7-8} = \frac{-6}{-1}$ or 6

Since opposite sides have the same slope, $\overline{KL} \parallel \overline{NM}$ and $\overline{KN} \parallel \overline{LM}$. Therefore, $KLMN$ is a parallelogram by definition.

GuidedPractice

Determine whether the quadrilateral is a parallelogram. Justify your answer using the given formula.

4A. $A(3, 3)$, $B(8, 2)$, $C(6, -1)$, $D(1, 0)$; Distance Formula

4B. $F(-2, 4)$, $G(4, 2)$, $H(4, -2)$, $J(-2, -1)$; Midpoint Formula

In Chapter 4, you learned that variable coordinates can be assigned to the vertices of triangles. Then the Distance, Slope, and Midpoint Formulas were used to write coordinate proofs of theorems. The same can be done with quadrilaterals.

ReviewVocabulary

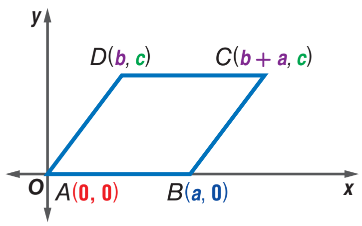
coordinate proof a proof that uses figures in the coordinate plane and algebra to prove geometric concepts

Example 5 Parallelograms and Coordinate Proofs

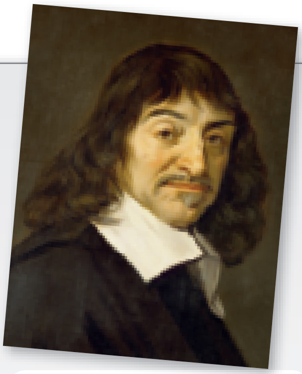
Write a coordinate proof for the following statement.

If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Step 1 Position quadrilateral $ABCD$ on the coordinate plane such that $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$.



- Begin by placing the vertex A at the **origin**.
- Let $\overline{AB}$ have a length of a units. Then B has coordinates $(a, 0)$.
- Since horizontal segments are parallel, position the endpoints of $\overline{DC}$ so that they have the same y-coordinate, c.
- So that the distance from D to C is also a units, let the x-coordinate of D be b and of C be $b + a$.



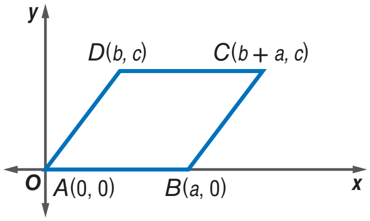
Math History Link

René Descartes
(1596–1650)
René Descartes was a French mathematician who was the first to use a coordinate grid. It has been said that he first thought of locating a point on a plane with a pair of numbers when he was watching a fly on the ceiling, but this is a myth.

Step 2 Use your figure to write a proof.

Given: quadrilateral $ABCD$, $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

Prove: $ABCD$ is a parallelogram.



Coordinate Proof:

By definition, a quadrilateral is a parallelogram if opposite sides are parallel. We are given that $\overline{AB} \parallel \overline{DC}$, so we need only show that $\overline{AD} \parallel \overline{BC}$.

Use the Slope Formula.

slope of $\overline{AD} = \frac{c-0}{b-0} = \frac{c}{b}$ slope of $\overline{BC} = \frac{c-0}{b+a-a} = \frac{c}{b}$

Since $\overline{AD}$ and $\overline{BC}$ have the same slope, $\overline{AD} \parallel \overline{BC}$. So quadrilateral $ABCD$ is a parallelogram because opposite sides are parallel.

Guided Practice

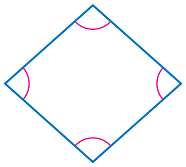
5. Write a coordinate proof of this statement: *If a quadrilateral is a parallelogram, then opposite sides are congruent.*

Check Your Understanding

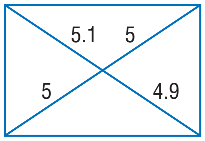
= Step-by-Step Solutions begin on page R14.

Example 1 **Determine whether each quadrilateral is a parallelogram. Justify your answer.**

1.

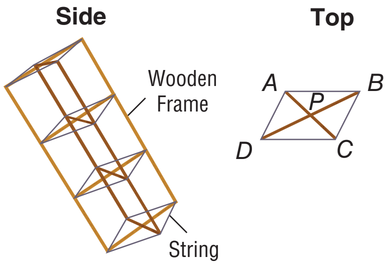


2.



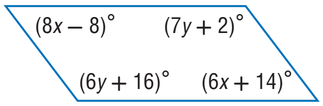
Example 2

3. KITES Charmaine is building the kite shown below. She wants to be sure that the string around her frame forms a parallelogram before she secures the material to it. How can she use the measures of the wooden portion of the frame to prove that the string forms a parallelogram? Explain your reasoning.



Example 3 **ALGEBRA** **Find x and y so that the quadrilateral is a parallelogram.**

4.



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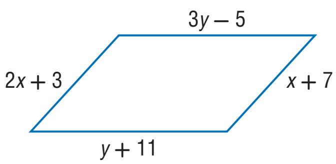


Example 4 **COORDINATE GEOMETRY** **Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.**

6. $A(-2, 4)$, $B(5, 4)$, $C(8, -1)$, $D(-1, -1)$; Slope Formula

7. $W(-5, 4)$, $X(3, 4)$, $Y(1, -3)$, $Z(-7, -3)$; Midpoint Formula

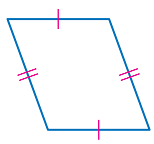
Example 5 **8.** Write a coordinate proof for the statement: *If a quadrilateral is a parallelogram, then its diagonals bisect each other.*

Practice and Problem Solving

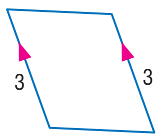
Extra Practice is on page R6.

Example 1 CCSS **ARGUMENTS** **Determine whether each quadrilateral is a parallelogram. Justify your answer.**

9.



10.



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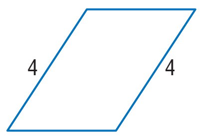

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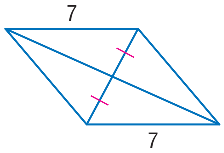

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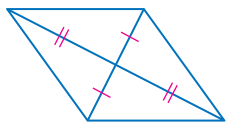

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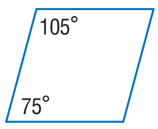
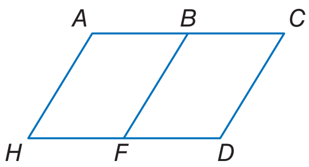
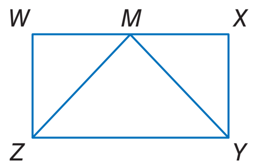


Example 2

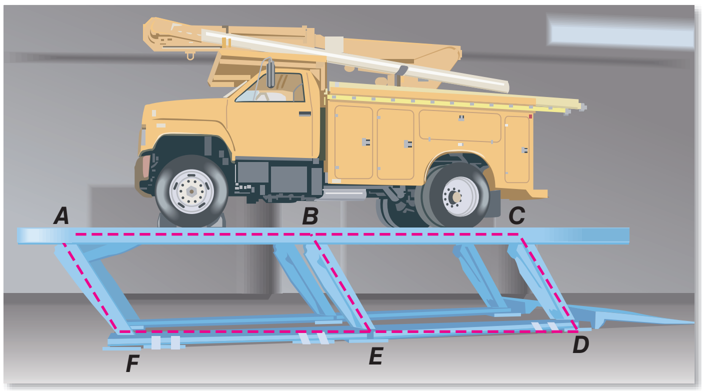
15. **PROOF** If *ACDH* is a parallelogram, *B* is the midpoint of $\overline{AC}$, and *F* is the midpoint of $\overline{HD}$, write a flow proof to prove that *ABFH* is a parallelogram.



16. **PROOF** If WXYZ is a parallelogram, $\angle W \cong \angle X$, and *M* is the midpoint of $\overline{WX}$, write a paragraph proof to prove that *ZMY* is an isosceles triangle.

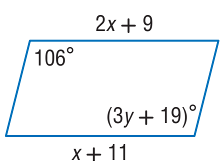


17. **REPAIR** Parallelogram lifts are used to elevate large vehicles for maintenance. In the diagram, *ABEF* and *BCDE* are parallelograms. Write a two-column proof to show that *ACDF* is also a parallelogram.



Example 3 **ALGEBRA** **Find *x* and *y* so that the quadrilateral is a parallelogram.**

18.



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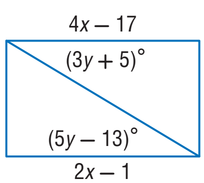

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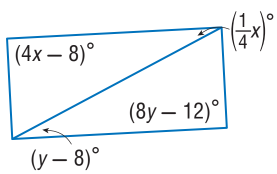
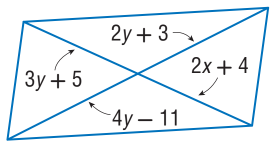
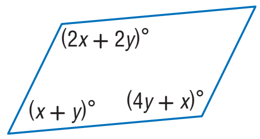


ALGEBRA **Find x and y so that the quadrilateral is a parallelogram.**

21.



22.



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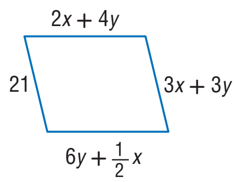


Example 4 **COORDINATE GEOMETRY** **Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.**

24. $A(-3, 4)$, $B(4, 5)$, $C(5, -1)$, $D(-2, -2)$; Slope Formula

25. $J(-4, -4)$, $K(-3, 1)$, $L(4, 3)$, $M(3, -3)$; Distance Formula

26. $V(3, 5)$, $W(1, -2)$, $X(-6, 2)$, $Y(-4, 7)$; Slope Formula

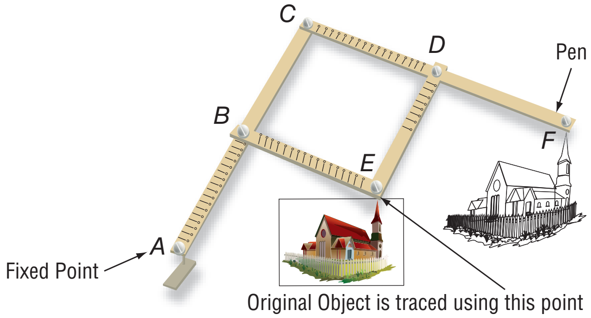
27. $Q(2, -4)$, $R(4, 3)$, $S(-3, 6)$, $T(-5, -1)$; Distance and Slope Formulas

Example 5 **28.** Write a coordinate proof for the statement: *If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.*

29. Write a coordinate proof for the statement: *If a parallelogram has one right angle, it has four right angles.*

30. **PROOF** Write a paragraph proof of Theorem 6.10.

31 **PANTOGRAPH** A pantograph is a device that can be used to copy an object and either enlarge or reduce it based on the dimensions of the pantograph.



a. If $\overline{AC} \cong \overline{CF}$, $\overline{AB} \cong \overline{CD} \cong \overline{BE}$, and $\overline{DF} \cong \overline{DE}$, write a paragraph proof to show that $\overline{BE} \parallel \overline{CD}$.

b. The scale of the copied object is the ratio of CF to BE. If AB is 12 inches, DF is 8 inches, and the width of the original object is 5.5 inches, what is the width of the copy?

PROOF **Write a two-column proof.**

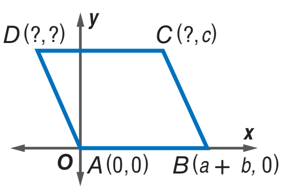
32. Theorem 6.11

33. Theorem 6.12

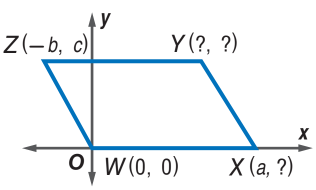
34. **CONSTRUCTION** Explain how you can use Theorem 6.11 to construct a parallelogram. Then construct a parallelogram using your method.

CCSS **REASONING** **Name the missing coordinates for each parallelogram.**

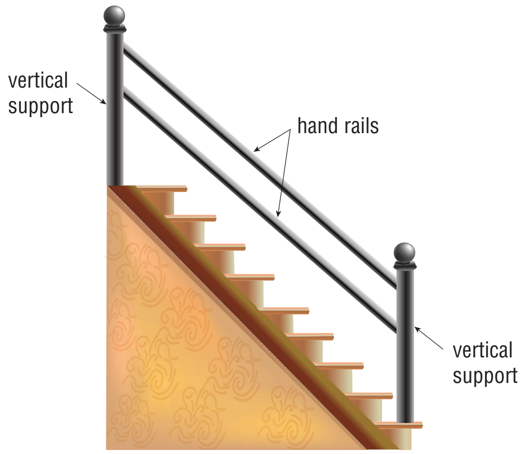
35



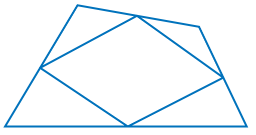
36.



37 **SERVICE** While replacing a hand rail, a contractor uses a carpenter's square to confirm that the vertical supports are perpendicular to the top step and the ground, respectively. How can the contractor prove that the two hand rails are parallel using the fewest measurements? Assume that the top step and the ground are both level.



38. **PROOF** Write a coordinate proof to prove that the segments joining the midpoints of the sides of any quadrilateral form a parallelogram.



39. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the properties of rectangles. A rectangle is a quadrilateral with four right angles.

a. Geometric Draw three rectangles with varying lengths and widths. Label one rectangle *ABCD*, one *MNOP*, and one *WXYZ*. Draw the two diagonals for each rectangle.

b. Tabular Measure the diagonals of each rectangle, and complete the table at the right.

c. Verbal Write a conjecture about the diagonals of a rectangle.

Rectangle	Side	Length
ABCD	$\overline{AC}$	
	$\overline{BD}$	
MNOP	$\overline{MO}$	
	$\overline{NP}$	
WXYZ	$\overline{WY}$	
	$\overline{XZ}$	

H.O.T. Problems Use Higher-Order Thinking Skills

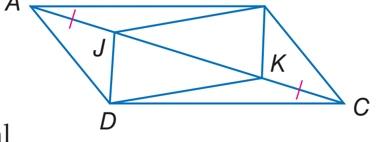
40. **CHALLENGE** The diagonals of a parallelogram meet at the point (0, 1). One vertex of the parallelogram is located at (2, 4), and a second vertex is located at (3, 1). Find the locations of the remaining vertices.

41. **WRITING IN MATH** Compare and contrast Theorem 6.9 and Theorem 6.3.

42. CCSS **ARGUMENTS** If two parallelograms have four congruent corresponding angles, are the parallelograms *sometimes*, *always*, or *never* congruent?

43. **OPEN ENDED** Position and label a parallelogram on the coordinate plane differently than shown in either Example 5, Exercise 35, or Exercise 36.

44. **CHALLENGE** If *ABCD* is a parallelogram and $\overline{AJ} \cong \overline{KC}$, show that quadrilateral *JBKD* is a parallelogram.



45. **WRITING IN MATH** How can you prove that a quadrilateral is a parallelogram?

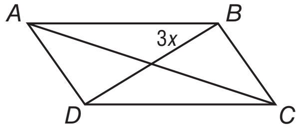
Standardized Test Practice

46. If sides $\overline{AB}$ and $\overline{DC}$ of quadrilateral $ABCD$ are parallel, which additional information would be sufficient to prove that quadrilateral $ABCD$ is a parallelogram?

A $\overline{AB} \cong \overline{AC}$ **C** $\overline{AC} \cong \overline{BD}$

B $\overline{AB} \cong \overline{DC}$ **D** $\overline{AD} \cong \overline{BC}$

47. SHORT RESPONSE Quadrilateral $ABCD$ is shown. AC is 40 and BD is $\frac{3}{5}AC$. $\overline{BD}$ bisects $\overline{AC}$. For what value of x is $ABCD$ a parallelogram?



48. ALGEBRA Jarod's average driving speed for a 5-hour trip was 58 miles per hour. During the first 3 hours, he drove 50 miles per hour. What was his average speed in miles per hour for the last 2 hours of his trip?

F 70 **H** 60

G 66 **J** 54

49. SAT/ACT A parallelogram has vertices at (0, 0), (3, 5), and (0, 5). What are the coordinates of the fourth vertex?

A (0, 3) **D** (0, −3)

B (5, 3) **E** (3, 0)

C (5, 0)

Spiral Review

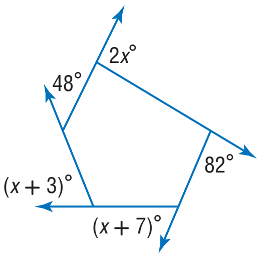
COORDINATE GEOMETRY **Find the coordinates of the intersection of the diagonals of ▱*ABCD* with the given vertices.** (Lesson 6-2)

50. $A(-3, 5)$, $B(6, 5)$, $C(5, -4)$, $D(-4, -4)$

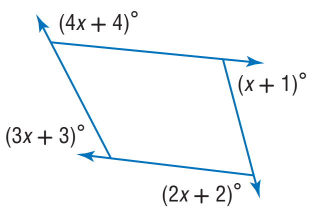
51. $A(2, 5)$, $B(10, 7)$, $C(7, -2)$, $D(-1, -4)$

Find the value of *x*. (Lesson 6-1)

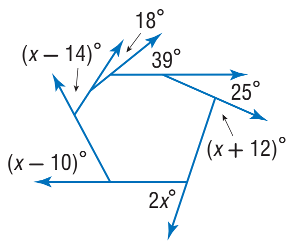
52.



53.



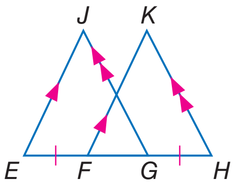
54.



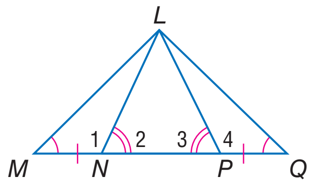
55. FITNESS Toshiro was at the gym for just over two hours. He swam laps in the pool and lifted weights. Prove that he did one of these activities for more than an hour. (Lesson 5-4)

PROOF **Write a flow proof.** (Lesson 4-5)

56. Given: $\overline{EJ} \parallel \overline{FK}$, $\overline{JG} \parallel \overline{KH}$, $\overline{EF} \cong \overline{GH}$
Prove: $\triangle EJG \cong \triangle FKH$



57. Given: $\overline{MN} \cong \overline{PQ}$, $\angle M \cong \angle Q$, $\angle 2 \cong \angle 3$
Prove: $\triangle MLP \cong \triangle QLN$



Skills Review

Use slope to determine whether *XY* and *YZ* are *perpendicular* or *not perpendicular*.

58. $X(-2, 2)$, $Y(0, 1)$, $Z(4, 1)$

59. $X(4, 1)$, $Y(5, 3)$, $Z(6, 2)$

CHAPTER 6 Mid-Chapter Quiz

Lessons 6-1 through 6-3

Find the sum of the measures of the interior angles of each convex polygon. (Lesson 6-1)

1. pentagon

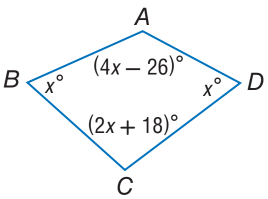
2. heptagon

3. 18-gon

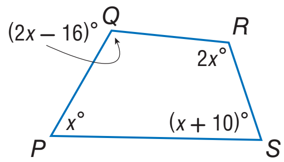
4. 23-gon

Find the measure of each interior angle. (Lesson 6-1)

5.



6.



The sum of the measures of the interior angles of a regular polygon is given. Find the number of sides in the polygon. (Lesson 6-1)

7. 720

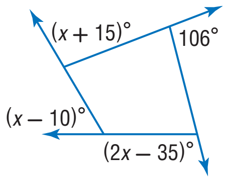
8. 1260

9. 1800

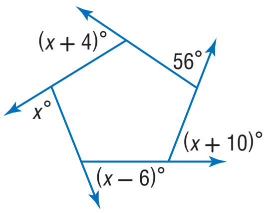
10. 4500

Find the value of x in each diagram. (Lesson 6-1)

11.



12.

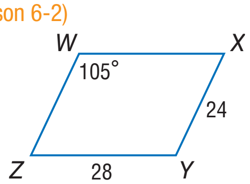


Use ▱$WXYZ$ to find each measure. (Lesson 6-2)

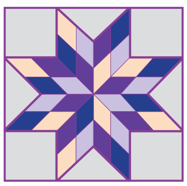
13. $m\angle WZY$

14. WZ

15. $m\angle XYZ$

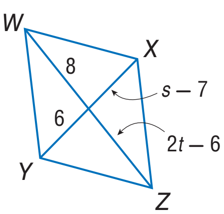


16. DESIGN Describe two ways to ensure that the pieces of the design at the right would fit properly together. (Lesson 6-2)

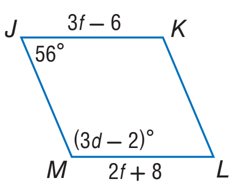


ALGEBRA Find the value of each variable in each parallelogram. (Lesson 6-2)

17.



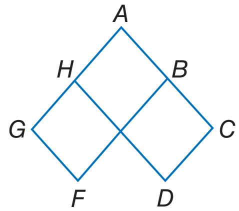
18.



19. PROOF Write a two-column proof. (Lesson 6-2)

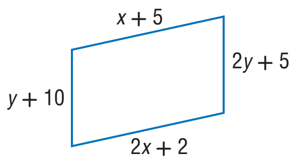
Given: ▱$GFBA$ and ▱$HACD$

Prove: $\angle F \cong \angle D$

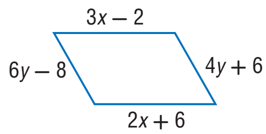


Find x and y so that each quadrilateral is a parallelogram. (Lesson 6-3)

20.



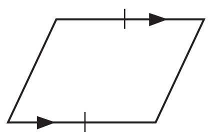
21.



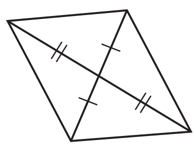
22. MUSIC Why will the keyboard stand with legs joined at the midpoints always remain parallel to the floor? (Lesson 6-3)

23. MULTIPLE CHOICE Which of the following quadrilaterals is not a parallelogram? (Lesson 6-3)

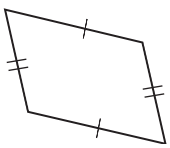
A



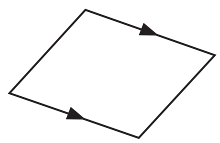
C



B



D



COORDINATE GEOMETRY Determine whether the figure is a parallelogram. Justify your answer with the method indicated. (Lesson 6-3)

24. $A(-6, -5)$, $B(-1, -4)$, $C(0, -1)$, $D(-5, -2)$; Distance Formula

25. $Q(-5, 2)$, $R(-3, -6)$, $S(2, 2)$, $T(-1, 6)$; Slope Formula

LESSON 6-4 Rectangles

Then

- You used properties of parallelograms and determined whether quadrilaterals were parallelograms.

Now

1. Recognize and apply properties of rectangles.
2. Determine whether parallelograms are rectangles.

Why?

- Leonardo is in charge of set design for a school play. He needs to use paint to create the appearance of a doorway on a lightweight solid wall. The doorway is to be a rectangle 36 inches wide and 80 inches tall. How can Leonardo be sure that he paints a rectangle?

NewVocabulary
rectangle

Common Core State Standards

Content Standards

G.CO.11 Prove theorems about parallelograms.

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

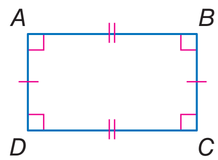
5 Use appropriate tools strategically.

1 Properties of Rectangles

A **rectangle** is a parallelogram with four right angles. By definition, a rectangle has the following properties.

- All four angles are right angles.
- Opposite sides are parallel and congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

In addition, the diagonals of a rectangle are congruent.



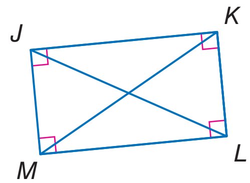
Rectangle *ABCD*

Theorem 6.13 Diagonals of a Rectangle

If a parallelogram is a rectangle, then its diagonals are congruent.

Abbreviation *If a ▱ is a rectangle, diag. are ≅.*

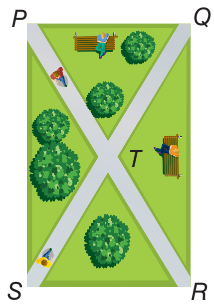
Example If ▱*JKLM* is a rectangle, then $\overline{JL} \cong \overline{MK}$.



You will prove Theorem 6.13 in Exercise 33.

Real-World Example 1 Use Properties of Rectangles

EXERCISE **A rectangular park has two walking paths as shown. If $PS = 180$ meters and $PR = 200$ meters, find QT.**



$\overline{QS} \cong \overline{PR}$ — If a ▱ is a rectangle, diag. are ≅.

$QS = PR$ — Definition of congruence

$QS = 200$ — Substitution

Since $PQRS$ is a rectangle, it is a parallelogram. The diagonals of a parallelogram bisect each other, so $QT = ST$.

$QT + ST = QS$ — Segment Addition

$QT + QT = QS$ — Substitution

$2QT = QS$ — Simplify.

$QT = \frac{1}{2}QS$ — Divide each side by 2.

$QT = \frac{1}{2}(200)$ or 100 — Substitution

GuidedPractice **Refer to the figure in Example 1.**

1A. If $TS = 120$ meters, find PR.

1B. If $m\angle PRS = 64$, find $m\angle SQR$.

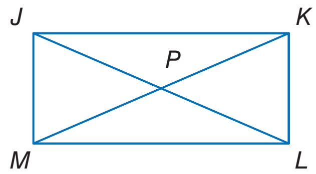
Siri Stafford/Stone/Getty Images

You can use the properties of rectangles along with algebra to find missing values.

PT

Example 2 Use Properties of Rectangles and Algebra

ALGEBRA Quadrilateral $JKLM$ is a rectangle. If $m\angle KJL = 2x + 4$ and $m\angle JLK = 7x + 5$, find x.



Since $JKLM$ is a rectangle, it has four right angles. So, $m\angle MLK = 90$. Since a rectangle is a parallelogram, opposite sides are parallel. Alternate interior angles of parallel lines are congruent, so $\angle JLM \cong \angle KJL$ and $m\angle JLM = m\angle KJL$.

$m\angle JLM + m\angle JLK = 90$	Angle Addition
$m\angle KJL + m\angle JLK = 90$	Substitution
$2x + 4 + 7x + 5 = 90$	Substitution
$9x + 9 = 90$	Add like terms.
$9x = 81$	Subtract 9 from each side.
$x = 9$	Divide each side by 9.

GuidedPractice

2. Refer to the figure in Example 2. If $JP = 3y - 5$ and $MK = 5y + 1$, find y.

> **StudyTip**
>
> **Right Angles** Recall from Theorem 6-6 that if a parallelogram has one right angle, then it has four right angles.

2 Prove that Parallelograms are Rectangles

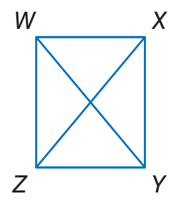
The converse of Theorem 6.13 is also true.

Theorem 6.14 Diagonals of a Rectangle

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Abbreviation *If diag. of a ▱ are ≅, then ▱ is a rectangle.*

Example If $\overline{WY} \cong \overline{XZ}$ in ▱$WXYZ$, then ▱$WXYZ$ is a rectangle.

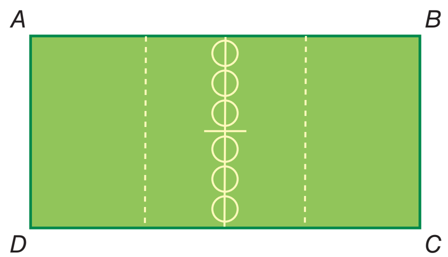


You will prove Theorem 6.14 in Exercise 34.

PT

Real-World Example 3 Providing Rectangle Relationships

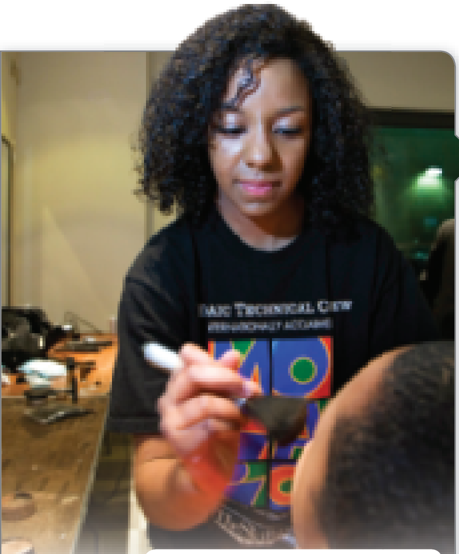
DODGEBALL A community recreation center has created an outdoor dodgeball playing field. To be sure that it meets the ideal playing field requirements, they measure the sides of the field and its diagonals. If $AB = 60$ feet, $BC = 30$ feet, $CD = 60$ feet, $AD = 30$ feet, $AC = 67$ feet, and $BD = 67$ feet, explain how the recreation center can be sure that the playing field is rectangular.



Since $AB = CD$, $BC = AD$, and $AC = BD$, $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$, and $\overline{AC} \cong \overline{BD}$. Because $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, $ABCD$ is a parallelogram. Since $\overline{AC}$ and $\overline{BD}$ are congruent diagonals in ▱$ABCD$, ▱$ABCD$ is a rectangle.

> **Real-WorldLink**
>
> The game of dodgeball is played on a rectangular playing field ideally 60 feet long and 30 feet wide. The field is divided into two equal sections by a center-line and attack-lines that are 3 meters (9.8 feet) from, and parallel to, the centerline.
>
> **Source:** National Amateur Dodgeball Assoc.

Micah Walter/Getty Images

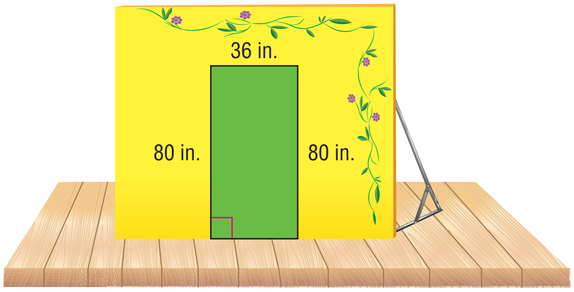


Real-WorldLink

The Mosaic Youth Theater in Detroit, Michigan, is a professional performing arts training program for young people ages 12 to 18. Students are involved in all aspects of performances, including set and lighting design, set construction, stage management, sound, and costumes.

GuidedPractice

3. SET DESIGN Refer to the beginning of the lesson. Leonardo measures the sides of his figure and confirms that they have the desired measures as shown. Using a carpenter's square, he also confirms that the measure of the bottom left corner of the figure is a right angle. Can he conclude that the figure is a rectangle? Explain.

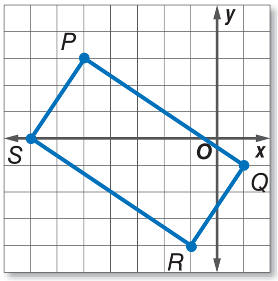


You can also use the properties of rectangles to prove that a quadrilateral positioned on a coordinate plane is a rectangle given the coordinates of the vertices.

Example 4 Rectangles and Coordinate Geometry

COORDINATE GEOMETRY Quadrilateral $PQRS$ has vertices $P(-5, 3)$, $Q(1, -1)$, $R(-1, -4)$, and $S(-7, 0)$. Determine whether $PQRS$ is a rectangle by using the Distance Formula.

Step 1 Use the Distance Formula to determine whether $PQRS$ is a parallelogram by determining if opposite sides are congruent.



$$PQ = \sqrt{(-5 - 1)^2 + [3 - (-1)]^2} \text{ or } \sqrt{52}$$

$$RS = \sqrt{[-1 - (-7)]^2 + (-4 - 0)^2} \text{ or } \sqrt{52}$$

$$PS = \sqrt{[-5 - (-7)]^2 + (3 - 0)^2} \text{ or } \sqrt{13}$$

$$QR = \sqrt{[1 - (-1)]^2 + [-1 - (-4)]^2} \text{ or } \sqrt{13}$$

Since opposite sides of the quadrilateral have the same measure, they are congruent. So, quadrilateral $PQRS$ is a parallelogram.

Step 2 Determine whether the diagonals of ▱$PQRS$ are congruent.

$$PR = \sqrt{[-5 - (-1)]^2 + [3 - (-4)]^2} \text{ or } \sqrt{65}$$

$$QS = \sqrt{[1 - (-7)]^2 + (-1 - 0)^2} \text{ or } \sqrt{65}$$

Since the diagonals have the same measure, they are congruent. So, ▱$PQRS$ is a rectangle.

StudyTip

Rectangles and Parallelograms A rectangle is a parallelogram, but a parallelogram is not necessarily a rectangle.

GuidedPractice

4. Quadrilateral $JKLM$ has vertices $J(-10, 2)$, $K(-8, -6)$, $L(5, -3)$, and $M(2, 5)$. Determine whether $JKLM$ is a rectangle using the Slope Formula.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

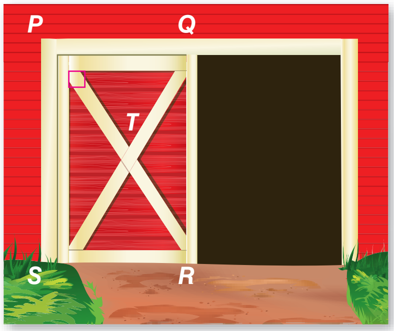
Example 1 **FARMING** **An X-brace on a rectangular barn door is both decorative and functional. It helps to prevent the door from warping over time. If $ST = 3\frac{13}{16}$ feet, $PS = 7$ feet, and $m\angle PTQ = 67$, find each measure.**

1. QR

2. SQ

3. $m\angle TQR$

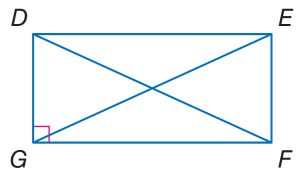
4. $m\angle TSR$



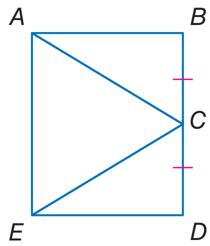
Example 2 **ALGEBRA** **Quadrilateral $DEFG$ is a rectangle.**

5. If $FD = 3x - 7$ and $EG = x + 5$, find EG.

6. If $m\angle EFD = 2x - 3$ and $m\angle DFG = x + 12$, find $m\angle EFD$.



Example 3 **7. PROOF** If $ABDE$ is a rectangle and $\overline{BC} \cong \overline{DC}$, prove that $\overline{AC} \cong \overline{EC}$.



Example 4 **COORDINATE GEOMETRY** **Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.**

8. $W(-4, 3)$, $X(1, 5)$, $Y(3, 1)$, $Z(-2, -2)$; Slope Formula

9. $A(4, 3)$, $B(4, -2)$, $C(-4, -2)$, $D(-4, 3)$; Distance Formula

Practice and Problem Solving

Extra Practice is on page R6.

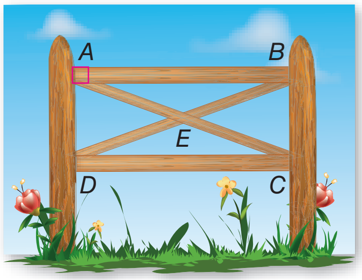
Example 1 **FENCING** **X-braces are also used to provide support in rectangular fencing. If $AB = 6$ feet, $AD = 2$ feet, and $m\angle DAE = 65$, find each measure.**

10. BC

11 DB

12. $m\angle CEB$

13. $m\angle EDC$



Example 2 CCSS **REGULARITY** **Quadrilateral $WXYZ$ is a rectangle.**

14. If $ZY = 2x + 3$ and $WX = x + 4$, find WX.

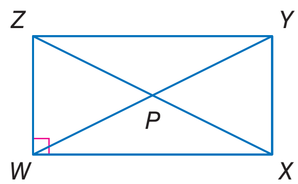
15. If $PY = 3x - 5$ and $WP = 2x + 11$, find ZP.

16. If $m\angle ZYW = 2x - 7$ and $m\angle WYX = 2x + 5$, find $m\angle ZYW$.

17. If $ZP = 4x - 9$ and $PY = 2x + 5$, find ZX.

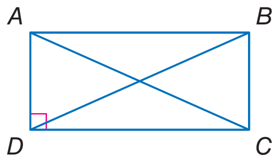
18. If $m\angle XZY = 3x + 6$ and $m\angle XZW = 5x - 12$, find $m\angle YXZ$.

19. If $m\angle ZXW = x - 11$ and $m\angle WZX = x - 9$, find $m\angle ZXY$.

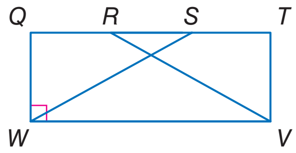


Example 3 **PROOF** **Write a two-column proof.**

20. **Given:** $ABCD$ is a rectangle.
Prove: $\triangle ADC \cong \triangle BCD$



21. **Given:** $QTVW$ is a rectangle.
$\overline{QR} \cong \overline{ST}$
Prove: $\triangle SWQ \cong \triangle RVT$



Example 4 **COORDINATE GEOMETRY** **Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.**

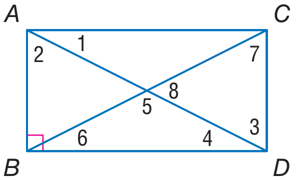
22. $W(-2, 4)$, $X(5, 5)$, $Y(6, -2)$, $Z(-1, -3)$; Slope Formula

23. $J(3, 3)$, $K(-5, 2)$, $L(-4, -4)$, $M(4, -3)$; Distance Formula

24. $Q(-2, 2)$, $R(0, -2)$, $S(6, 1)$, $T(4, 5)$; Distance Formula

25. $G(1, 8)$, $H(-7, 7)$, $J(-6, 1)$, $K(2, 2)$; Slope Formula

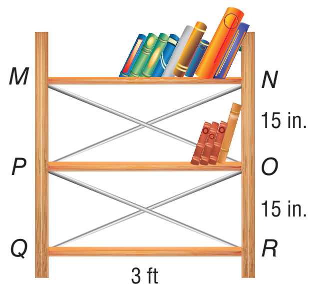
Quadrilateral $ABCD$ is a rectangle. Find each measure if $m\angle 2 = 40$.



26. $m\angle 1$ **27.** $m\angle 7$ **28.** $m\angle 3$

29 $m\angle 5$ **30.** $m\angle 6$ **31.** $m\angle 8$

32. CCSS **MODELING** Jody is building a new bookshelf using wood and metal supports like the one shown. To what length should she cut the metal supports in order for the bookshelf to be *square*, which means that the angles formed by the shelves and the vertical supports are all right angles? Explain your reasoning.



PROOF **Write a two-column proof.**

33. Theorem 6.13 **34.** Theorem 6.14

PROOF **Write a paragraph proof of each statement.**

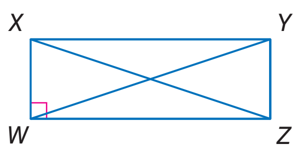
35. If a parallelogram has one right angle, then it is a rectangle.

36. If a quadrilateral has four right angles, then it is a rectangle.

37. **CONSTRUCTION** Construct a rectangle using the construction for congruent segments and the construction for a line perpendicular to another line through a point on the line. Justify each step of the construction.

38. **SPORTS** The end zone of a football field is 160 feet wide and 30 feet long. Kyle is responsible for painting the field. He has finished the end zone. Explain how Kyle can confirm that the end zone is the regulation size and be sure that it is also a rectangle using only a tape measure.

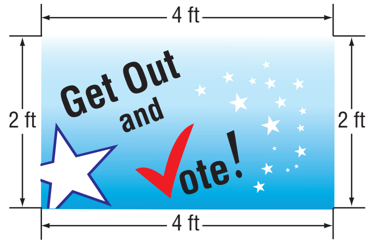
ALGEBRA **Quadrilateral $WXYZ$ is a rectangle.**



39. If $XW = 3$, $WZ = 4$, and $XZ = b$, find YW.

40. If $XZ = 2c$ and $ZY = 6$, and $XY = 8$, find WY.

41. **SIGNS** The sign below is in the foyer of Nyoko's school. Based on the dimensions given, can Nyoko be sure that the sign is a rectangle? Explain your reasoning.



PROOF **Write a coordinate proof of each statement.**

42. The diagonals of a rectangle are congruent.

43 If the diagonals of a parallelogram are congruent, then it is a rectangle.

44. **MULTIPLE REPRESENTATIONS** In the problem, you will explore properties of other special parallelograms.

a. **Geometric** Draw three parallelograms, each with all four sides congruent. Label one parallelogram *ABCD*, one *MNOP*, and one *WXYZ*. Draw the two diagonals of each parallelogram and label the intersections *R*.

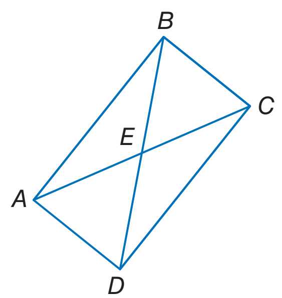
b. **Tabular** Use a protractor to measure the appropriate angles and complete the table below.

Parallelogram	*ABCD*		*MNOP*		*WXYZ*	
Angle	$\angle ARB$	$\angle BRC$	$\angle MRN$	$\angle NRO$	$\angle WRX$	$\angle XRY$
Angle Measure						

c. **Verbal** Make a conjecture about the diagonals of a parallelogram with four congruent sides.

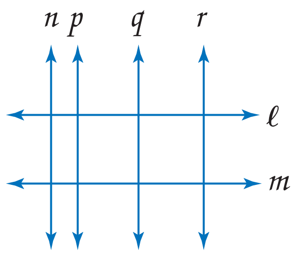
H.O.T. Problems Use Higher-Order Thinking Skills

45. **CHALLENGE** In rectangle *ABCD*, $m\angle EAB = 4x + 6$, $m\angle DEC = 10 - 11y$, and $m\angle EBC = 60$. Find the values of x and y.



46. **CCSS** **CRITIQUE** Parker says that any two congruent acute triangles can be arranged to make a rectangle. Tamika says that only two congruent right triangles can be arranged to make a rectangle. Is either of them correct? Explain your reasoning.

47. **REASONING** In the diagram at the right, lines $n, p, q,$ and r are parallel and lines ℓ and m are parallel. How many rectangles are formed by the intersecting lines?

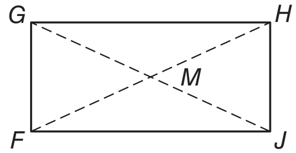


48. **OPEN ENDED** Write the equations of four lines having intersections that form the vertices of a rectangle. Verify your answer using coordinate geometry.

49. **WRITING IN MATH** Why are all rectangles parallelograms, but all parallelograms are not rectangles? Explain.

Standardized Test Practice

50. If $FJ = -3x + 5y$, $FM = 3x + y$, $GH = 11$, and $GM = 13$, what values of x and y make parallelogram $FGHJ$ a rectangle?

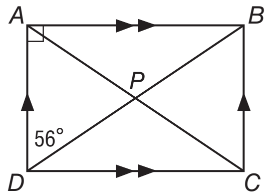


A $x = 3, y = 4$ **C** $x = 7, y = 8$
B $x = 4, y = 3$ **D** $x = 8, y = 7$

51. ALGEBRA A rectangular playground is surrounded by an 80-foot fence. One side of the playground is 10 feet longer than the other. Which of the following equations could be used to find r, the shorter side of the playground?

F $10r + r = 80$ **H** $r(r + 10) = 80$
G $4r + 10 = 80$ **J** $2(r + 10) + 2r = 80$

52. SHORT RESPONSE What is the measure of $\angle APB$?



53. SAT/ACT If p is odd, which of the following must also be odd?

A $2p$

B $2p + 2$

C $\frac{p}{2}$

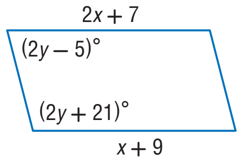
D $2p - 2$

E $p + 2$

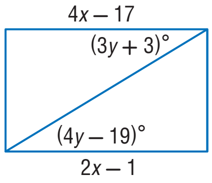
Spiral Review

ALGEBRA **Find x and y so that the quadrilateral is a parallelogram.** (Lesson 6-3)

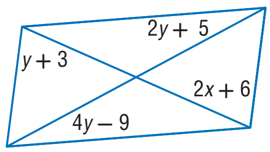
54.



55.

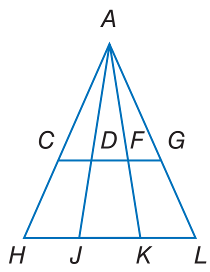


56.



57. COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of ▱$ABCD$ with vertices $A(1, 3)$, $B(6, 2)$, $C(4, -2)$, and $D(-1, -1)$. (Lesson 6-2)

Refer to the figure at the right. (Lesson 4-6)



58. If $\overline{AC} \cong \overline{AF}$, name two congruent angles.

59. If $\angle AHJ \cong \angle AJH$, name two congruent segments.

60. If $\angle AJL \cong \angle ALJ$, name two congruent segments.

61. If $\overline{JA} \cong \overline{KA}$, name two congruent angles.

Skills Review

Find the distance between each pair of points.

62. $(4, 2), (2, -5)$ **63.** $(0, 6), (-1, -4)$ **64.** $(-4, 3), (3, -4)$

LESSON 6-5 Rhombi and Squares

Then

- You determined whether quadrilaterals were parallelograms and/or rectangles.

Now

1. Recognize and apply the properties of rhombi and squares.
2. Determine whether quadrilaterals are rectangles, rhombi, or squares.

Why?

- Some fruits, nuts, and vegetables are packaged using bags made out of rhombus-shaped tubular netting. Similar shaped nylon netting is used for goals in such sports as soccer, hockey, and football. A rhombus and a square are both types of equilateral parallelograms.

NewVocabulary
rhombus
square

Common Core State Standards

Content Standards

G.CO.11 Prove theorems about parallelograms.

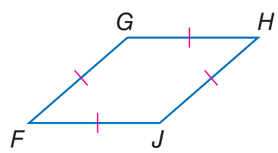
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.
2 Reason abstractly and quantitatively.

1 Properties of Rhombi and Squares

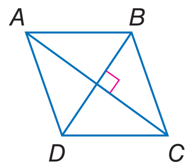
A **rhombus** is a parallelogram with all four sides congruent. A rhombus has all the properties of a parallelogram and the two additional characteristics described in the theorems below.



Theorems Diagonals of a Rhombus

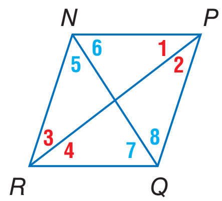
6.15 If a parallelogram is a rhombus, then its diagonals are perpendicular.

Example If $\square ABCD$ is a rhombus, then $\overline{AC} \perp \overline{BD}$.



6.16 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

Example If $\square NPQR$ is a rhombus, then $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, and $\angle 7 \cong \angle 8$.

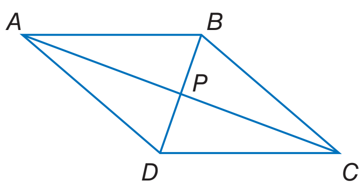


You will prove Theorem 6.16 in Exercise 34.

Proof Theorem 6.15

Given: *ABCD* is a rhombus.

Prove: $\overline{AC} \perp \overline{BD}$



Paragraph Proof:

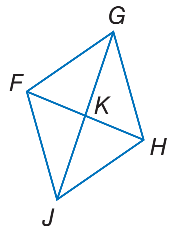
Since *ABCD* is a rhombus, by definition $\overline{AB} \cong \overline{BC}$. A rhombus is a parallelogram and the diagonals of a parallelogram bisect each other, so $\overline{BD}$ bisects $\overline{AC}$ at *P*. Thus, $\overline{AP} \cong \overline{PC}$. $\overline{BP} \cong \overline{BP}$ by the Reflexive Property. So, $\triangle APB \cong \triangle CPB$ by SSS. $\angle APB \cong \angle CPB$ by CPCTC. $\angle APB$ and $\angle CPB$ also form a linear pair. Two congruent angles that form a linear pair are right angles. $\angle APB$ is a right angle, so $\overline{AC} \perp \overline{BD}$ by the definition of perpendicular lines.

Reading Math

Rhombi The plural form of rhombus is *rhombi*, pronounced ROM-bye.

Example 1 Use Properties of a Rhombus

The diagonals of rhombus $FGHJ$ intersect at K. Use the given information to find each measure or value.



a. If $m\angle FJH = 82$, find $m\angle KHJ$.

Since $FGHJ$ is a rhombus, diagonal $\overline{JG}$ bisects $\angle FJH$. Therefore, $m\angle KJH = \frac{1}{2}m\angle FJH$. So $m\angle KJH = \frac{1}{2}(82)$ or 41. Since the diagonals of a rhombus are perpendicular, $m\angle JKH = 90$ by the definition of perpendicular lines.

$m\angle KJH + m\angle JKH + m\angle KHJ = 180$	Triangle Sum Theorem
$41 + 90 + m\angle KHJ = 180$	Substitution
$131 + m\angle KHJ = 180$	Simplify.
$m\angle KHJ = 49$	Subtract 131 from each side.

b. ALGEBRA If $GH = x + 9$ and $JH = 5x - 2$, find x.

$\overline{GH} \cong \overline{JH}$	By definition, all sides of a rhombus are congruent.
$GH = JH$	Definition of congruence
$x + 9 = 5x - 2$	Substitution
$9 = 4x - 2$	Subtract x from each side.
$11 = 4x$	Add 2 to each side.
$2.75 = x$	Divide each side by 4.

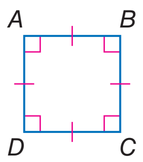
Guided Practice

Refer to rhombus $FGHJ$ above.

1A. If $FK = 5$ and $FG = 13$, find KJ.

1B. **ALGEBRA** If $m\angle JFK = 6y + 7$ and $m\angle KFG = 9y - 5$, find y.

A **square** is a parallelogram with four congruent sides and four right angles. Recall that a parallelogram with four right angles is a rectangle, and a parallelogram with four congruent sides is a rhombus. Therefore, a parallelogram that is both a rectangle and a rhombus is also a square.

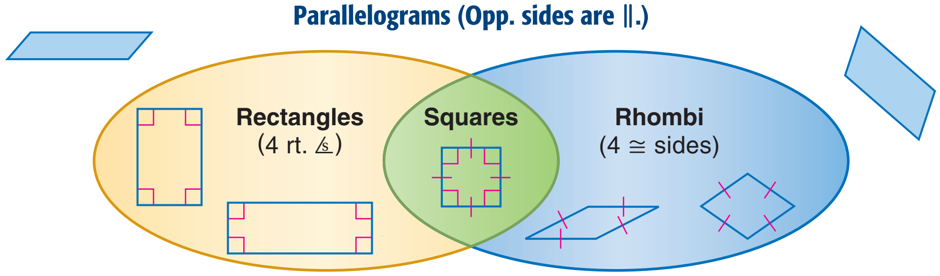


Square *ABCD*

The Venn diagram summarizes the relationships among parallelograms, rhombi, rectangles, and squares.

Concept Summary Parallelograms

Parallelograms (Opp. sides are ∥.)



All of the properties of parallelograms, rectangles, and rhombi apply to squares. For example, the diagonals of a square bisect each other (parallelogram), are congruent (rectangle), and are perpendicular (rhombus).

2 Prove that Quadrilaterals are Rhombi or Squares

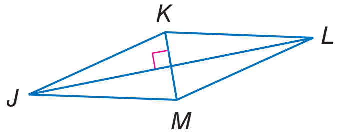
The theorems below provide conditions for rhombi and squares.

> **StudyTip**
> **Common Misconception**
> Theorems 6.17, 6.18, and 6.19 apply only if you already know that a quadrilateral is a parallelogram.

Theorems Conditions for Rhombi and Squares

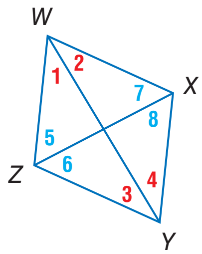
6.17 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem. 6.15)

Example If $\overline{JL} \perp \overline{KM}$, then $\square JKLM$ is a rhombus.



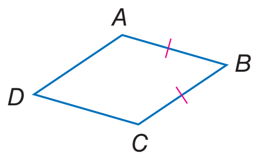
6.18 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (Converse of Theorem. 6.16)

Example If $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, or $\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$, then $\square WXYZ$ is a rhombus.



6.19 If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

Example If $\overline{AB} \cong \overline{BC}$, then $\square ABCD$ is a rhombus.



6.20 If a quadrilateral is both a rectangle and a rhombus, then it is a square.

You will prove Theorems 6.17–6.20 in Exercises 35–38, respectively.

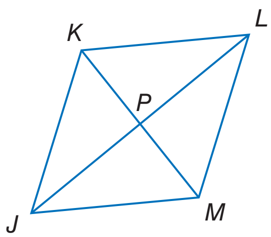
You can use the properties of rhombi and squares to write proofs.

Example 2 Proofs Using Properties of Rhombi and Squares

Write a paragraph proof.

Given: $JKLM$ is a parallelogram.
$\triangle JKL$ is isosceles.

Prove: $JKLM$ is a rhombus.



Paragraph Proof:

Since it is given that $\triangle JKL$ is isosceles, $\overline{KL} \cong \overline{JK}$ by definition. These are consecutive sides of the given parallelogram $JKLM$. So, by Theorem 6.19, $JKLM$ is a rhombus.

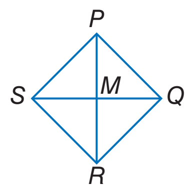
> **StudyTip**
> **Congruent Triangles**
> Since a rhombus has four congruent sides, one diagonal separates the rhombus into two congruent isosceles triangles. Drawing two diagonals separates the rhombus into four congruent right triangles.

GuidedPractice

2. Write a paragraph proof.

Given: $\overline{SQ}$ is the perpendicular bisector of $\overline{PR}$.
$\overline{PR}$ is the perpendicular bisector of $\overline{SQ}$.
$\triangle RMS$ is isosceles.

Prove: $PQRS$ is a square.



Real-WorldLink

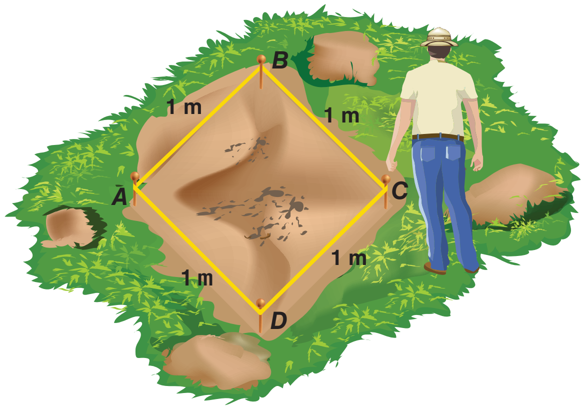
Archaeology is the study of artifacts that provide information about human life and activities in the past. Since humans only began writing about 5000 years ago, information from periods before that time must be gathered from the objects that archeologists locate.

Source: Encyclopeadia Britannica

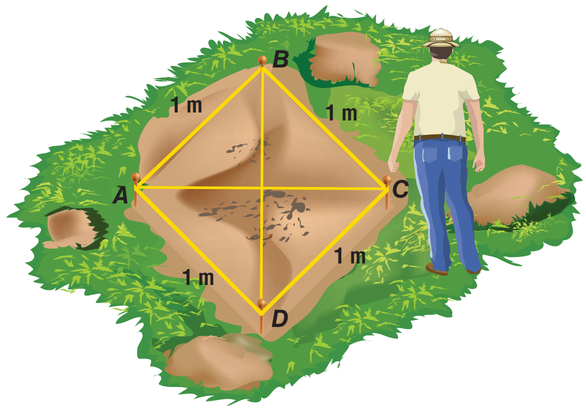
PT

Real-World Example 3 Use Conditions for Rhombi and Squares

ARCHAEOLOGY The key to the successful excavation of an archaeological site is accurate mapping. How can archaeologists be sure that the region they have marked off is a 1-meter by 1-meter square?



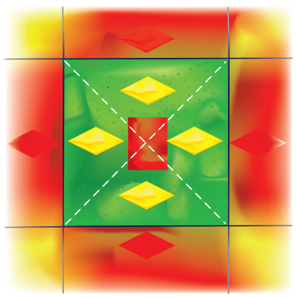
Each side of quadrilateral *ABCD* measures 1 meter. Since opposite sides are congruent, *ABCD* is a parallelogram. Since consecutive sides of ▱*ABCD* are congruent, it is a rhombus. If the archaeologists can show that ▱*ABCD* is also a rectangle, then by Theorem 6.20, ▱*ABCD* is a square.



If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. So if the archeologists measure the length of string needed to form each diagonal and find that these lengths are equal, then *ABCD* is a square.

GuidedPractice

3. QUILTING Kathy is designing a quilt with blocks like the one shown.



A. If she marks the diagonals of each yellow piece and determines that each pair of diagonals is perpendicular, can she conclude that each yellow piece is a rhombus? Explain.

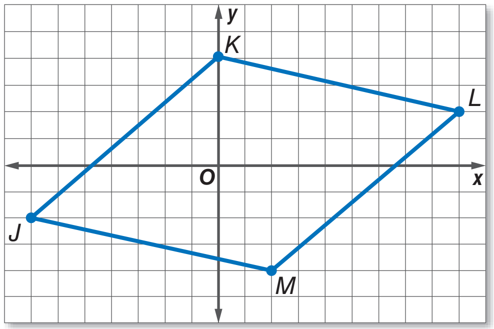
B. If all four angles of the green piece have the same measure and the bottom and left sides have the same measure, can she conclude that the green piece is a square? Explain.

In Chapter 4, you used coordinate geometry to classify triangles. Coordinate geometry can also be used to classify quadrilaterals.

Example 4 Classify Quadrilaterals Using Coordinate Geometry

COORDINATE GEOMETRY **Determine whether ▱*JKLM* with vertices *J*(−7, −2), *K*(0, 4), *L*(9, 2), and *M*(2, −4) is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**

Understand Plot and connect the vertices on a coordinate plane.



It appears from the graph that the parallelogram has four congruent sides, but no right angles. So, it appears that the figure is a rhombus, but not a square or a rectangle.

Plan If the diagonals of the parallelogram are congruent, then it is a rectangle. If they are perpendicular, then it is a rhombus. If they are both congruent and perpendicular, the parallelogram is a rectangle, a rhombus, and a square.

Solve **Step 1** Use the Distance Formula to compare the diagonal lengths.

$KM = \sqrt{(2-0)^2 + (-4-4)^2} = \sqrt{68}$ or $2\sqrt{17}$

$JL = \sqrt{[9-(-7)]^2 + [2-(-2)]^2} = \sqrt{272}$ or $4\sqrt{17}$

Since $2\sqrt{17} \neq 4\sqrt{17}$, the diagonals are not congruent. So, ▱*JKLM* is *not* a rectangle. Since the figure is not a rectangle, it also *cannot* be a square.

Step 2 Use the Slope Formula to determine whether the diagonals are perpendicular.

slope of $\overline{KM} = \frac{-4-4}{2-0} = \frac{-8}{2}$ or -4

slope of $\overline{JL} = \frac{2-(-2)}{9-(-7)} = \frac{4}{16}$ or $\frac{1}{4}$

Since the product of the slopes of the diagonals is −1, the diagonals are perpendicular, so ▱*JKLM* is a rhombus.

Check $JK = \sqrt{[4-(-2)]^2 + [0-(-7)]^2}$ or $\sqrt{85}$

$KL = \sqrt{(9-0)^2 + (2-4)^2}$ or $\sqrt{85}$

So, ▱*JKLM* is a rhombus by Theorem 6.20.

Since the slope of $\overline{JK} = \frac{4-(-2)}{0-(-7)}$ or $\frac{6}{7}$, the slope of $\overline{KL} = \frac{2-4}{9-0}$ or $-\frac{2}{9}$, and the product of these slopes is not −1, consecutive sides $\overline{JK}$ and $\overline{KL}$ are not perpendicular. Therefore, ∠*JKL* is not a right angle. So ▱*JKLM* is not a rectangle or a square. ✔

Guided Practice

4. Given *J*(5, 0), *K*(8, −11), *L*(−3, −14), *M*(−6, −3), determine whether parallelogram *JKLM* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

Problem-Solving Tip

Make a Graph When analyzing a figure using coordinate geometry, graph the figure to help formulate a conjecture and also to help check the reasonableness of the answer you obtain algebraically.

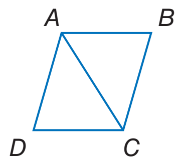
Study Tip

Square and Rhombus A square is a rhombus, but a rhombus is not necessarily a square.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

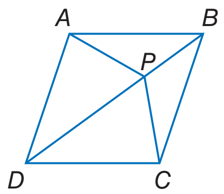
Example 1 **ALGEBRA** **Quadrilateral *ABCD* is a rhombus. Find each value or measure.**



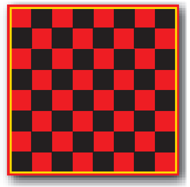
1. If $m\angle BCD = 64$, find $m\angle BAC$.
2. If $AB = 2x + 3$ and $BC = x + 7$, find CD.

Examples 2–3

3. **PROOF** Write a two-column proof to prove that if $ABCD$ is a rhombus with diagonal $\overline{DB}$, then $\overline{AP} \cong \overline{CP}$.



4. **GAMES** The checkerboard below is made up of 64 congruent black and red squares. Use this information to prove that the board itself is a square.



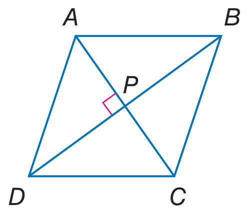
Example 4 **COORDINATE GEOMETRY** **Given each set of vertices, determine whether $\square QRST$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**

5. $Q(1, 2)$, $R(-2, -1)$, $S(1, -4)$, $T(4, -1)$
6. $Q(-2, -1)$, $R(-1, 2)$, $S(4, 1)$, $T(3, -2)$

Practice and Problem Solving

Extra Practice is on page R6.

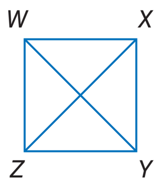
Example 1 **ALGEBRA** **Quadrilateral *ABCD* is a rhombus. Find each value or measure.**



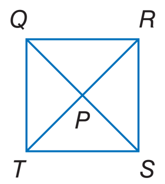
7. If $AB = 14$, find BC.
8. If $m\angle BCD = 54$, find $m\angle BAC$.
9. If $AP = 3x - 1$ and $PC = x + 9$, find AC.
10. If $DB = 2x - 4$ and $PB = 2x - 9$, find PD.
11. If $m\angle ABC = 2x - 7$ and $m\angle BCD = 2x + 3$, find $m\angle DAB$.
12. If $m\angle DPC = 3x - 15$, find x.

Example 2 CCSS **ARGUMENTS** **Write a two-column proof.**

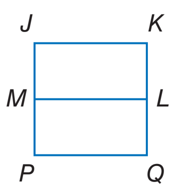
13. **Given:** $\overline{WZ} \parallel \overline{XY}$, $\overline{WX} \parallel \overline{ZY}$, $\overline{WZ} \cong \overline{ZY}$
Prove: $WXYZ$ is a rhombus.



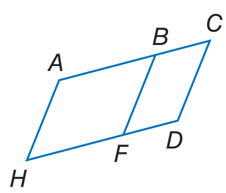
14. **Given:** $QRST$ is a parallelogram. $\overline{TR} \cong \overline{QS}$, $m\angle QPR = 90$
Prove: $QRST$ is a square.



15. **Given:** $JKQP$ is a square. $\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$.
Prove: $JKLM$ is a parallelogram.

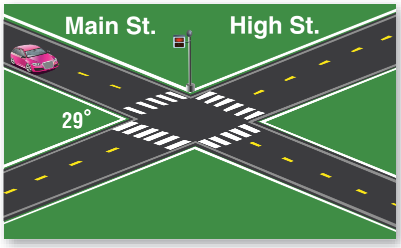


16. **Given:** $ACDH$ and $BCDF$ are parallelograms; $\overline{BF} \cong \overline{AB}$.
Prove: $ABFH$ is a rhombus.

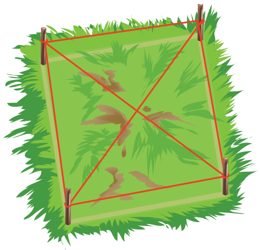


Example 3

17. **ROADWAYS** Main Street and High Street intersect as shown in the diagram. Each of the crosswalks is the same length. Classify the quadrilateral formed by the crosswalks. Explain your reasoning.



18. **CCSS MODELING** A landscaper has staked out the area for a square garden as shown. She has confirmed that each side of the quadrilateral formed by the stakes is congruent and that the diagonals are perpendicular. Is this information enough for the landscaper to be sure that the garden is a square? Explain your reasoning.



Example 4

COORDINATE GEOMETRY **Given each set of vertices, determine whether $\square JKLM$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**

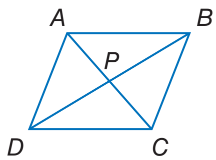
19. $J(-4, -1), K(1, -1), L(4, 3), M(-1, 3)$

20. $J(-3, -2), K(2, -2), L(5, 2), M(0, 2)$

21. $J(-2, -1), K(-4, 3), L(1, 5), M(3, 1)$

22. $J(-1, 1), K(4, 1), L(4, 6), M(-1, 6)$

***ABCD* is a rhombus. If $PB = 12$, $AB = 15$, and $m\angle ABD = 24$, find each measure.**



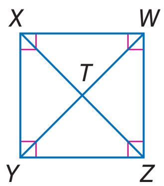
23 AP

24. CP

25. $m\angle BDA$

26. $m\angle ACB$

***WXYZ* is a square. If $WT = 3$, find each measure.**



27. ZX

28. XY

29. $m\angle WTZ$

30. $m\angle WYX$

Classify each quadrilateral.

31.

32.

33.

PROOF **Write a paragraph proof.**

34. Theorem 6.16

35. Theorem 6.17

36. Theorem 6.18

37. Theorem 6.19

38. Theorem 6.20

CONSTRUCTION **Use diagonals to construct each figure. Justify each construction.**

39. rhombus

40. square

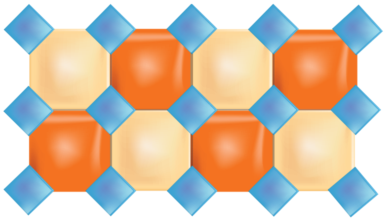
PROOF **Write a coordinate proof of each statement.**

41. The diagonals of a square are perpendicular.

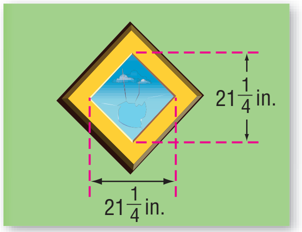
42. The segments joining the midpoints of the sides of a rectangle form a rhombus.

(l)Exactostock/SuperStock, (c)Rick Brady/The McGraw-Hill Companies, (r)Neil Beer/Photodisc/Getty Images

43. **DESIGN** The tile pattern below consists of regular octagons and quadrilaterals. Classify the quadrilaterals in the pattern and explain your reasoning.

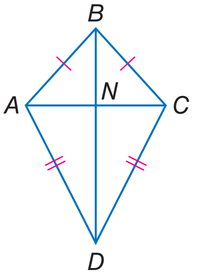


44. **REPAIR** The window pane shown needs to be replaced. What are the dimensions of the replacement pane?



45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the properties of kites, which are quadrilaterals with exactly two distinct pairs of adjacent congruent sides.

a. **Geometric** Draw three kites with varying side lengths. Label one kite *ABCD*, one *PQRS*, and one *WXYZ*. Then draw the diagonals of each kite, labeling the point of intersection *N* for each kite.



Kite *ABCD*

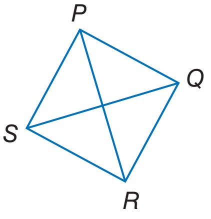
b. **Tabular** Measure the distance from *N* to each vertex. Record your results in a table like the one shown.

Figure	Distance from *N* to Each Vertex Along Shorter Diagonal		Distance from *N* to Each Vertex Along Longer Diagonal	
ABCD				
PQRS				
WXYZ				

c. **Verbal** Make a conjecture about the diagonals of a kite.

H.O.T. Problems Use Higher-Order Thinking Skills

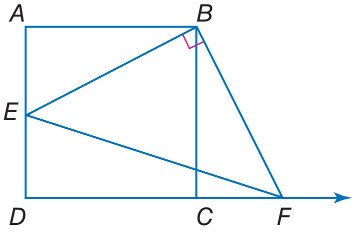
46. **ERROR ANALYSIS** In parallelogram *PQRS*, $\overline{PR} \cong \overline{QS}$. Lola thinks that the parallelogram is a square, and Xavier thinks that it is a rhombus. Is either of them correct? Explain your reasoning.



47. **CCSS ARGUMENTS** Determine whether the statement is *true* or *false*. Then write the converse, inverse, and contrapositive of the statement and determine the truth value of each. Explain your reasoning.

If a quadrilateral is a square, then it is a rectangle.

48. **CHALLENGE** The area of square *ABCD* is 36 square units and the area of $\triangle EBF$ is 20 square units. If $\overline{EB} \perp \overline{BF}$ and $\overline{AE} = 2$, find the length of $\overline{CF}$.

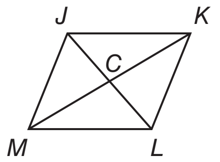


49. **OPEN ENDED** Find the vertices of a square with diagonals that are contained in the lines $y = x$ and $y = -x + 6$. Justify your reasoning.

50. **WRITING IN MATH** Compare all of the properties of the following quadrilaterals: parallelograms, rectangles, rhombi, and squares.

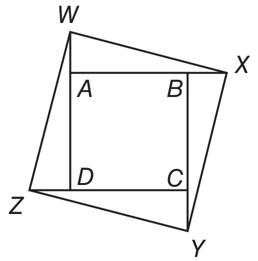
Standardized Test Practice

51. *JKLM* is a rhombus. If $CK = 8$ and $JK = 10$, find *JC*.



A 4 **C** 8
B 6 **D** 10

52. EXTENDED RESPONSE The sides of square *ABCD* are extended by sides of equal length to form square *WXYZ*.

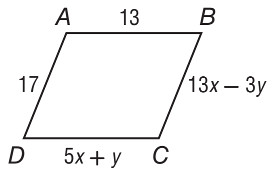


a. If $CY = 3$ cm and the area of *ABCD* is 81 cm^2, find the area of *WXYZ*.

b. If the areas of *ABCD* and *WXYZ* are 49 cm^2 and 169 cm^2 respectively, find *DZ*.

c. If $AB = 2CY$ and the area of $ABCD = g$ square meters, find the area of *WXYZ* in square meters.

53. ALGEBRA What values of x and y make quadrilateral *ABCD* a parallelogram?



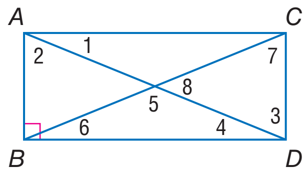
F $x = 3, y = 2$
G $x = \frac{3}{2}, y = -1$
H $x = 2, y = 3$
J $x = 3, y = -1$

54. SAT/ACT What is 6 more than the product of -3 and a certain number x?

A $-3x - 6$ **D** $-3x + 6$
B $-3x$ **E** $6 + 3x$
C $-x$

Spiral Review

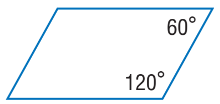
Quadrilateral *ABDC* is a rectangle. Find each measure if $m\angle 1 = 38$. (Lesson 6-4)



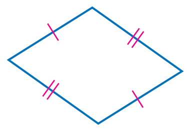
55. $m\angle 2$ **56.** $m\angle 5$ **57.** $m\angle 6$

Determine whether each quadrilateral is a parallelogram. Justify your answer. (Lesson 6-3)

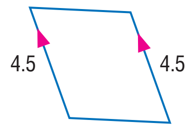
58.



59.

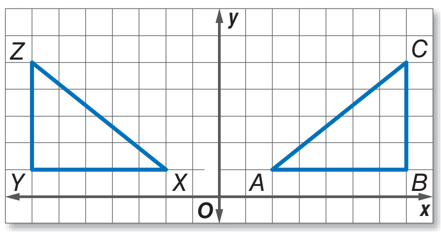


60.



61. MEASUREMENT Monifa says that her backyard is shaped like a triangle and that the lengths of its sides are 22 feet, 23 feet, and 45 feet. Do you think these measurements are correct? Explain your reasoning. (Lesson 5-5)

62. COORDINATE GEOMETRY Identify the transformation and verify that it is a congruence transformation. (Lesson 4-7)



Skills Review

Solve each equation.

63. $\frac{1}{2}(5x + 7x - 1) = 11.5$ **64.** $\frac{1}{2}(10x + 6x + 2) = 7$ **65.** $\frac{1}{2}(12x + 6 - 8x + 7) = 9$

LESSON 6-6 Trapezoids and Kites

Then

- You used properties of special parallelograms.

Now

1. Apply properties of trapezoids.
2. Apply properties of kites.

Why?

- In gymnastics, vaulting boxes made out of high compression foam are used as spotting platforms, vaulting horses, and steps. The left and right side of each section is a *trapezoid*.

NewVocabulary

trapezoid
bases
legs of a trapezoid
base angles
isosceles trapezoid
midsegment of a trapezoid
kite

Common Core State Standards

Content Standards

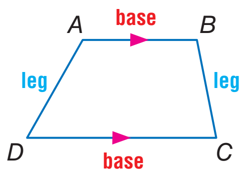
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

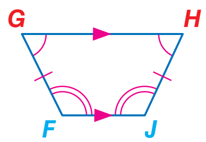
1 Properties of Trapezoids A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**. The nonparallel sides are called **legs**. The **base angles** are formed by the base and one of the legs. In trapezoid $ABCD$, $\angle A$ and $\angle B$ are one pair of base angles and $\angle C$ and $\angle D$ are the other pair. If the legs of a trapezoid are congruent, then it is an **isosceles trapezoid**.



Theorems Isosceles Trapezoids

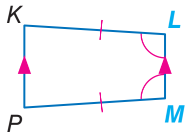
6.21 If a trapezoid is isosceles, then each pair of base angles is congruent.

Example If trapezoid $FGHJ$ is isosceles, then $\angle G \cong \angle H$ and $\angle F \cong \angle J$.



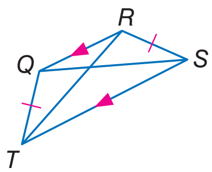
6.22 If a trapezoid has one pair of congruent base angles, then it is an isosceles trapezoid.

Example If $\angle L \cong \angle M$, then trapezoid $KLMP$ is isosceles.



6.23 A trapezoid is isosceles if and only if its diagonals are congruent.

Example If trapezoid $QRST$ is isosceles, then $\overline{QS} \cong \overline{RT}$. Likewise, if $\overline{QS} \cong \overline{RT}$, then trapezoid $QRST$ is isosceles.

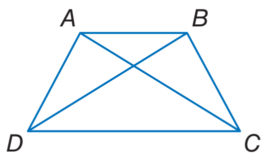


You will prove Theorem 6.21, Theorem 6.22, and the other part of Theorem 6.23 in Exercises 28, 29, and 30.

Proof Part of Theorem 6.23

Given: $ABCD$ is an isosceles trapezoid.

Prove: $\overline{AC} \cong \overline{BD}$



$ABCD$ is an isosceles trapezoid.
Given

$\overline{DC} \cong \overline{CD}$
Reflexive Property

$\overline{AD} \cong \overline{BC}$
Def. Isos. Trapezoid

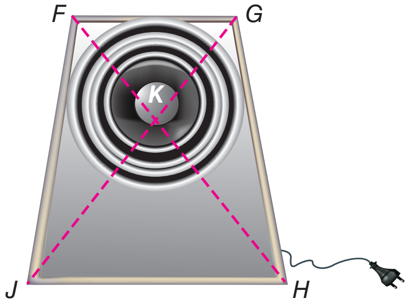
$\angle ADC \cong \angle BCD$
Base ∠s of trapezoid are ≅.

$\triangle ADC \cong \triangle BCD$
SAS

$\overline{AC} \cong \overline{BD}$
CPCTC

Real-World Example 1 Use Properties of Isosceles Trapezoids

MUSIC The speaker shown is an isosceles trapezoid. If $m\angle FJH = 85$, $FK = 8$ inches, and $JG = 19$ inches, find each measure.



a. $m\angle FGH$

Since $FGHJ$ is an isosceles trapezoid, $\angle FJH$ and $\angle GHJ$ are congruent base angles. So, $m\angle GHJ = m\angle FJH = 85$.

Since $FGHJ$ is a trapezoid, $\overline{FG} \parallel \overline{JH}$.

$m\angle FGH + m\angle GHJ = 180$	Consecutive Interior Angles Theorem
$m\angle FGH + 85 = 180$	Substitution
$m\angle FGH = 95$	Subtract 85 from each side.

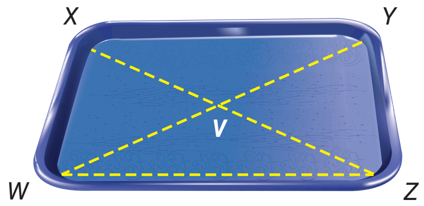
b. KH

Since $FGHJ$ is an isosceles trapezoid, diagonals $\overline{FH}$ and $\overline{JG}$ are congruent.

$FH = JG$	Definition of congruent
$FK + KH = JG$	Segment Addition
$8 + KH = 19$	Substitution
$KH = 11$ cm	Subtract 8 from each side.

Guided Practice

1. CAFETERIA TRAYS To save space at a square table, cafeteria trays often incorporate trapezoids into their design. If $WXYZ$ is an isosceles trapezoid and $m\angle YZW = 45$, $WV = 15$ centimeters, and $VY = 10$ centimeters, find each measure.



A. $m\angle XWZ$ **B.** $m\angle WXY$

C. XZ **D.** XV

Real-World Link

Speakers are amplifiers that intensify sound waves so that they are audible to the unaided ear. Amplifiers exist in devices such as televisions, stereos, and computers.

Source: How Stuff Works

Study Tip

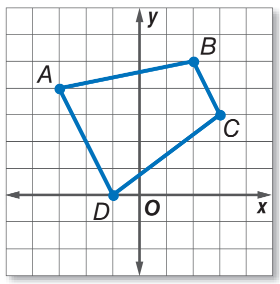
Isosceles Trapezoids The base angles of a trapezoid are only congruent if the trapezoid is isosceles.

You can use coordinate geometry to determine whether a trapezoid is an isosceles trapezoid.

Example 2 Isosceles Trapezoids and Coordinate Geomerty

COORDINATE GEOMETRY Quadrilateral $ABCD$ has vertices $A(-3, 4)$, $B(2, 5)$, $C(3, 3)$, and $D(-1, 0)$. Show that $ABCD$ is a trapezoid and determine whether it is an isosceles trapezoid.

Graph and connect the vertices of $ABCD$.



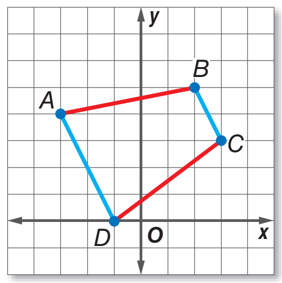
Step 1 Use the Slope Formula to compare the slopes of opposite sides $\overline{BC}$ and $\overline{AD}$ and of opposite sides $\overline{AB}$ and $\overline{DC}$. A quadrilateral is a trapezoid if exactly one pair of opposite sides are parallel.

Opposite sides $\overline{BC}$ and $\overline{AD}$:

slope of $\overline{BC} = 3 - \frac{5}{3} - 2 = -\frac{2}{1}$ or -2

slope of $\overline{AD} = \frac{0-4}{-1-(-3)} = \frac{-4}{2}$ or -2

Since the slopes of $\overline{BC}$ and $\overline{AD}$ are equal, $\overline{BC} \parallel \overline{AD}$.



Opposite sides $\overline{AB}$ and $\overline{DC}$:

slope of $\overline{AB} = \frac{5-4}{2-(-3)} = \frac{1}{5}$ slope of $\overline{DC} = \frac{0-3}{-1-3} = \frac{-3}{-4}$ or $\frac{3}{4}$

Since the slopes of $\overline{AB}$ and $\overline{DC}$ are *not* equal, $\overline{BC} \nparallel \overline{AD}$. Since quadrilateral $ABCD$ has only one pair of opposite sides that are parallel, quadrilateral $ABCD$ is a trapezoid.

ReadingMath

Symbols Recall that the symbol $\nparallel$ means *is not parallel to.*

Step 2 Use the Distance Formula to compare the lengths of legs $\overline{AB}$ and $\overline{DC}$. A trapezoid is isosceles if its legs are congruent.

$AB = \sqrt{(-3-2)^2 + (4-5)^2}$ or $\sqrt{26}$

$DC = \sqrt{(-1-3)^2 + (0-3)^2} = \sqrt{25}$ or 5

Since $AB \neq DC$, legs $\overline{AB}$ abd $\overline{DC}$ are *not* congruent. Therefore, trapezoid $ABCD$ is not isosceles.

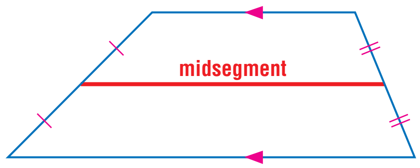
GuidedPractice

2. Quadrilateral $QRST$ has vertices $Q(-8, -4)$, $R(0, 8)$, $S(6, 8)$, and $T(-6, -10)$. Show that $QRST$ is a trapezoid and determine whether $QRST$ is an isosceles trapezoid.

ReadingMath

Midsegment A midsegment of a trapezoid can also be called a *median.*

The **midsegment of a trapezoid** is the segment that connects the midpoints of the legs of the trapezoid.

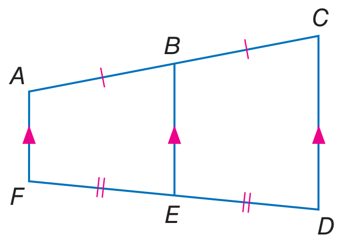


The theorem below relates the midsegment and the bases of a trapezoid.

Theorem 6.24 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

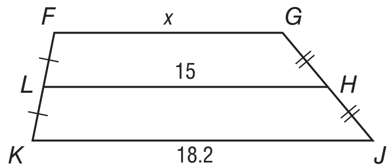
Example If $\overline{BE}$ is the midsegment of trapezoid $ACDF$, then $\overline{AF} \parallel \overline{BE}$, $\overline{CD} \parallel \overline{BE}$, and $BE = \frac{1}{2}(AF + CD)$.



You will prove Theorem 6.24 in Exercise 33.

Standardized Test Example 3 Midsegment of a Trapezoid

GRIDDED RESPONSE **In the figure, $\overline{LH}$ is the midsegment of trapezoid *FGJK*. What is the value of x?**



Note: The figure is not drawn to scale.

Read the Test Item

You are given the measure of the midsegment of a trapezoid and the measure of one of its bases. You are asked to find the measure of the other base.

Solve the Test Item

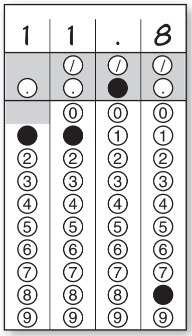
$LH = \frac{1}{2}(FG + KJ)$ Trapezoid Midsegment Theorem

$5 = \frac{1}{2}(x + 18.2)$ Substitution

$30 = x + 18.2$ Multiply each side by 2.

$11.8 = x$ Subtract 18.2 from each side.

Grid In Your Answer



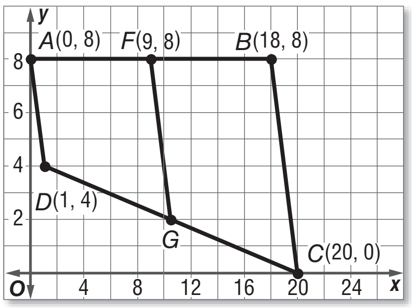
- You can align the numerical answer by placing the first digit in the left answer box or by putting the last digit in the right answer box.
- Do not leave blank boxes in the middle of an answer.
- Fill in **one** bubble for each filled answer box. Do not fill more than one bubble for an answer box. Do not fill in a bubble for blank answer boxes.

Test-Taking Tip

Gridded Responses Rational answers can often be gridded in more than one way. An answer such as $\frac{8}{5}$ could be gridded as 8/5 or 1.6, but not as 1 3/5.

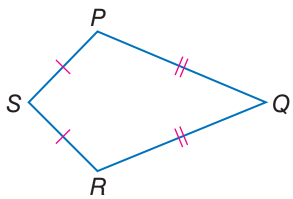
Guided Practice

3. GRIDDED RESPONSE Trapezoid *ABCD* is shown below. If $\overline{FG}$ is parallel to $\overline{AD}$, what is the x-coordinate of point G?



2 Properties of Kites

A **kite** is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.



Study Tip

Kites The congruent angles of a kite are included by the non-congruent adjacent sides.

Real-World Link

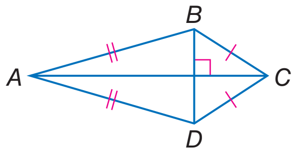
The fastest recorded speed of a kite is over 120 miles per hour. The record for the highest single kite flown is 12,471 feet.

Source: Borealis Kites

Theorems Kites

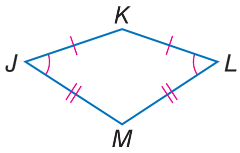
6.25 If a quadrilateral is a kite, then its diagonals are perpendicular.

Example If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.



6.26 If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

Example If quadrilateral $JKLM$ is a kite, $\overline{JK} \cong \overline{KL}$, and $\overline{JM} \cong \overline{LM}$, then $\angle J \cong \angle L$ and $\angle K \ncong \angle M$.

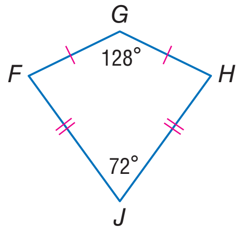


You will prove Theorems 6.25 and 6.26 in Exercises 31 and 32, respectively.

You can use the theorems above, the Pythagorean Theorem, and the Polygon Interior Angles Sum Theorem to find missing measures in kites.

Example 4 Use Properties of Kites

a. If $FGHJ$ is a kite, find $m\angle GFJ$.



Since a kite can only have one pair of opposite congruent angles and $\angle G \ncong \angle J$, then $\angle F \cong \angle H$. So, $m\angle F = m\angle H$. Write and solve an equation to find $m\angle F$.

$m\angle F + m\angle G + m\angle H + m\angle J = 360$ Polygon Interior Angles Sum Theorem

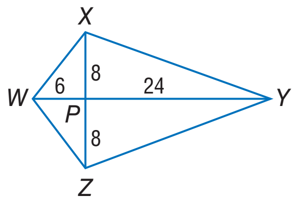
$m\angle F + 128 + m\angle F + 72 = 360$ Substitution

$2m\angle F + 200 = 360$ Simplify.

$2m\angle F = 160$ Subtract 200 from each side.

$m\angle F = 80$ Divide each side by 2.

b. If $WXYZ$ is a kite, find ZY.



Since the diagonals of a kite are perpendicular, they divide $WXYZ$ into four right triangles. Use the Pythagorean Theorem to find ZY, the length of the hypotenuse of right $\triangle YPZ$.

$PZ^2 + PY^2 = ZY^2$ Pythagorean Theorem

$8^2 + 24^2 = ZY^2$ Substitution

$640 = ZY^2$ Simplify.

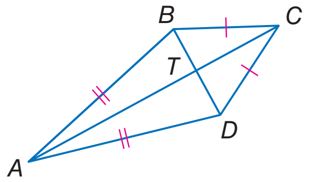
$\sqrt{640} = ZY$ Take the square root of each side.

$8\sqrt{10} = ZY$ Simplify.

Guided Practice

4A. If $m\angle BAD = 38$ and $m\angle BCD = 50$, find $m\angle ADC$.

4B. If $BT = 5$ and $TC = 8$, find CD.



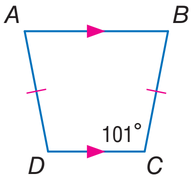
Guy Grenier/Masterfile

Check Your Understanding

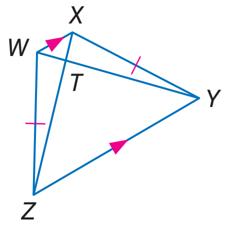
= Step-by-Step Solutions begin on page R14.

Example 1 **Find each measure.**

1. $m\angle D$



2. WT, if $ZX = 20$ and $TY = 15$

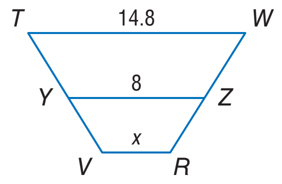


Example 2 **COORDINATE GEOMETRY** **Quadrilateral $ABCD$ has vertices $A(-4, -1)$, $B(-2, 3)$, $C(3, 3)$, and $D(5, -1)$.**

3. Verify that $ABCD$ is a trapezoid.

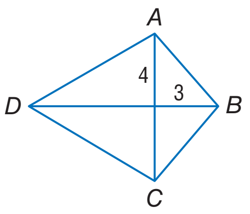
4. Determine whether $ABCD$ is an isosceles trapezoid. Explain.

Example 3 **5. GRIDDED RESPONSE** In the figure at the right, $\overline{YZ}$ is the midsegment of trapezoid $TWRV$. Determine the value of x.

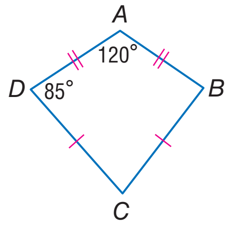


Example 4 CCSS **SENSE-MAKING** **If $ABCD$ is a kite, find each measure.**

6. AB



7. $m\angle C$

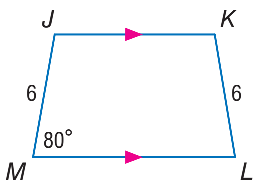


Practice and Problem Solving

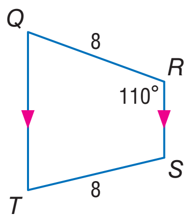
Extra Practice is on page R6.

Example 1 **Find each measure.**

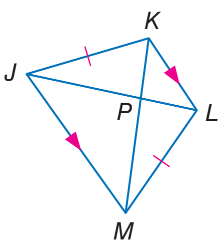
8. $m\angle K$



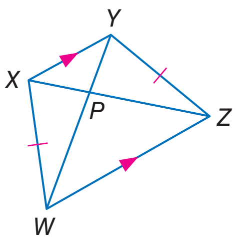
9. $m\angle Q$



10. JL, if $KP = 4$ and $PM = 7$



11 PW, if $XZ = 18$ and $PY = 3$



Example 2 **COORDINATE GEOMETRY** **For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.**

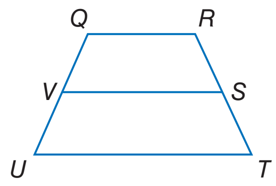
12. $A(-2, 5)$, $B(-3, 1)$, $C(6, 1)$, $D(3, 5)$

13. $J(-4, -6)$, $K(6, 2)$, $L(1, 3)$, $M(-4, -1)$

14. $Q(2, 5)$, $R(-2, 1)$, $S(-1, -6)$, $T(9, 4)$

15. $W(-5, -1)$, $X(-2, 2)$, $Y(3, 1)$, $Z(5, -3)$

Example 3 **For trapezoid $QRTU$, V and S are midpoints of the legs.**



16. If $QR = 12$ and $UT = 22$, find VS.

17. If $QR = 4$ and $UT = 16$, find VS.

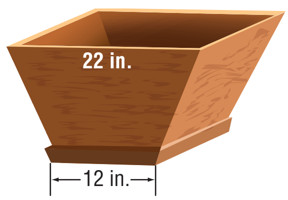
18. If $VS = 9$ and $UT = 12$, find QR.

19. If $TU = 26$ and $SV = 17$, find QR.

20. If $QR = 2$ and $VS = 7$, find UT.

21. If $RQ = 5$ and $VS = 11$, find UT.

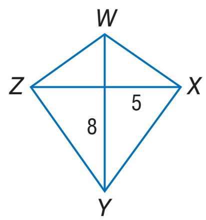
22. **DESIGN** Juana is designing a window box. She wants the end of the box to be a trapezoid with the dimensions shown. If she wants to put a shelf in the middle for the plants to rest on, about how wide should she make the shelf?



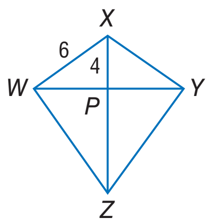
23 **MUSIC** The keys of the xylophone shown form a trapezoid. If the length of the lower pitched C is 6 inches long, and the higher pitched D is 1.8 inches long, how long is the G key?

Example 4 **CCSS SENSE-MAKING** **If $WXYZ$ is a kite, find each measure.**

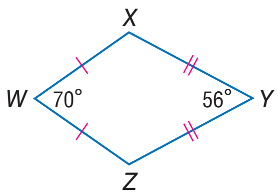
24. YZ



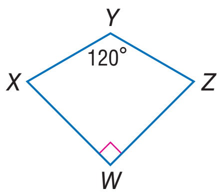
25. WP



26. $m\angle X$



27. $m\angle Z$



PROOF **Write a paragraph proof for each theorem.**

28. Theorem 6.21 **29.** Theorem 6.22 **30.** Theorem 6.23

31. Theorem 6.25 **32.** Theorem 6.26

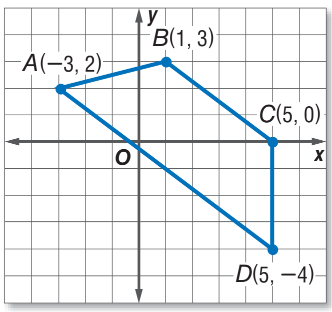
33. **PROOF** Write a coordinate proof for Theorem 6.24.

34. **COORDINATE GEOMETRY** Refer to quadrilateral $ABCD$.

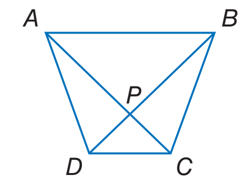
a. Determine whether the figure is a trapezoid. If so, is it isosceles? Explain.

b. Is the midsegment contained in the line with equation $y = -x + 1$? Justify your answer.

c. Find the length of the midsegment.



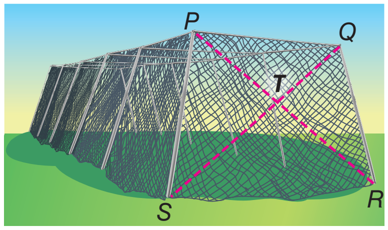
ALGEBRA ***ABCD* is a trapezoid.**



35. If $AC = 3x - 7$ and $BD = 2x + 8$, find the value of x so that $ABCD$ is isosceles.

36. If $m\angle ABC = 4x + 11$ and $m\angle DAB = 2x + 33$, find the value of x so that $ABCD$ is isosceles.

SPORTS **The end of the batting cage shown is an isosceles trapezoid. If $PT = 12$ feet, $ST = 28$ feet, and $m\angle PQR = 110$, find each measure.**



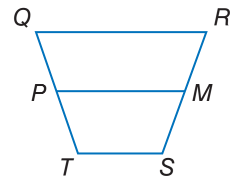
37. TR

38. SQ

39. $m\angle QRS$

40. $m\angle QPS$

ALGEBRA **For trapezoid *QRST*, *M* and *P* are midpoints of the legs.**



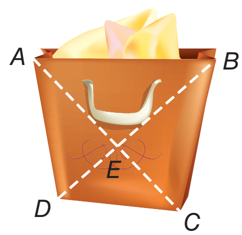
41. If $QR = 16$, $PM = 12$, and $TS = 4x$, find x.

42. If $TS = 2x$, $PM = 20$, and $QR = 6x$, find x.

43. If $PM = 2x$, $QR = 3x$, and $TS = 10$, find PM.

44. If $TS = 2x + 2$, $QR = 5x + 3$, and $PM = 13$, find TS.

SHOPPING **The side of the shopping bag shown is an isosceles trapezoid. If $EC = 9$ inches, $DB = 19$ inches, $m\angle ABE = 40$, and $m\angle EBC = 35$, find each measure.**



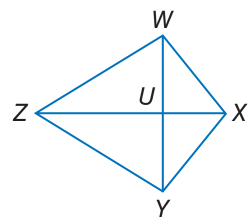
45. AE

46. AC

47. $m\angle BCD$

48. $m\angle EDC$

ALGEBRA ***WXYZ* is a kite.**



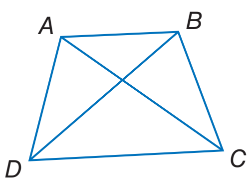
49. If $m\angle WXY = 120$, $m\angle WZY = 4x$, and $m\angle ZWX = 10x$, find $m\angle ZYX$.

50. If $m\angle WXY = 13x + 24$, $m\angle WZY = 35$, and $m\angle ZWX = 13x + 14$, find $m\angle ZYX$.

CCSS ARGUMENTS **Write a two-column proof.**

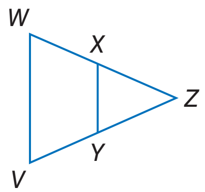
51. Given: $ABCD$ is an isosceles trapezoid.

Prove: $\angle DAC \cong \angle CBD$



52. Given: $\overline{WZ} \cong \overline{ZV}$, $\overline{XY}$ bisects $\overline{WZ}$ and $\overline{ZV}$, and $\angle W \cong \angle ZXY$.

Prove: $WXYV$ is an isosceles trapezoid.



Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

53. The opposite angles of a trapezoid are supplementary.

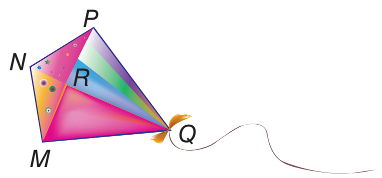
54. One pair of opposite sides are parallel in a kite.

55. A square is a rhombus.

56. A rectangle is a square.

57. A parallelogram is a rectangle.

58. **KITES** Refer to the kite at the right. Using the properties of kites, write a two-column proof to show that $\triangle MNR$ is congruent to $\triangle PNR$.



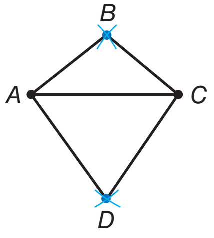
59. **VENN DIAGRAM** Create a Venn diagram that incorporates all quadrilaterals, including trapezoids, isosceles trapezoids, kites, and quadrilaterals that cannot be classified as anything other than quadrilaterals.

COORDINATE GEOMETRY **Determine whether each figure is a *trapezoid*, a *parallelogram*, a *square*, a *rhombus*, or a *quadrilateral* given the coordinates of the vertices. Choose the most specific term. Explain.**

60. $A(-1, 4)$, $B(2, 6)$, $C(3, 3)$, $D(0, 1)$

61. $W(-3, 4)$, $X(3, 4)$, $Y(5, 3)$, $Z(-5, 1)$

62. **MULTIPLE REPRESENTATIONS** In this problem, you will explore proportions in kites.



a. **Geometric** Draw a segment. Construct a noncongruent segment that perpendicularly bisects the first segment. Connect the endpoints of the segments to form a quadrilateral $ABCD$. Repeat the process two times. Name the additional quadrilaterals $PQRS$ and $WXYZ$.

b. **Tabular** Copy and complete the table below.

Figure	Side	Length	Side	Length	Side	Length	Side	Length
ABCD	AB		BC		CD		DA	
PQRS	PQ		QR		RS		SP	
WXYZ	WX		XY		YZ		ZW	

c. **Verbal** Make a conjecture about a quadrilateral in which the diagonals are perpendicular, exactly one diagonal is bisected, and the diagonals are not congruent.

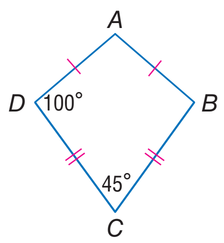
PROOF **Write a coordinate proof of each statement.**

63. The diagonals of an isosceles trapezoid are congruent.

64. The median of an isosceles trapezoid is parallel to the bases.

H.O.T. Problems Use Higher-Order Thinking Skills

65. **ERROR ANALYSIS** Bedagi and Belinda are trying to determine $m\angle A$ in kite $ABCD$ shown. Is either of them correct? Explain.



Bedagi

$m\angle A = 45$

Belinda

$m\angle A = 115$

66. **CHALLENGE** If the parallel sides of a trapezoid are contained by the lines $y = x + 4$ and $y = x - 8$, what equation represents the line contained by the midsegment?

67. **CCSS ARGUMENTS** Is it *sometimes*, *always*, or *never* true that a square is also a kite? Explain.

68. **OPEN ENDED** Sketch two noncongruent trapezoids $ABCD$ and $FGHJ$ in which $\overline{AC} \cong \overline{FH}$ and $\overline{BD} \cong \overline{GJ}$.

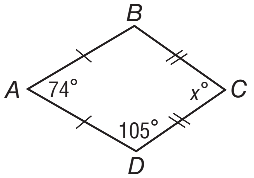
69. **WRITING IN MATH** Describe the properties a quadrilateral must possess in order for the quadrilateral to be classified as a trapezoid, an isosceles trapezoid, or a kite. Compare the properties of all three quadrilaterals.

Standardized Test Practice

70. ALGEBRA All of the items on a breakfast menu cost the same whether ordered with something else or alone. Two pancakes and one order of bacon costs \$4.92. If two orders of bacon cost \$3.96, what does one pancake cost?

A \$0.96 **C** \$1.98
B \$1.47 **D** \$2.94

71. GRIDDED RESPONSE If quadrilateral *ABCD* is a kite, what is $m\angle C$?

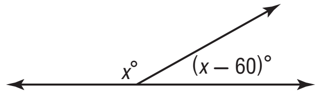


72. Which figure can serve as a counterexample to the conjecture below?

If the diagonals of a quadrilateral are congruent, then the quadrilateral is a rectangle.

F square **H** parallelogram
G rhombus **J** isosceles trapezoid

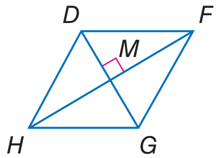
73. SAT/ACT In the figure below, what is the value of x?



A 60 **D** 240
B 120 **E** 300
C 180

Spiral Review

ALGEBRA Quadrilateral *DFGH* is a rhombus. Find each value or measure. (Lesson 6-5)



74. If $m\angle FGH = 118$, find $m\angle MHG$.

75. If $DM = 4x - 3$ and $MG = x + 6$, find DG.

76. If $DF = 10$, find FG.

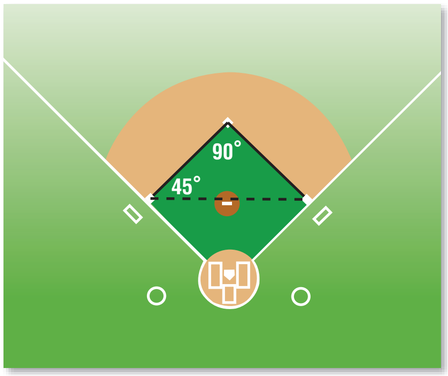
77. If $HM = 12$ and $HD = 15$, find MG.

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula. (Lesson 6-4)

78. $A(4, 2)$, $B(-4, 1)$, $C(-3, -5)$, $D(5, -4)$; Distance Formula

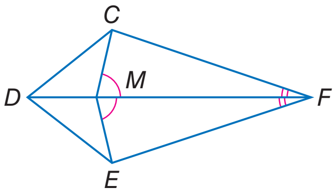
79. $J(0, 7)$, $K(-8, 6)$, $L(-7, 0)$, $M(1, 1)$; Slope Formula

80. BASEBALL A batter hits the ball to the third baseman and begins to run toward first base. At the same time, the runner on first base runs toward second base. If the third baseman wants to throw the ball to the nearest base, to which base should he throw? Explain. (Lesson 5-3)



81. PROOF Write a two-column proof. (Lesson 4-5)

Given: $\angle CMF \cong \angle EMF$,
$\angle CFM \cong \angle EFM$
Prove: $\triangle DMC \cong \triangle DME$



Skills Review

Write an expression for the slope of each segment given the coordinates and endpoints.

82. $(x, 4y)$, $(-x, 4y)$ **83.** $(-x, 5x)$, $(0, 6x)$ **84.** (y, x), (y, y)

CHAPTER 6 Study Guide and Review

Study Guide

KeyConcepts

Angles of Polygons (Lesson 6-1)

- The sum of the measures of the interior angles of a polygon is given by the formula $S = (n - 2)180$.
- The sum of the measures of the exterior angles of a convex polygon is 360.

Properties of Parallelograms (Lessons 6-2 and 6-3)

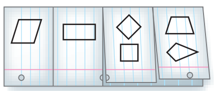
- Opposite sides are congruent and parallel.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- If a parallelogram has one right angle, it has four right angles.
- Diagonals bisect each other.

Properties of Rectangles, Rhombi, Squares, and Trapezoids (Lesson 6-4 through 6-6)

- A rectangle has all the properties of a parallelogram. Diagonals are congruent and bisect each other. All four angles are right angles.
- A rhombus has all the properties of a parallelogram. All sides are congruent. Diagonals are perpendicular. Each diagonal bisects a pair of opposite angles.
- A square has all the properties of a parallelogram, a rectangle, and a rhombus.
- In an isosceles trapezoid, both pairs of base angles are congruent and the diagonals are congruent.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

base (p. 439)

base angle (p. 439)

diagonal (p. 393)

isosceles trapezoid (p. 439)

kite (p. 442)

legs (p. 439)

midsegment of a trapezoid (p. 441)

parallelogram (p. 403)

rectangle (p. 423)

rhombus (p. 430)

square (p. 431)

trapezoid (p. 439)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. No angles in an isosceles trapezoid are congruent.
2. If a parallelogram is a rectangle, then the diagonals are congruent.
3. A midsegment of a trapezoid is a segment that connects any two nonconsecutive vertices.
4. The base of a trapezoid is one of the parallel sides.
5. The diagonals of a rhombus are perpendicular.
6. The diagonal of a trapezoid is the segment that connects the midpoints of the legs.
7. A rectangle is not always a parallelogram.
8. A quadrilateral with only one set of parallel sides is a parallelogram.
9. A rectangle that is also a rhombus is a square.
10. The leg of a trapezoid is one of the parallel sides.

Lesson-by-Lesson Review

6-1 Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

11. decagon

12. 15-gon

13. SNOWFLAKES The snowflake decoration at the right suggests a regular hexagon. Find the sum of the measures of the interior angles of the hexagon.

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

14. 135

15. ≈ 166.15

Example 1

Find the sum of the measures of the interior angles of a convex 22-gon.

$m = (n - 2)180$	Write an equation.
$= (22 - 2)180$	Substitution
$= 20 \cdot 180$	Subtract.
$= 3600$	Multiply.

Example 2

The measure of an interior angle of a regular polygon is 157.5. Find the number of sides in the polygon.

$157.5n = (n - 2)180$	Write an equation.
$157.5n = 180n - 360$	Distributive Property
$-22.5n = -360$	Subtract.
$n = 16$	Divide.

The polygon has 16 sides.

6-2 Parallelograms

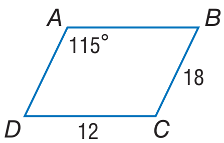
Use ▱$ABCD$ to find each measure.

16. $m\angle ADC$

17. AD

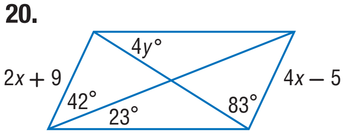
18. AB

19. $m\angle BCD$

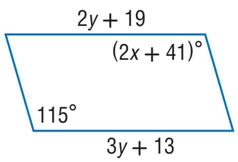


ALGEBRA **Find the value of each variable in each parallelogram.**

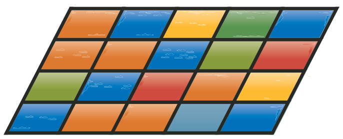
20.



21.

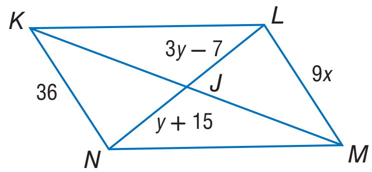


22. DESIGN What type of information is needed to determine whether the shapes that make up the stained glass window below are parallelograms?



Example 3

ALGEBRA **If $KLMN$ is a parallelogram, find the value of the indicated variable.**



a. x

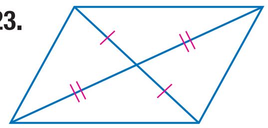
$\overline{KN} \cong \overline{LM}$	Opp. sides of a ▱ are ≅.
$KN = LM$	Definition of congruence
$36 = 9x$	Substitution
$4 = x$	Divide.

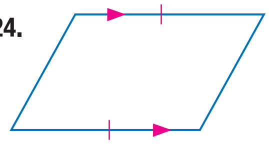
b. y

$\overline{NJ} \cong \overline{JL}$	Diag. of a ▱ bisect each other.
$NJ = JL$	Definition of congruence
$y + 15 = 3y - 7$	Substitution
$-2y = -22$	Subtract.
$y = 11$	Divide.

6-3 Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

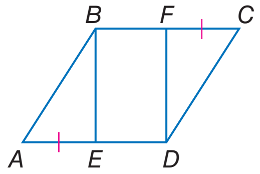
23. 

24. 

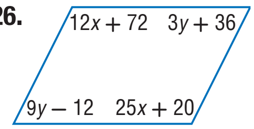
25. **PROOF** Write a two-column proof.

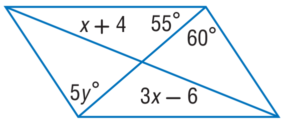
Given: $\square ABCD$, $\overline{AE} \cong \overline{CF}$

Prove: Quadrilateral *EBFD* is a parallelogram.



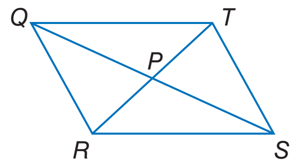
ALGEBRA Find x and y so that the quadrilateral is a parallelogram.

26. 

27. 

Example 4

If $TP = 4x + 2$, $QP = 2y - 6$, $PS = 5y - 12$, and $PR = 6x - 4$, find x and y so that the quadrilateral is a parallelogram.



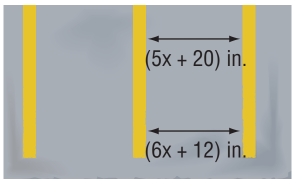
Find x such that $\overline{TP} \cong \overline{PR}$ and y such that $\overline{QP} \cong \overline{PS}$.

$TP = PR$	Definition of $\cong$
$4x + 2 = 6x - 4$	Substitution
$-2x = -6$	Subtract.
$x = 3$	Divide.

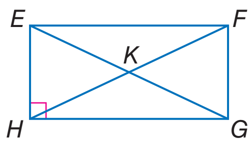
$QP = PS$	Definition of $\cong$
$2y - 6 = 5y - 12$	Substitution
$-3y = -6$	Subtract.
$y = 2$	Divide.

6-4 Rectangles

28. **PARKING** The lines of the parking space shown below are parallel. How wide is the space (in inches)?



ALGEBRA Quadrilateral *EFGH* is a rectangle.



29. If $m\angle FEG = 57$, find $m\angle GEH$.

30. If $m\angle HGE = 13$, find $m\angle FGE$.

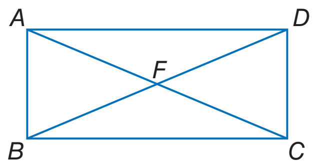
31. If $FK = 32$ feet, find EG.

32. Find $m\angle HEF + m\angle EFG$.

33. If $EF = 4x - 6$ and $HG = x + 3$, find EF.

Example 5

ALGEBRA Quadrilateral *ABCD* is a rectangle. If $m\angle ADB = 4x + 8$ and $m\angle DBA = 6x + 12$, find x.



ABCD is a rectangle, so $m\angle ABC = 90$. Since the opposite sides of a rectangle are parallel, and the alternate interior angles of parallel lines are congruent, $\angle DBC \cong \angle ADB$ and $m\angle DBC = m\angle ADB$.

$m\angle DBC + m\angle DBA = 90$	Angle Addition
$m\angle ADB + m\angle DBA = 90$	Substitution
$4x + 8 + 6x + 12 = 90$	Substitution
$10x + 20 = 90$	Add.
$10x = 70$	Subtract.
$x = 7$	Divide.

CHAPTER 6 Study Guide and Review *Continued*

6-5 Rhombi and Squares

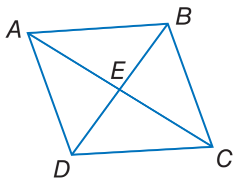
ALGEBRA *ABCD* is a rhombus. If $EB = 9$, $AB = 12$ and $m\angle ABD = 55$, find each measure.

34. AE

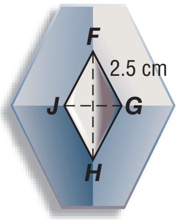
35. $m\angle BDA$

36. CE

37. $m\angle ACB$



38. LOGOS A car company uses the symbol shown at the right for their logo. If the inside space of the logo is a rhombus, what is the length of FJ?



COORDINATE GEOMETRY Given each set of vertices, determine whether ▱*QRST* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

39. $Q(12, 0)$, $R(6, -6)$, $S(0, 0)$, $T(6, 6)$

40. $Q(-2, 4)$, $R(5, 6)$, $S(12, 4)$, $T(5, 2)$

Example 6

The diagonals of rhombus *QRST* intersect at *P*. Use the information to find each measure or value.

a. ALGEBRA If $QT = x + 7$ and $TS = 2x - 9$, find x.

$\overline{QT} \cong \overline{TS}$	Def. of rhombus
$QT = TS$	Def. of congruence
$x + 7 = 2x - 9$	Substitution
$-x = -16$	Subtract.
$x = 16$	Divide.

b. If $m\angle QTS = 76$, find $m\angle TSP$.

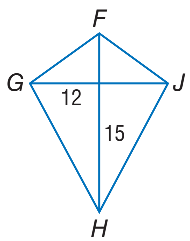
$\overline{TR}$ bisects $\angle QTS$. Therefore, $m\angle PTS = \frac{1}{2}m\angle QTS$. So $m\angle PTS = \frac{1}{2}(76)$ or 38. Since the diagonals of a rhombus are perpendicular, $m\angle TPS = 90$.

$m\angle PTS + m\angle TPS + m\angle TSP = 180$	△ Sum Thm.
$38 + 90 + m\angle TSP = 180$	Substitution
$128 + m\angle TSP = 180$	Add.
$m\angle TSP = 52$	Subtract.

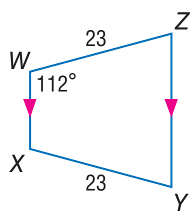
6-6 Trapezoids and Kites

Find each measure.

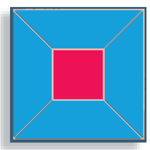
41. GH



42. $m\angle Z$



43. DESIGN Renee designed the square tile as an art project.



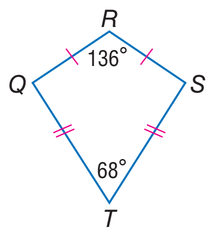
a. Describe a way to determine if the trapezoids in the design are isosceles.

b. If the perimeter of the tile is 48 inches and the perimeter of the red square is 16 inches, what is the perimeter of one of the trapezoids?

Example 7

If *QRST* is a kite, find $m\angle RST$.

Since $\angle Q \cong \angle S$, $m\angle Q = m\angle S$. Write and solve an equation to find $m\angle S$.



$m\angle Q + m\angle R + m\angle S + m\angle T = 360$	Polygon Int. ∠s Sum Thm
$m\angle Q + 136 + m\angle S + 68 = 360$	Substitution
$2m\angle S + 204 = 360$	Simplify.
$2m\angle S = 156$	Subtract.
$m\angle S = 78$	Divide.

CHAPTER 6 Practice Test

Find the sum of the measures of the interior angles of each convex polygon.

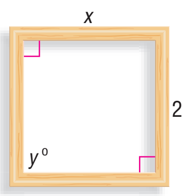
1. hexagon

2. 16-gon

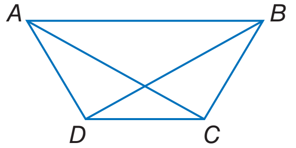
3. ART Jen is making a frame to stretch a canvas over for a painting. She nailed four pieces of wood together at what she believes will be the four vertices of a square.

a. How can she be sure that the canvas will be a square?

b. If the canvas has the dimensions shown below, what are the missing measures?



Quadrilateral *ABCD* is an isosceles trapezoid.



4. Which angle is congruent to $\angle C$?

5. Which side is parallel to $\overline{AB}$?

6. Which segment is congruent to $\overline{AC}$?

The measure of the interior angles of a regular polygon is given. Find the number of sides in the polygon.

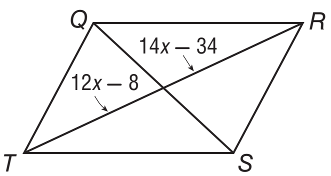
7. 900

8. 1980

9. 2880

10. 5400

11. MULTIPLE CHOICE If *QRST* is a parallelogram, what is the value of x?



A 11

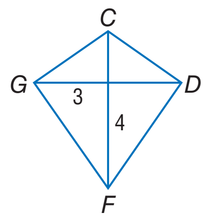
B 12

C 13

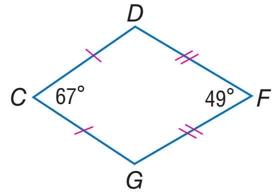
D 14

If *CDFG* is a kite, find each measure.

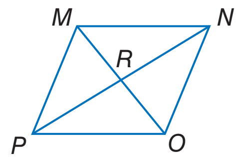
12. GF



13. $m\angle D$



ALGEBRA Quadrilateral *MNOP* is a rhombus. Find each value or measure.



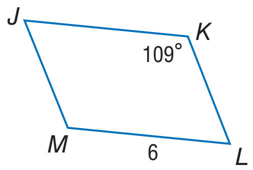
14. $m\angle MRN$

15. If $PR = 12$, find RN.

16. If $m\angle PON = 124$, find $m\angle POM$.

17. CONSTRUCTION The Smiths are building an addition to their house. Mrs. Smith is cutting an opening for a new window. If she measures to see that the opposite sides are congruent and that the diagonal measures are congruent, can Mrs. Smith be sure that the window opening is rectangular? Explain.

Use ▱*JKLM* to find each measure.

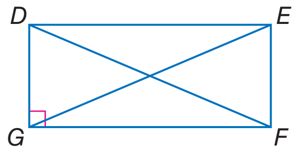


18. $m\angle JML$

19. JK

20. $m\angle KLM$

ALGEBRA Quadrilateral *DEFG* is a rectangle.



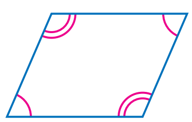
21. If $DF = 2(x + 5) - 7$ and $EG = 3(x - 2)$, find EG.

22. If $m\angle EDF = 5x - 3$ and $m\angle DFG = 3x + 7$, find $m\angle EDF$.

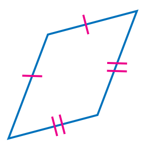
23. If $DE = 14 + 2x$ and $GF = 4(x - 3) + 6$, find GF.

Determine whether each quadrilateral is a parallelogram. Justify your answer.

24.



25.



Preparing for Standardized Tests

Apply Definitions and Properties

Many geometry problems on standardized tests require the application of definitions and properties in order to solve them. Use this section to practice applying definitions to help you solve extended-response test items.

Strategies for Applying Definitions and Properties

Step 1

Read the problem statement carefully.

- Determine what you are being asked to solve.
- Study any figures given in the problem.
- **Ask yourself:** What principles or properties of this figure can I apply to solve the problem?

Step 2

Solve the problem.

- Identify any definitions or geometric concepts you can use to help you find the unknowns in the problem.
- Use definitions and properties of figures to set up and solve an equation.

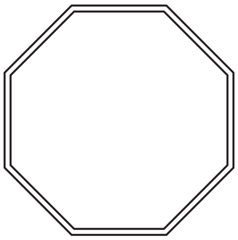
Step 3

- Check your answer.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

A performing arts group is building a theater in the round for upcoming productions. The stage will be a regular octagon with a perimeter of 76 feet.



a. What length should each board be to form the sides of the stage?

b. What angle should the end of each board be cut so that they will fit together properly to form the stage? Explain.

Read the problem carefully. You are told that the boards form a regular octagon with a perimeter of 76 feet. You need to find the length of each board and the angle that they should be cut to fit together properly.

To find the length of each board, divide the perimeter by the number of boards.

$76 \div 8 = 9.5$

So, each board should be 9.5 feet, or 9 feet 6 inches, long.

Use the property of the interior angle sum of convex polygons to find the measure of an interior angle of a regular octagon. First find the sum S of the interior angles.

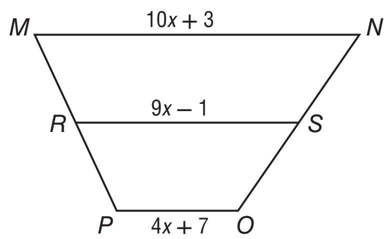
$$S = (n - 2) \cdot 180$$
$$= (8 - 2) \cdot 180$$
$$= 1080$$

So, the measure of an interior angle of a regular octagon is $1080 \div 8$, or 135°. Since two boards are used to form each vertex of the stage, the end of each board should be cut at an angle of $135 \div 2$, or 67.5°.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

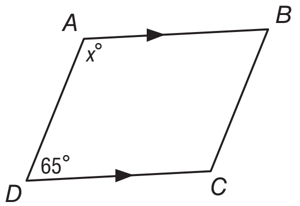
1. $\overline{RS}$ is the midsegment of trapezoid $MNOP$. What is the length of $\overline{RS}$?



A 14 units **C** 23 units

B 19 units **D** 26 units

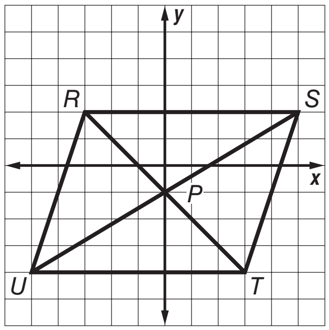
2. If $\overline{AB} \parallel \overline{DC}$, find x.



F 32.5 **H** 105

G 65 **J** 115

3. Use the graph shown below to answer each question.



a. Do the diagonals of quadrilateral $RSTU$ bisect each other? Use the Distance Formula to verify your answer.

b. What type of quadrilateral is $RSTU$? Explain using the properties and/or definitions of this type of quadrilateral.

4. What is the sum of the measures of the exterior angles of a regular octagon?

A 45

B 135

C 360

D 1080

CHAPTER 6

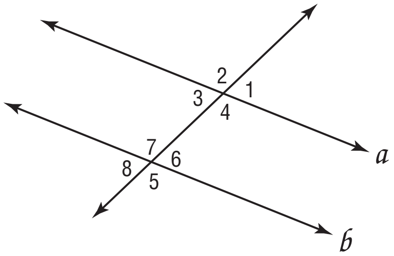
Standardized Test Practice

Cumulative, Chapters 1 through 6

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. If $a \parallel b$, which of the following might not be true?



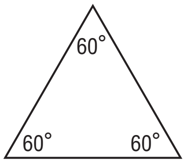
A $\angle 1 \cong \angle 3$

B $\angle 4 \cong \angle 7$

C $\angle 2 \cong \angle 5$

D $\angle 8 \cong \angle 2$

2. Classify the triangle below according to its angle measures. Choose the most appropriate term.



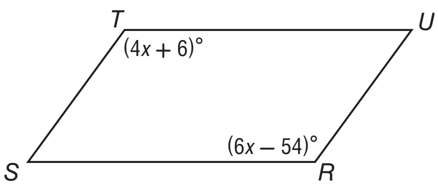
F acute

G equiangular

H obtuse

J right

3. Solve for x in parallelogram $RSTU$.



A 12

B 18

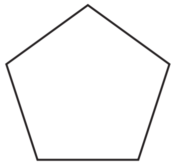
C 25

D 30

Test-Taking Tip

Question 3 Use the properties of parallelograms to solve the problem. Opposite angles are congruent.

4. What is the measure of an interior angle of a regular pentagon?



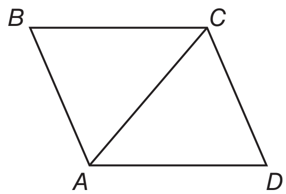
F 96

G 108

H 120

J 135

5. Quadrilateral $ABCD$ is a rhombus. If $m\angle BCD = 120$, find $m\angle DAC$.



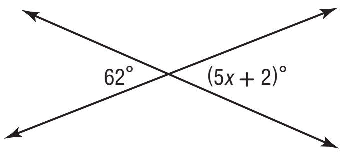
A 30

B 60

C 90

D 120

6. What is the value of x in the figure below?



F 10

G 12

H 14

J 15

7. Which of the following statements is true?

A All rectangles are squares.

B All rhombi are squares.

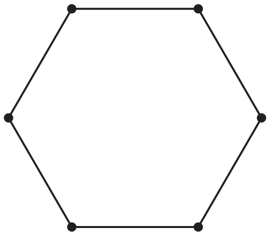
C All rectangles are parallelograms.

D All parallelograms are rectangles.

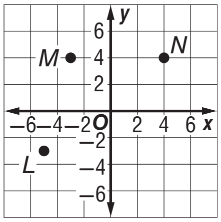
Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or a sheet of paper.

8. **GRIDDED RESPONSE** The posts for Nancy's gazebo form a regular hexagon. What is the measure of the angle formed at each corner of the gazebo?



9. What are the coordinates of point P, the fourth vertex of an isosceles trapezoid? Show your work.



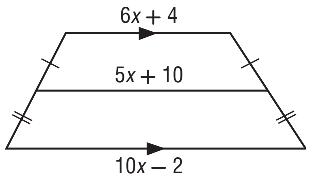
10. What do you know about a parallelogram if its diagonals are perpendicular? Explain.

11. Determine whether the stated conclusion is valid based on the given information below. If not, write *invalid*. Explain your reasoning.

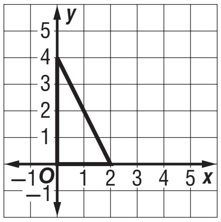
Given: If a number is divisible by 9, then the number is divisible by 3. The number 144 is divisible by 9.

Conclusion: The number 144 is divisible by 3.

12. **GRIDDED RESPONSE** Solve for x in the figure below. Round to the nearest tenth if necessary.



13. What are the coordinates of the circumcenter of the triangle below?

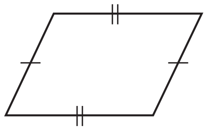


Extended Response

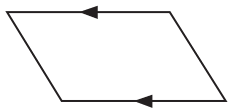
Record your answers on a sheet of paper. Show your work.

14. Determine whether you can prove each figure is a parallelogram. If not, tell what additional information would be needed to prove that it is a parallelogram. Explain your reasoning.

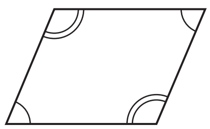
a.



b.



c.



Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Go to Lesson...	3-2	4-1	6-2	6-1	6-5	2-8	6-5	6-1	6-6	6-5	2-4	6-6	5-1	6-3