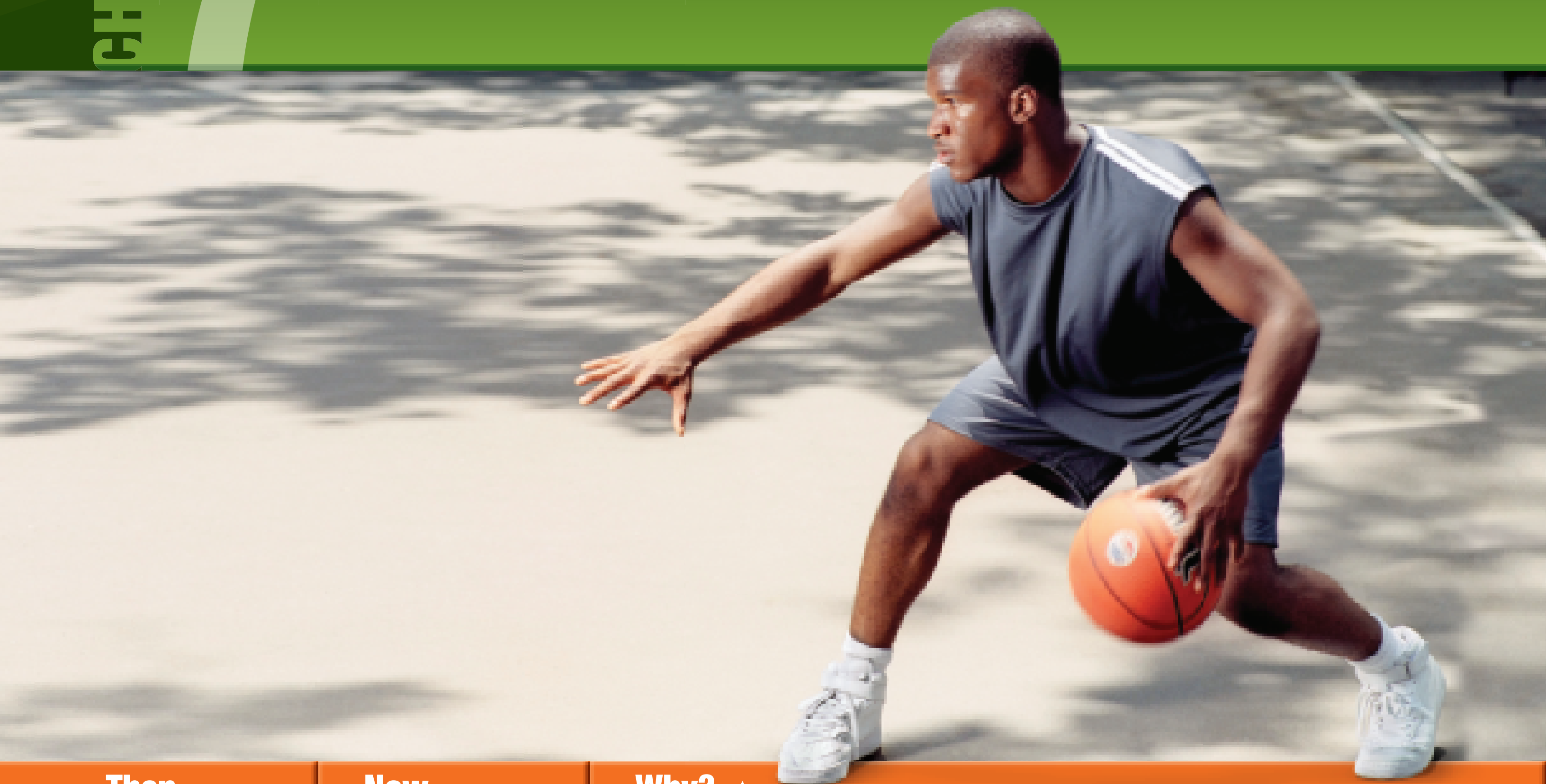
CHAPTER 7

# Proportions and Similarity



## Then

You learned about ratios and proportions and applied them to real-world applications.

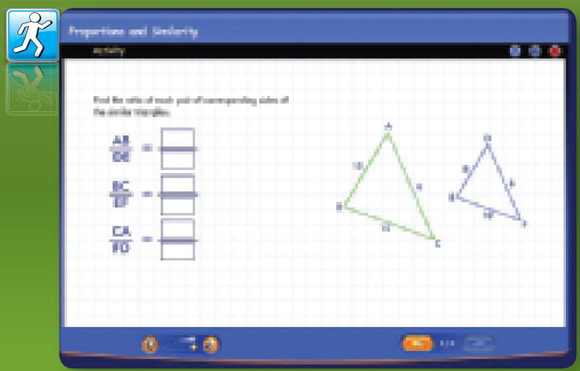
## Now

In this chapter, you will:

- Identify similar polygons and use ratios and proportions to solve problems.
- Identify and apply similarity transformations.
- Use scale models and drawings to solve problems.

## Why? ▲

**SPORTS** Similar triangles can be used in sports to describe the path of a ball, such as a bounce pass from one person to another.



Barry Rosenthal/The Image Bank/Getty Images

connectED.mcgraw-hill.com Your Digital Math Portal

Animation | Vocabulary | eGlossary | Personal Tutor | Virtual Manipulatives | Graphing Calculator | Audio | Foldables | Self-Check Practice | Worksheets

# **Get Ready** for the Chapter

**Diagnose** Readiness | You have two options for checking prerequisite skills.

## 1 Textbook Option

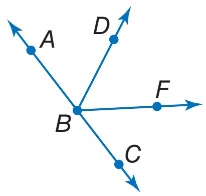
Take the Quick Check below. Refer to the Quick Review for help.

### QuickCheck

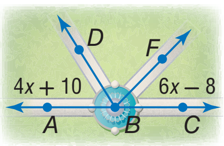
Solve each equation.

1. $\frac{3x}{8} = \frac{6}{x}$
2. $\frac{7}{3} = \frac{x-4}{6}$
3. $\frac{x+9}{2} = \frac{3x-1}{8}$
4. $\frac{3}{2x} = \frac{3x}{8}$
5. **EDUCATION** The student to teacher ratio at Elder High School is 17 to 1. If there are 1088 students in the school, how many teachers are there?

**ALGEBRA** In the figure, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays and $\overrightarrow{BD}$ bisects $\angle ABF$.




6. If $m\angle ABF = 3x - 8$ and $m\angle ABD = x + 14$, find $m\angle ABD$.
7. If $m\angle FBC = 2x + 25$ and $m\angle ABF = 10x - 1$, find $m\angle DBF$.
8. **LANDSCAPING** A landscape architect is planning to add sidewalks around a fountain as shown below. If $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays and $\overrightarrow{BD}$ bisects $\angle ABF$, find $m\angle FBC$.




### QuickReview

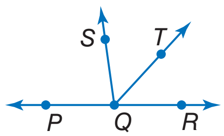
**Example 1**

Solve $\frac{4x-3}{5} = \frac{2x+11}{3}$.

| | |
|---|---|
| $\frac{4x-3}{5} = \frac{2x+11}{3}$ | Original equation |
| $3(4x - 3) = 5(2x + 11)$ | Cross multiplication |
| $12x - 9 = 10x + 55$ | Distributive Property |
| $2x = 64$ | Add. |
| $x = 32$ | Simplify. |

**Example 2**

In the figure, $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are opposite rays, and $\overrightarrow{QT}$ bisects $\angle SQR$. If $m\angle SQR = 6x + 8$ and $m\angle TQR = 4x - 14$, find $m\angle SQT$.




Since $\overrightarrow{TQ}$ bisects $\angle SQR$, $m\angle SQR = 2(m\angle TQR)$.

| | |
|---|---|
| $m\angle SQR = 2(m\angle TQR)$ | Def. of ∠ bisector |
| $6x + 8 = 2(4x - 14)$ | Substitution |
| $6x + 8 = 8x - 28$ | Distributive Property |
| $-2x = -36$ | Subtract. |
| $x = 18$ | Simplify. |

Since $\overrightarrow{TQ}$ bisects $\angle SQR$, $m\angle SQT = m\angle TQR$.

| | |
|---|---|
| $m\angle SQT = m\angle TQR$ | Def. of ∠ bisector |
| $m\angle SQT = 4x - 14$ | Substitution |
| $m\angle SQT = 58$ | $x = 18$ |

## 2 Online Option

Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

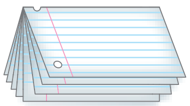
# Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 7. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

## FOLDABLES® StudyOrganizer

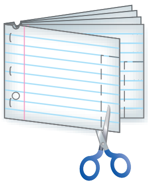
**Proportions and Similarity** Make this Foldable to help you organize your Chapter 7 notes about proportions, similar polygons, and similarity transformations. Begin with four sheets of notebook paper.

1 **Fold** the four sheets of paper in half.

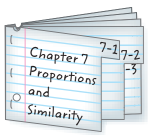


2 **Cut** along the top fold of the papers. Staple along the side to form a book.

3 **Cut** the right sides of each paper to create a tab for each lesson.



4 **Label** each tab with a lesson number, as shown.




## NewVocabulary

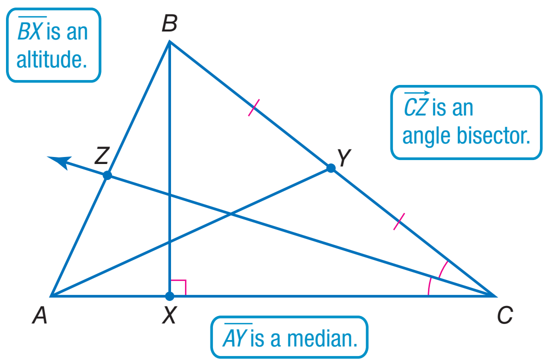
| English | | Español |
|---|---|---|
| ratio | p. 461 | razón |
| proportion | p. 462 | proporción |
| extremes | p. 462 | extremos |
| means | p. 462 | medias |
| cross products | p. 462 | productos cruzados |
| similar polygons | p. 469 | polígonos semejantes |
| scale factor | p. 470 | factor de escala |
| dilation | p. 511 | homotecia |
| similarity transformation | p. 511 | transformación de semejanza |
| enlargement | p. 511 | ampliación |
| reduction | p. 511 | reducción |
| scale model | p. 518 | modelo a escala |
| scale drawing | p. 518 | dibujo a escala |

## ReviewVocabulary

altitude altura a segment drawn from a vertex of a triangle perpendicular to the line containing the other side

angle bisector bisectriz de un ángulo a ray that divides an angle into two congruent angles

median mediana a segment drawn from a vertex of a triangle to the midpoint of the opposite side



LESSON 7-1

# Ratios and Proportions

**Then**

- You solved problems by writing and solving equations.

**Now**

1. Write ratios.
2. Write and solve proportions.

**Why?**

- The aspect ratio of a television or computer screen is the screen's width divided by its height. A standard television screen has an aspect ratio of $\frac{4}{3}$ or 4:3, while a high definition television screen (HDTV) has an aspect ratio of 16:9.

**New Vocabulary**

ratio
extended ratios
proportion
extremes
means
cross products

**Common Core State Standards**

**Content Standards**

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

**Mathematical Practices**

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

## 1 Write and Use Ratios

A **ratio** is a comparison of two quantities using division. The ratio of quantities $a$ and $b$ can be expressed as $a$ to $b$, $a:b$, or $\frac{a}{b}$, where $b \neq 0$. Ratios are usually expressed in simplest form.

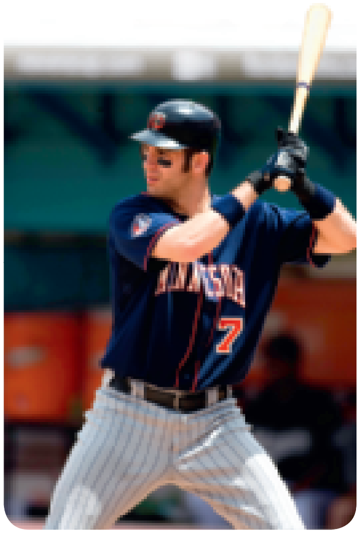
The aspect ratios 32:18 and 16:9 are equivalent.

$$\frac{\text{width of screen}}{\text{height of screen}} = \frac{32 \text{ in.}}{18 \text{ in.}}$$ Divide out units.

$$= \frac{32 \div 2}{18 \div 2} \text{ or } \frac{16}{9}$$ Divide out common factors.

### Real-World Example 1 Write and Simplify Ratios

**SPORTS A baseball player's batting average is the ratio of the number of base hits to the number of at-bats, not including walks. Minnesota Twins' Joe Mauer had the highest batting average in Major League Baseball in 2006. If he had 521 official at-bats and 181 hits, find his batting average.**



Divide the number of hits by the number of at-bats.

$$\frac{\text{number of hits}}{\text{number of at-bats}} = \frac{181}{521}$$

$$\approx \frac{0.347}{1}$$ A ratio in which the denominator is 1 is called a *unit ratio*.

Joe Mauer's batting average was 0.347.

**Guided Practice**

1. **SCHOOL** In Logan's high school, there are 190 teachers and 2650 students. What is the approximate student-teacher ratio at his school?

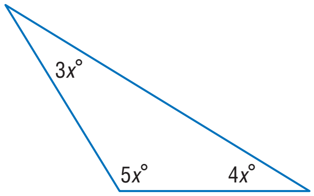
**Extended ratios** can be used to compare three or more quantities. The expression $a:b:c$ means that the ratio of the first two quantities is $a:b$, the ratio of the last two quantities is $b:c$, and the ratio of the first and last quantities is $a:c$.

## Example 2 Use Extended Ratios

**The ratio of the measures of the angles in a triangle is 3:4:5. Find the measures of the angles.**

Just as the ratio $\frac{3}{4}$ or 3:4 is equivalent to $\frac{3x}{4x}$ or $3x:4x$, the extended ratio 3:4:5 can be written as $3x:4x:5x$.

Sketch and label the angle measures of the triangle. Then write and solve an equation to find the value of $x$.




$3x + 4x + 5x = 180$ Triangle Sum Theorem

$12x = 180$ Combine like terms.

$x = 15$ Divide each side by 12.

So the measures of the angles are 3(15) or 45, 4(15) or 60, and 5(15) or 75.

**CHECK** The sum of the angle measures should be 180.

$45 + 60 + 75 = 180$ ✓

### GuidedPractice

**2.** In a triangle, the ratio of the measures of the sides is 2:2:3 and the perimeter is 392 inches. Find the length of the longest side of the triangle.

### ReadingMath

**Proportion** When a proportion is written using colons, it is read using the word *to* for the colon. For example, 2:3 is read *2 to 3*. The means are the inside numbers, and the extremes are the outside numbers.

extremes

2:3 = 6:9

means

## 2 Use Properties of Proportions

An equation stating that two ratios are equal is called a **proportion**. In the proportion $\frac{a}{b} = \frac{c}{d}$, the numbers $a$ and $d$ are called the **extremes** of the proportion, while the numbers $b$ and $c$ are called the **means** of the proportion.

extreme → $a$ ... $c$ ← mean

mean → $b$ ... $d$ ← extreme

$$\frac{a}{b} = \frac{c}{d}$$

The product of the extremes $ad$ and the product of the means $bc$ are called **cross products**.

### KeyConcept Cross Products Property

| | |
|---|---|
| Words | In a proportion, the product of the extremes equals the product of the means. |
| Symbols | If $\frac{a}{b} = \frac{c}{d}$ when $b \neq 0$ and $d \neq 0$, then $ad = bc$. |
| Example | If $\frac{4}{10} = \frac{6}{15}$, then $4 \cdot 15 = 10 \cdot 6$. |

You will prove the Cross Products Property in Exercise 41.

The converse of the Cross Products Property is also true. If $ad = bc$ and $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} = \frac{c}{d}$. That is, $\frac{a}{b}$ and $\frac{c}{d}$ form a proportion. You can use the Cross Products Property to solve a proportion.

## Example 3 Use Cross Products to Solve Proportions

**Solve each proportion.**

**a.** $\frac{6}{x} = \frac{21}{31.5}$

| | |
|---|---|
| $\frac{6}{x} = \frac{21}{31.5}$ | Original proportion |
| $6(31.5) = x(21)$ | Cross Products Property |
| $189 = 21x$ | Simplify. |
| $9 = x$ | Solve for $x$. |

**b.** $\frac{x+3}{2} = \frac{4x}{5}$

$$\frac{x+3}{2} = \frac{4x}{5}$$
$$(x+3)5 = 2(4x)$$
$$5x + 15 = 8x$$
$$15 = 3x$$
$$5 = x$$

### Study Tip

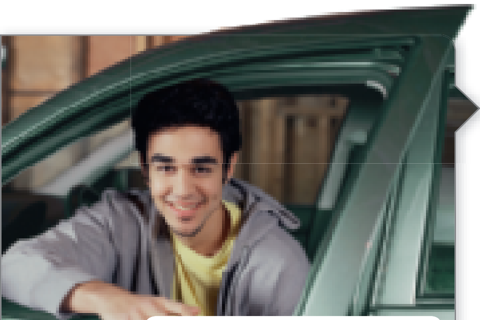
**CCSS Perseverance** Example 3b could also be solved by multiplying each side of the equation by 10, the least common denominator.

$$10\left(\frac{x+3}{2}\right) = \frac{4x}{5}(10)$$
$$5(x+3) = 2(4x)$$
$$5x + 15 = 8x$$
$$15 = 3x$$
$$5 = x$$

### Guided Practice

**3A.** $\frac{x}{4} = \frac{11}{-6}$ **3B.** $\frac{-4}{7} = \frac{6}{2y+5}$ **3C.** $\frac{7}{z-1} = \frac{9}{z+4}$

Proportions can be used to make predictions.



Ocean/CORBIS

### Real-World Link

The percent of driving-age teens (ages 15 to 20) with their own vehicles nearly doubled nationwide from 22 percent in 1985 to 42 percent in 2003.

**Source:** CNW Marketing Research

## Real-World Example 4 Use Proportions to Make Predictions

**CAR OWNERSHIP** **Fernando conducted a survey of 50 students driving to school and found that 28 owned cars. If 755 students drive to his school, predict the total number of students who own cars.**

Write and solve a proportion that compares the number of students who own cars to the number who drive to school.

| | |
|---|---|
| $\frac{28}{50} = \frac{x}{755}$ | ← students owning cars / ← students driving to school |
| $28 \cdot 755 = 50 \cdot x$ | Cross Products Property |
| $21{,}140 = 50x$ | Simplify. |
| $422.8 = x$ | Divide each side by 50. |

Based on Fernando's survey, about 423 students at his school own cars.

### Guided Practice

**4. BIOLOGY** In an experiment, students netted butterflies, recorded the number with tags on their wings, and then released them. The students netted 48 butterflies and 3 of those had tagged wings. Predict the number of butterflies that would have tagged wings out of 100 netted.

The proportion shown in Example 4 is not the only correct proportion for that situation. Equivalent forms of a proportion all have identical cross products.

### Key Concept Equivalent Proportions

**Symbols** The following proportions are equivalent.

$$\frac{a}{b} = \frac{c}{d}, \quad \frac{b}{a} = \frac{d}{c}, \quad \frac{a}{c} = \frac{b}{d}, \quad \frac{c}{a} = \frac{d}{b}$$

**Examples** $\frac{28}{50} = \frac{x}{755}$, $\frac{50}{28} = \frac{755}{x}$, $\frac{28}{x} = \frac{50}{755}$, $\frac{x}{28} = \frac{755}{50}$.

## Check Your Understanding

= Step-by-Step Solutions begin on page R14.

**Example 1**

1. **PETS** Out of a survey of 1000 households, 460 had at least one dog or cat as a pet. What is the ratio of pet owners to households?

2. **SPORTS** Thirty girls tried out for 15 spots on the basketball team. What is the ratio of open spots to the number of girls competing?

**Example 2**

3. The ratio of the measures of three sides of a triangle is 2:5:4, and its perimeter is 165 units. Find the measure of each side of the triangle.

4. The ratios of the measures of three angles of a triangle are 4:6:8. Find the measure of each angle of the triangle.

**Example 3**

**Solve each proportion.**

5. $\frac{2}{3} = \frac{x}{24}$

6. $\frac{x}{5} = \frac{28}{100}$

7. $\frac{2.2}{x} = \frac{26.4}{96}$

8. $\frac{x-3}{3} = \frac{5}{8}$

**Example 4**

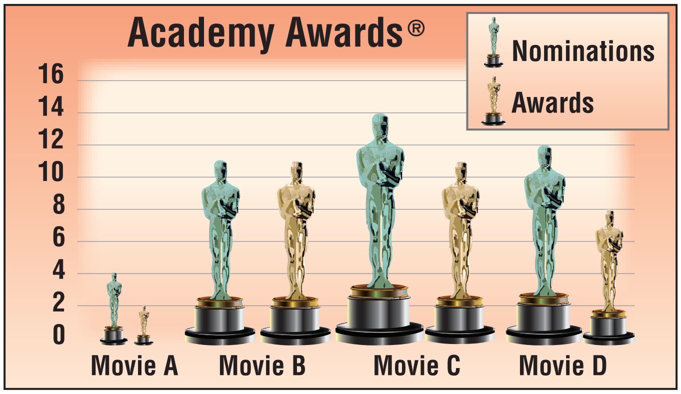
9. CCSS **MODELING** Ella is baking apple muffins for the Student Council bake sale. The recipe that she is using calls for 2 eggs per dozen muffins, and she needs to make 108 muffins. How many eggs will she need?

## Practice and Problem Solving

Extra Practice is on page R7.

**Example 1**

**MOVIES** **For Exercises 10 and 11, refer to the graphic below.**




10. Of the films listed, which had the greatest ratio of Academy Awards to number of nominations?

11. Which film listed had the lowest ratio of awards to nominations?

**Example 2**

12. **GAMES** A video game store has 60 games to choose from, including 40 sports games. What is the ratio of sports games to video games?

13. The ratio of the measures of the three sides of a triangle is 9 : 7 : 5. Its perimeter is 191.1 inches. Find the measure of each side.

14. The ratio of the measures of the three sides of a triangle is 3:7:5, and its perimeter is 156.8 meters. Find the measure of each side.

15. The ratio of the measures of the three sides of a triangle is $\frac{1}{4}:\frac{1}{8}:\frac{1}{6}$. Its perimeter is 4.75 feet. Find the length of the longest side.

16. The ratio of the measures of the three sides of a triangle is $\frac{1}{4}:\frac{1}{3}:\frac{1}{6}$, and its perimeter is 31.5 centimeters. Find the length of the shortest side.

**Find the measures of the angles of each triangle.**

17. The ratio of the measures of the three angles is 3:6:1.

18. The ratio of the measures of the three angles is 7:5:8.

19. The ratio of the measures of the three angles is 10:8:6.

20. The ratio of the measures of the three angles is 5:4:7.

**Example 3** **Solve each proportion.**

21. $\frac{5}{8} = \frac{y}{3}$ 22. $\frac{w}{6.4} = \frac{1}{2}$ 23. $\frac{4x}{24} = \frac{56}{112}$ 24. $\frac{11}{20} = \frac{55}{20x}$

25. $\frac{2x + 5}{10} = \frac{42}{20}$ 26. $\frac{a + 2}{a - 2} = \frac{3}{2}$ 27. $\frac{3x - 1}{4} = \frac{2x + 4}{5}$ 28. $\frac{3x - 6}{2} = \frac{4x - 2}{4}$

**Example 4** 29. **NUTRITION** According to a recent study, 7 out of every 500 Americans aged 13 to 17 years are vegetarian. In a group of 350 13- to 17-year-olds, about how many would you expect to be vegetarian?

30. **CURRENCY** Your family is traveling to Mexico on vacation. You have saved \$500 to use for spending money. If 269 Mexican pesos is equivalent to 25 United States dollars, how much money will you get when you exchange your \$500 for pesos?

**ALGEBRA** **Solve each proportion. Round to the nearest tenth.**

31. $\frac{2x + 3}{3} = \frac{6}{x - 1}$ 32. $\frac{x^2 + 4x + 4}{40} = \frac{x + 2}{10}$ 33. $\frac{9x + 6}{18} = \frac{20x + 4}{3x}$

34. The perimeter of a rectangle is 98 feet. The ratio of its length to its width is 5:2. Find the area of the rectangle.

35. The perimeter of a rectangle is 220 inches. The ratio of its length to its width is 7:3. Find the area of the rectangle.

36. The ratio of the measures of the side lengths of a quadrilateral is 2:3:5:4. Its perimeter is 154 feet. Find the length of the shortest side.

37. The ratio of the measures of the angles of a quadrilateral is 2:4:6:3. Find the measures of the angles of the quadrilateral.

38. **SUMMER JOBS** In June of 2000, 60.2% of American teens 16 to 19 years old had summer jobs. By June of 2006, 51.6% of teens in that age group were a part of the summer work force.

    a. Has the number of 16- to 19-year-olds with summer jobs increased or decreased since 2000? Explain your reasoning.

    b. In June 2006, how many 16- to 19-year-olds would you expect to have jobs out of 700 in that age group? Explain your reasoning.

39. CCSS **MODELING** In a golden rectangle, the ratio of the length to the width is about 1.618. This is known as the *golden ratio*.

    a. Recall from page 461 that a standard television screen has an aspect ratio of 4:3, while a high-definition television screen has an aspect ratio of 16:9. Is either type of screen a golden rectangle? Explain.

    b. The golden ratio can also be used to determine column layouts for Web pages. Consider a site with two columns, the left for content and the right as a sidebar. The ratio of the left to right column widths is the golden ratio. Determine the width of each column if the page is 960 pixels wide.

40. **SCHOOL ACTIVITIES** A survey of club involvement showed that, of the 36 students surveyed, the ratio of French Club members to Spanish Club members to Drama Club members was 2:3:7. How many of those surveyed participate in Spanish Club? Assume that each student is active in only one club.

**41. PROOF** Write an algebraic proof of the Cross Products Property.

**42. SPORTS** Jane jogs the same path every day in the winter to stay in shape for track season. She runs at a constant rate, and she spends a total of 39 minutes jogging. If the ratio of the times of the four legs of the jog is 3:5:1:4, how long does the second leg of the jog take her?

**43. MULTIPLE REPRESENTATIONS** In this problem, you will explore proportional relationships in triangles.

**a. Geometric** Draw an isosceles triangle *ABC*. Measure and label the legs and the vertex angle. Draw a second triangle *MNO* with a congruent vertex angle and legs twice as long as *ABC*. Draw a third triangle *PQR* with a congruent vertex angle and legs half as long as *ABC*.

**b. Tabular** Copy and complete the table below using the appropriate measures.

| Triangle | *ABC* | *MNO* | *PQR* |
|---|---|---|---|
| Leg length | | | |
| Perimeter | | | |

**c. Verbal** Make a conjecture about the change in the perimeter of an isosceles triangle if the vertex angle is held constant and the leg length is increased or decreased by a factor.

## H.O.T. Problems Use Higher-Order Thinking Skills

**44. ERROR ANALYSIS** Mollie and Eva have solved the proportion $\frac{x-3}{4} = \frac{1}{2}$. Is either of them correct? Explain your reasoning.

| Mollie | Eva |
|---|---|
| $(x-3)1 = 4(2)$ | $x - 3(2) = 4(1)$ |
| $x - 3 = 8$ | $x - 3 = 4$ |
| $x = 11$ | $x = 7$ |

**45. CHALLENGE** The dimensions of a rectangle are $y$ and $y^2 + 1$ and the perimeter of the rectangle is 14 units. Find the ratio of the longer side of the rectangle to the shorter side of the rectangle.

**46. CCSS REASONING** The ratio of the lengths of the diagonals of a quadrilateral is 1:1. The ratio of the lengths of the consecutive sides of the quadrilateral is 3:4:3:5. Classify the quadrilateral. Explain.

**47. WHICH ONE DOESN'T BELONG?** Identify the proportion that does not belong with the other three. Explain your reasoning.

$\frac{3}{8} = \frac{8.4}{22.4}$ $\frac{2}{3} = \frac{5}{7.5}$ $\frac{5}{6} = \frac{14}{16.8}$ $\frac{7}{9} = \frac{19.6}{25.2}$

**48. OPEN ENDED** Write four ratios that are equivalent to the ratio 2:5. Explain why all of the ratios are equivalent.

**49. WRITING IN MATH** Compare and contrast a ratio and a proportion. Explain how you use both to solve a problem.

## Standardized Test Practice

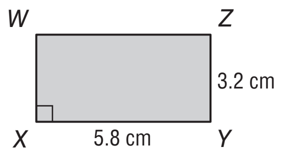
**50.** Solve the following proportion.

$$\frac{x}{-8} = \frac{12}{6}$$

**A** $-12$ **C** $-16$

**B** $-14$ **D** $-18$

**51.** What is the area of rectangle *WXYZ*?




**F** 18.6 $cm^2$ **H** 21.2 $cm^2$

**G** 20.4 $cm^2$ **J** 22.8 $cm^2$

**52. GRIDDED RESPONSE** Mrs. Sullivan's rectangular bedroom measures 12 feet by 10 feet. She wants to purchase carpet for the bedroom that costs \$2.56 per square foot, including tax. How much will it cost in dollars to carpet her bedroom?

**53. SAT/ACT** Kamilah has 5 more than 4 times the number of DVDs that Mercedes has. If Mercedes has $x$ DVDs, then in terms of $x$, how many DVDs does Kamilah have?

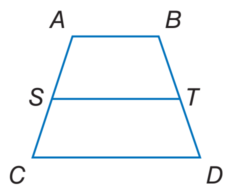
**A** $4(x + 5)$ **D** $4x + 5$

**B** $4(x + 3)$ **E** $5x + 4$

**C** $9x$

## Spiral Review

**For trapezoid *ABCD*, *S* and *T* are midpoints of the legs.** (Lesson 6-6)

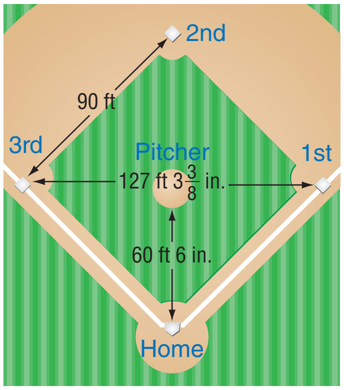



**54.** If $CD = 14$, $ST = 10$, and $AB = 2x$, find $x$.

**55.** If $AB = 3x$, $ST = 15$, and $CD = 9x$, find $x$.

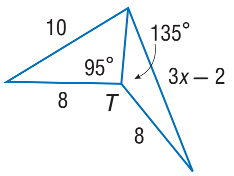
**56.** If $AB = x + 4$, $CD = 3x + 2$, and $ST = 9$, find $AB$.

**57. SPORTS** The infield of a baseball diamond is a square, as shown at the right. Is the pitcher's mound located in the center of the infield? Explain. (Lesson 6-5)

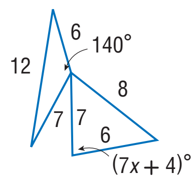



**Write an inequality for the range of values for $x$.** (Lesson 5-6)

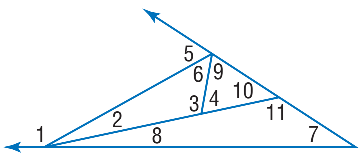
**58.**




**59.**




**Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.** (Lesson 5-3)




**60.** measures less than $m\angle 5$

**61.** measures greater than $m\angle 6$

**62.** measures greater than $m\angle 10$

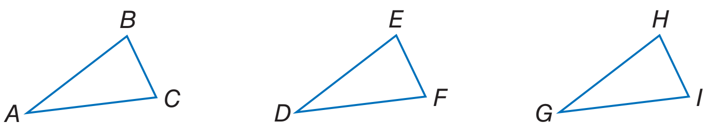
**63.** measures less than $m\angle 11$

**64. REASONING** Find a counterexample for the following statement. (Lesson 3-5)
*If lines p and m are cut by transversal t so that consecutive interior angles are congruent, then lines p and m are parallel and t is perpendicular to both lines.*

## Skills Review

**Write a paragraph proof.**

**65. Given:** $\triangle ABC \cong \triangle DEF$; $\triangle DEF \cong \triangle GHI$
**Prove:** $\triangle ABC \cong \triangle GHI$



EXTEND 7-1

## Graphing Technology Lab

# Fibonacci Sequence and Ratios

Leonardo Pisano (c. 1170–c. 1250), or Fibonacci, was born in Italy but educated in North Africa. As a result, his work is similar to that of other North African authors of that time. His book *Liber abaci*, published in 1202, introduced what is now called the Fibonacci sequence, in which each term after the first two terms is the sum of the two numbers before it.

| Term | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Fibonacci Number | 1 | 1 | 2 | 3 | 5 | 8 | 13 |
| | | | ↑ 1 + 1 | ↑ 1 + 2 | ↑ 2 + 3 | ↑ 3 + 5 | ↑ 5 + 8 |

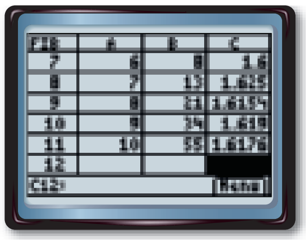
## Activity

**You can use CellSheet on a TI-83/84 Plus graphing calculator to calculate terms of the Fibonacci sequence. Then compare each term with its preceding term.**

**Step 1** Access the **CellSheet** application by pressing the APPS key. Choose the number for **CellSheet** and press ENTER.

**Step 2** Enter the column headings in row 1. Use the **ALPHA** key to enter letters and press **[ " ]** at the beginning of each label.

**Step 3** Enter **1** into cell **A2.** Then insert the formula **=A2+1** in cell **A3.** Press STO to insert the = in the formula. Then use F3 to copy this formula and use F4 to paste it in each cell in the column. This will automatically calculate the number of the term.



**Step 4** In **column B,** we will record the Fibonacci numbers. Enter 1 in cells **B2** and **B3** since you do not have two previous terms to add. Then insert the formula **=B2+B3** in cell **B4.** Copy this formula down the column.

**Step 5** In **column C,** we will find the ratio of each term to its preceding term. Enter 1 in cell **C2** since there is no preceding term. Then enter B3/B2 in cell **C3.** Copy this formula down the column. *The screens show the results for terms 1 through 11.*

### Analyze the Results

1. What happens to the Fibonacci number as the number of the term increases?
2. What pattern of odd and even numbers do you notice in the Fibonacci sequence?
3. As the number of terms gets greater, what pattern do you notice in the ratio column?
4. Extend the spreadsheet to calculate fifty terms of the Fibonacci sequence. Describe any differences in the patterns you described in Exercises 1–3.
5. **MAKE A CONJECTURE** How might the Fibonacci sequence relate to the golden ratio?

# LESSON 7-2 Similar Polygons

**Then**

- You used proportions to solve problems.

**Now**

1. Use proportions to identify similar polygons.
2. Solve problems using the properties of similar polygons.

**Why?**

- People often customize their computer desktops using photos, centering the images at their original size or stretching them to fit the screen. This second method distorts the image, because the original and new images are not geometrically similar.

**NewVocabulary**
similar polygons
scale factor

**Common Core State Standards**

**Content Standards**
G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**Mathematical Practices**
7 Look for and make use of structure.
3 Construct viable arguments and critique the reasoning of others.

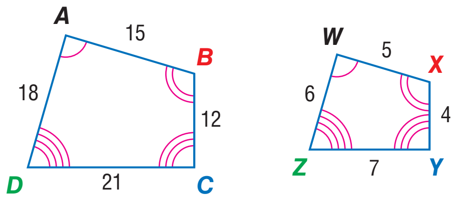
## 1 Identify Similar Polygons

**Similar polygons** have the same shape but not necessarily the same size.

### KeyConcept Similar Polygons

Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

**Example** In the diagram below, *ABCD* is similar to *WXYZ*.




Corresponding angles

$\angle A \cong \angle W$, $\angle B \cong \angle X$, $\angle C \cong \angle Y$, and $\angle D \cong \angle Z$

Corresponding sides

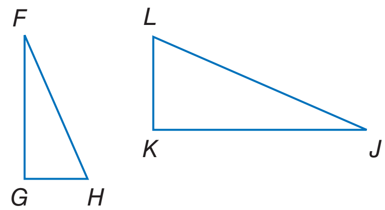
$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{3}{1}$

**Symbols** $ABCD \sim WXYZ$

As with congruence statements, the order of vertices in a similarity statement like $ABCD \sim WXYZ$ is important. It identifies the corresponding angles and sides.

### Example 1 Use a Similarity Statement

**If $\triangle FGH \sim \triangle JKL$, list all pairs of congruent angles, and write a proportion that relates the corresponding sides.**




Use the similarity statement.

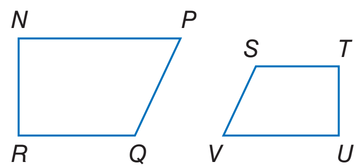
$\triangle FGH \sim \triangle JKL$

Congruent angles: $\angle F \cong \angle J$, $\angle G \cong \angle K$, $\angle H \cong \angle L$

Proportion: $\frac{FG}{JK} = \frac{GH}{KL} = \frac{HF}{LJ}$

**GuidedPractice**

**1.** In the diagram, $NPQR \sim UVST$. List all pairs of congruent angles, and write a proportion that relates the corresponding sides.



**StudyTip**

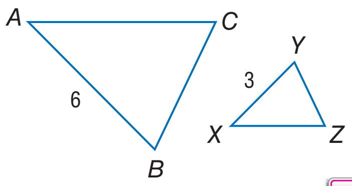
**Similarity Ratio** The scale factor between two similar polygons is sometimes called the *similarity ratio*.

The ratio of the lengths of the corresponding sides of two similar polygons is called the **scale factor**. The scale factor depends on the order of comparison.

In the diagram, $\triangle ABC \sim \triangle XYZ$.

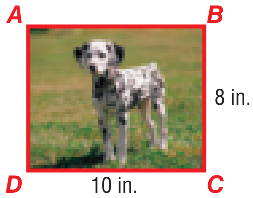
The scale factor of $\triangle ABC$ to $\triangle XYZ$ is $\frac{6}{3}$ or 2.

The scale factor of $\triangle XYZ$ to $\triangle ABC$ is $\frac{3}{6}$ or $\frac{1}{2}$.

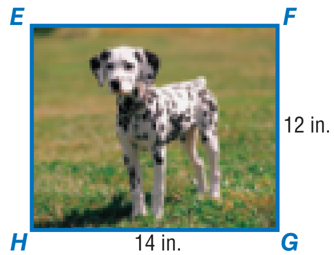



## Real-World Example 2 Identify Similar Polygons

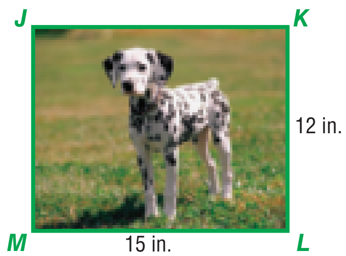
**PHOTO EDITING** **Kuma wants to use the rectangular photo shown as the background for her computer's desktop, but she needs to resize it. Determine whether the following rectangular images are similar. If so, write the similarity statement and scale factor. Explain your reasoning.**




**a.**




**b.**




**a. Step 1** Compare corresponding angles.

Since all angles of a rectangle are right angles and right angles are congruent, corresponding angles are congruent.

**Step 2** Compare corresponding sides.

$\frac{DC}{HG} = \frac{10}{14}$ or $\frac{5}{7}$ $\qquad$ $\frac{BC}{FG} = \frac{8}{12}$ or $\frac{2}{3}$ $\qquad$ $\frac{5}{7} \neq \frac{2}{3}$

Since corresponding sides are not proportional, $ABCD \nsim EFGH$. So the photos are not similar.

**b. Step 1** Since $ABCD$ and $JKLM$ are both rectangles, corresponding angles are congruent.

**Step 2** Compare corresponding sides.

$\frac{DC}{ML} = \frac{10}{15}$ or $\frac{2}{3}$ $\qquad$ $\frac{BC}{KL} = \frac{8}{12}$ or $\frac{2}{3}$ $\qquad$ $\frac{2}{3} = \frac{2}{3}$

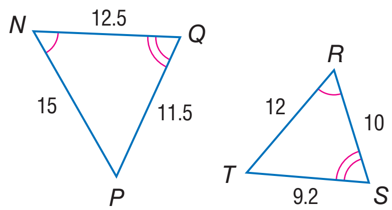
Since corresponding sides are proportional, $ABCD \sim JKLM$. So the rectangles are similar with a scale factor of $\frac{2}{3}$.

**ReadingMath**

**Similarity Symbol** The symbol $\nsim$ is read as *is not similar to*.

### GuidedPractice

**2.** Determine whether the triangles shown are similar. If so, write the similarity statement and scale factor. Explain your reasoning.




Stockbyte/Getty Images

**StudyTip**

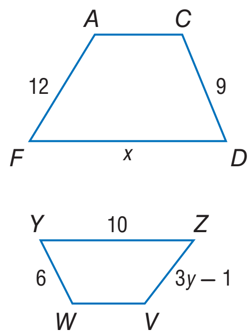
**Similarity and Congruence** If two polygons are congruent, they are also similar. All of the corresponding angles are congruent, and the lengths of the corresponding sides have a ratio of 1:1.

## 2 Use Similar Figures

You can use scale factors and proportions to solve problems involving similar figures.

### Example 3 Use Similar Figures to Find Missing Measures

**In the diagram, $ACDF \sim VWYZ$.**




**a. Find $x$.**

Use the corresponding side lengths to write a proportion.

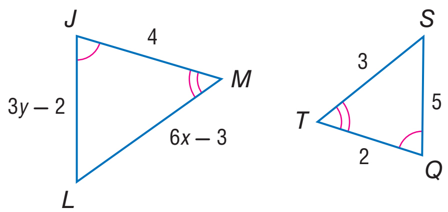
| | |
|---|---|
| $\frac{CD}{WY} = \frac{DF}{YZ}$ | Similarity proportion |
| $\frac{9}{6} = \frac{x}{10}$ | $CD = 9$, $WY = 6$, $DF = x$, $YZ = 10$ |
| $9(10) = 6(x)$ | Cross Products Property |
| $90 = 6x$ | Multiply. |
| $15 = x$ | Divide each side by 6. |

**b. Find $y$.**

| | |
|---|---|
| $\frac{CD}{WY} = \frac{FA}{ZV}$ | Similarity proportion |
| $\frac{9}{6} = \frac{12}{3y - 1}$ | $CD = 9$, $WY = 6$, $FA = 12$, $ZV = 3y - 1$ |
| $9(3y - 1) = 6(12)$ | Cross Products Property |
| $27y - 9 = 72$ | Multiply. |
| $27y = 81$ | Add 9 to each side. |
| $y = 3$ | Divide each side by 27. |

**GuidedPractice**

**Find the value of each variable if $\triangle JLM \sim \triangle QST$.**




**3A.** $x$

**3B.** $y$

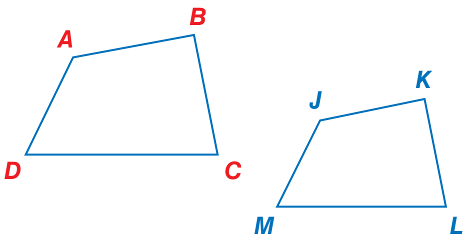
**StudyTip**

**Identifying Similar Triangles** When only two congruent angles of a triangle are given, remember that you can use the Third Angles Theorem to establish that the remaining corresponding angles are also congruent.

In similar polygons, the ratio of any two corresponding lengths is proportional to the scale factor between them. This leads to the following theorem about the perimeters of two similar polygons.

### Theorem 7.1 Perimeters of Similar Polygons

If two polygons are similar, then their perimeters are proportional to the scale factor between them.

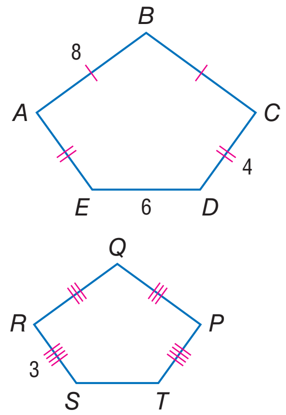



**Example** If $ABCD \sim JKLM$, then

$$\frac{AB + BC + CD + DA}{JK + KL + LM + MJ} = \frac{AB}{JK} = \frac{BC}{KL} = \frac{CD}{LM} = \frac{DA}{MJ}.$$

## Example 4 Use a Scale Factor to Find Perimeter

**If $ABCDE \sim PQRST$, find the scale factor of $ABCDE$ to $PQRST$ and the perimeter of each polygon.**




The scale factor of $ABCDE$ to $PQRST$ is $\frac{CD}{RS}$ or $\frac{4}{3}$.

Since $\overline{BC} \cong \overline{AB}$ and $\overline{AE} \cong \overline{CD}$, the perimeter of $ABCDE$ is $8 + 8 + 4 + 6 + 4$ or 30.

Use the perimeter of $ABCDE$ and the scale factor to write a proportion. Let $x$ represent the perimeter of $PQRST$.

$\frac{4}{3} = \frac{\text{perimeter of } ABCDE}{\text{perimeter of } PQRST}$ Theorem 7.1

$\frac{4}{3} = \frac{30}{x}$ Substitution

$(3)(30) = 4x$ Cross Products Property

$22.5 = x$ Solve.

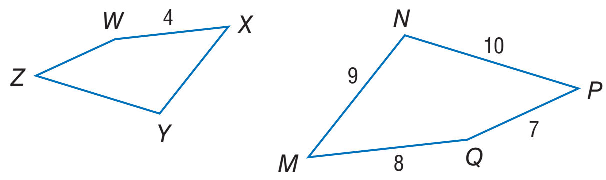
So, the perimeter of $PQRST$ is 22.5.

**WatchOut!**

**Perimeter** Remember that perimeter is the distance around a figure. Be sure to find the sum of all side lengths when finding the perimeter of a polygon. You may need to use other markings or geometric principles to find the length of unmarked sides.

### GuidedPractice

**4.** If $MNPQ \sim XYZW$, find the scale factor of $MNPQ$ to $XYZW$ and the perimeter of each polygon.

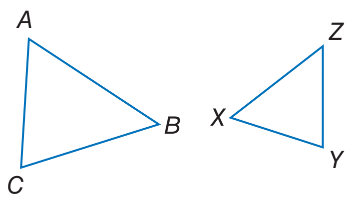



## Check Your Understanding

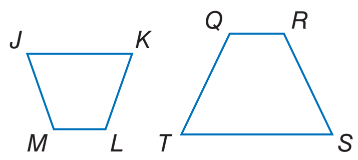
= Step-by-Step Solutions begin on page R14.

**Example 1** **List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.**

**1.** $\triangle ABC \sim \triangle ZYX$

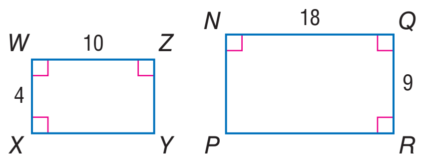



**2.** $JKLM \sim TSRQ$

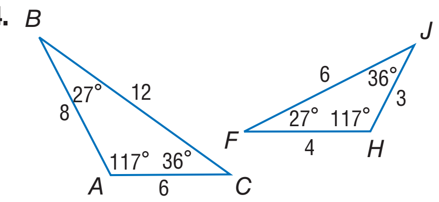



**Example 2** **Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.**

**3.**

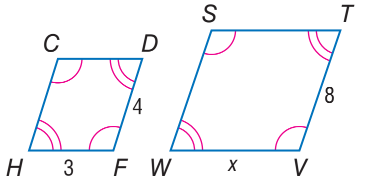



**4.**

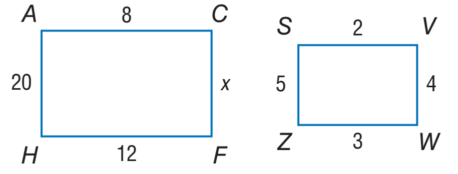


**Example 3** **Each pair of polygons is similar. Find the value of $x$.**

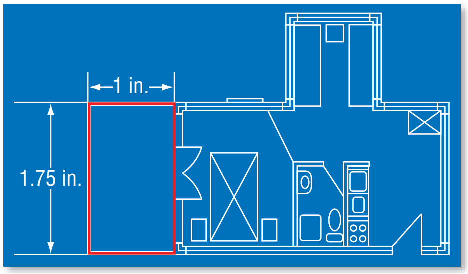
**5.**




**6.**




**Example 4** **7. DESIGN** On the blueprint of the apartment shown, the balcony measures 1 inch wide by 1.75 inches long. If the actual length of the balcony is 7 feet, what is the perimeter of the balcony?

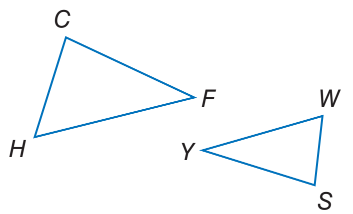



## Practice and Problem Solving

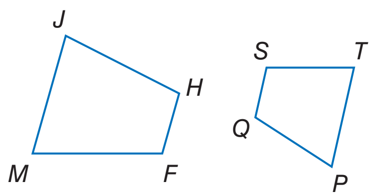
Extra Practice is on page R7.

**Example 1** **List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.**

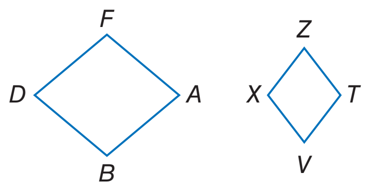
**8.** $\triangle CHF \sim \triangle YWS$



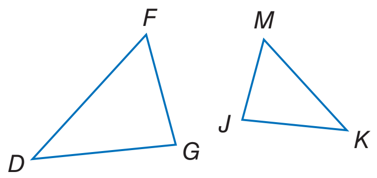

**9.** $JHFM \sim PQST$




**10.** $ABDF \sim VXZT$

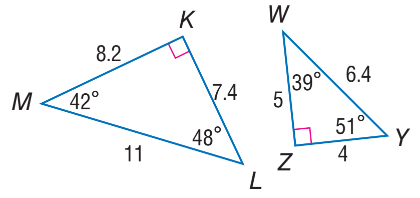



**11.** $\triangle DFG \sim \triangle KMJ$

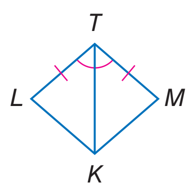



**Example 2** **CCSS ARGUMENTS** **Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.**

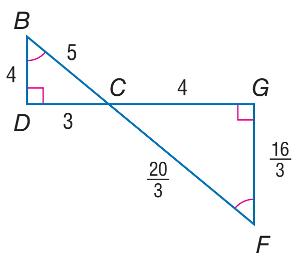
**12.**



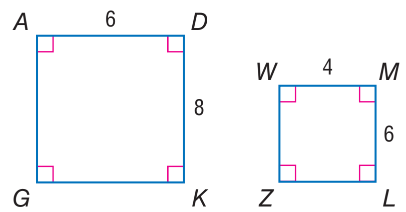

**13**



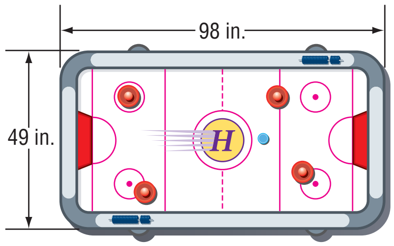

**14.**




**15.**



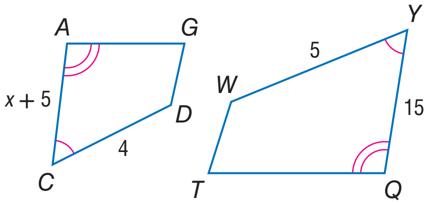
**16.** **GAMES** The dimensions of a hockey rink are 200 feet by 85 feet. Are the hockey rink and the air hockey table shown similar? Explain your reasoning.



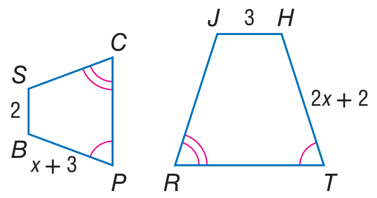

**17.** **COMPUTERS** The dimensions of a 17-inch flat panel computer screen are approximately $13\frac{1}{4}$ by $10\frac{3}{4}$ inches. The dimensions of a 19-inch flat panel computer screen are approximately $14\frac{1}{2}$ by 12 inches. To the nearest tenth, are the computer screens similar? Explain your reasoning.

**Example 3** CCSS **REGULARITY** **Each pair of polygons is similar. Find the value of *x*.**

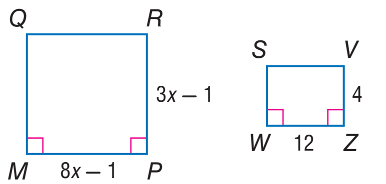
**18.**



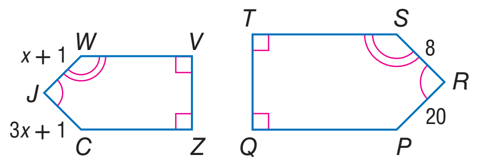

**19.**




**20.**



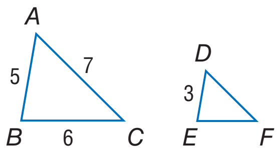

**21.**



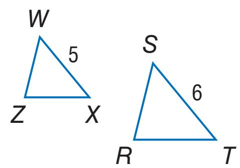

**Example 4** **22.** Rectangle *ABCD* has a width of 8 yards and a length of 20 yards. Rectangle *QRST*, which is similar to rectangle *ABCD*, has a length of 40 yards. Find the scale factor of rectangle *ABCD* to rectangle *QRST* and the perimeter of each rectangle.

**Find the perimeter of the given triangle.**

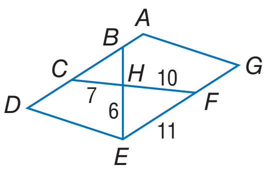
**23.** $\triangle DEF$, if $\triangle ABC \sim \triangle DEF$, $AB = 5$, $BC = 6$, $AC = 7$, and and $DE = 3$



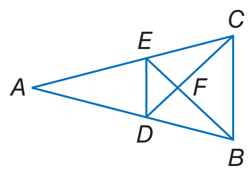

**24.** $\triangle WZX$, if $\triangle WZX \sim \triangle SRT$, $ST = 6$, $WX = 5$, and the perimeter of $\triangle SRT = 15$




**25.** $\triangle CBH$, if $\triangle CBH \sim \triangle FEH$, *ADEG* is a parallelogram, $CH = 7$, $FH = 10$, $FE = 11$, and $EH = 6$




**26.** $\triangle DEF$, if $\triangle DEF \sim \triangle CBF$, perimeter of $\triangle CBF = 27$, $DF = 6$, $FC = 8$

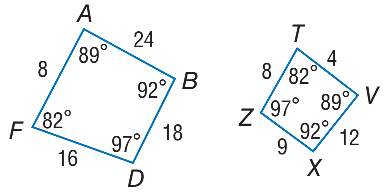



**27.** Two similar rectangles have a scale factor of 2:4. The perimeter of the large rectangle is 80 meters. Find the perimeter of the small rectangle.

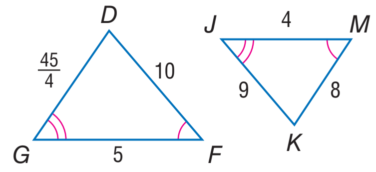
**28.** Two similar rectangles have a scale factor of 3:2. The perimeter of the small rectangle is 50 feet. Find the perimeter of the large rectangle.

**List all pairs of congruent angles, and write a proportion that relates the corresponding sides.**

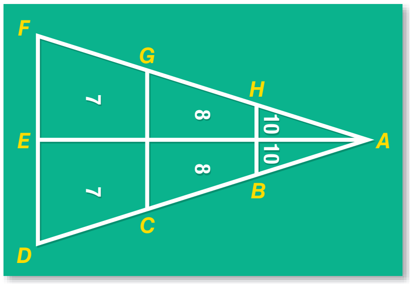
**29.**




**30.**




**SHUFFLEBOARD** **A shuffleboard court forms three similar triangles in which ∠*AHB* ≅ ∠*AGC* ≅ ∠*AFD*. For the given sides or angles, find the corresponding side(s) or angle(s) that are congruent.**

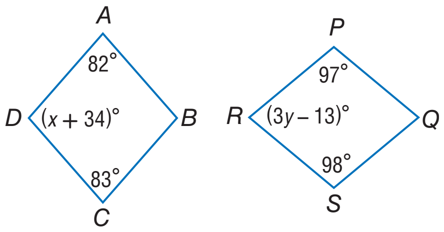



**31.** $\overline{AB}$ **32.** $\overline{FD}$

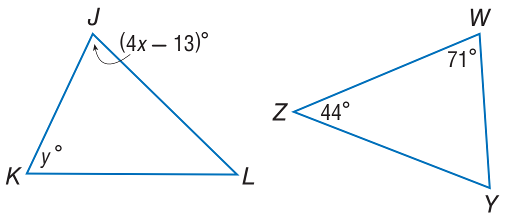
**33.** ∠*ACG* **34.** ∠*A*

**Find the value of each variable.**

**35** *ABCD* ~ *QSRP*




**36.** △*JKL* ~ △*WYZ*




**37.** **SLIDE SHOW** You are using a digital projector for a slide show. The photos are 13 inches by $9\frac{1}{4}$ inches on the computer screen, and the scale factor of the computer image to the projected image is 1:4. What are the dimensions of the projected image?

**COORDINATE GEOMETRY** **For the given vertices, determine whether rectangle *ABCD* is similar to rectangle *WXYZ*. Justify your answer.**

**38.** *A*(−1, 5), *B*(7, 5), *C*(7, −1), *D*(−1, −1);
*W*(−2, 10), *X*(14, 10), *Y*(14, −2), *Z*(−2, −2)

**39.** *A*(5, 5), *B*(0, 0), *C*(5, −5), *D*(10, 0);
*W*(1, 6), *X*(−3, 2), *Y*(2, −3), *Z*(6, 1)

**CCSS ARGUMENTS** **Determine whether the polygons are *always*, *sometimes*, or *never* similar. Explain your reasoning.**

**40.** two obtuse triangles

**41.** a trapezoid and a parallelogram

**42.** two right triangles

**43.** two isosceles triangles

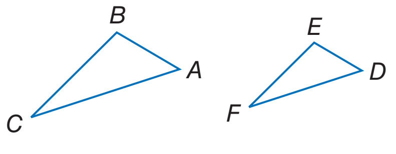
**44.** a scalene triangle and an isosceles triangle

**45.** two equilateral triangles

**46.** **PROOF** Write a paragraph proof of Theorem 7.1.

**Given:** △*ABC* ~ △*DEF* and $\frac{AB}{DE} = \frac{m}{n}$

**Prove:** $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{m}{n}$



**47** **PHOTOS** You are enlarging the photo shown at the right for your school yearbook. If the dimensions of the original photo are $2\frac{1}{3}$ inches by $1\frac{2}{3}$ inches and the scale factor of the old photo to the new photo is 2:3, what are the dimensions of the new photo?

**48.** **CHANGING DIMENSIONS** Rectangle *QRST* is similar to rectangle *JKLM* with sides in a ratio of 4:1.

**a.** What is the ratio of the areas of the two rectangles?

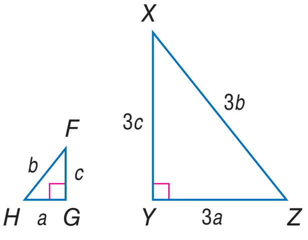
**b.** Suppose the dimension of each rectangle is tripled. What is the new ratio of the sides of the rectangles?

**c.** What is the ratio of the areas of these larger rectangles?

**49.** **CHANGING DIMENSIONS** In the figure shown, $\triangle FGH \sim \triangle XYZ$.

**a.** Show that the perimeters of $\triangle FGH$ and $\triangle XYZ$ have the same ratio as their corresponding sides.

**b.** If 6 units are added to the lengths of each side, are the new triangles similar? Explain.




**50.** **MULTIPLE REPRESENTATIONS** In this problem, you will investigate similarity in squares.

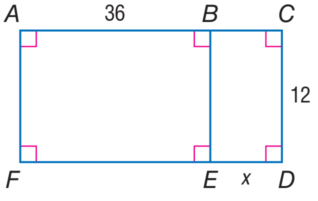
**a. Geometric** Draw three different-sized squares. Label them *ABCD*, *PQRS*, and *WXYZ*. Measure and label each square with its side length.

**b. Tabular** Calculate and record in a table the ratios of corresponding sides for each pair of squares: *ABCD* and *PQRS*, *PQRS* and *WXYZ*, and *WXYZ* and *ABCD*. Is each pair of squares similar?

**c. Verbal** Make a conjecture about the similarity of all squares.

## H.O.T. Problems Use Higher-Order Thinking Skills

**51.** **CHALLENGE** For what value(s) of $x$ is $BEFA \sim EDCB$?




**52.** **REASONING** Recall that an *equivalence relation* is any relationship that satisfies the Reflexive, Symmetric, and Transitive Properties. Is similarity an equivalence relation? Explain.

**53.** **OPEN ENDED** Find a counterexample for the following statement.

*All rectangles are similar.*

**54.** **CCSS REASONING** Draw two regular pentagons of different sizes. Are the pentagons similar? Will any two regular polygons with the same number of sides be similar? Explain.

**55.** **WRITING IN MATH** How can you describe the relationship between two figures?

## Standardized Test Practice

**56. ALGEBRA** If the arithmetic mean of $4x$, $3x$, and 12 is 18, then what is the value of $x$?

**A** 6 **C** 4
**B** 5 **D** 3

**57.** Two similar rectangles have a scale factor of 3:5. The perimeter of the large rectangle is 65 meters. What is the perimeter of the small rectangle?

**F** 29 m **H** 49 m
**G** 39 m **J** 59 m

**58. SHORT RESPONSE** If a jar contains 25 dimes and 7 quarters, what is the probability that a coin selected from the jar at random will be a dime?

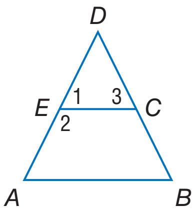
**59. SAT/ACT** If the side of a square is $x + 3$, then what is the diagonal of the square?

**A** $x^2 + 3$ **D** $x\sqrt{3} + 3\sqrt{3}$
**B** $3x + 3$ **E** $x\sqrt{2} + 3\sqrt{2}$
**C** $2x + 6$

## Spiral Review

**60. COMPUTERS** In a survey of 5000 households, 4200 had at least one computer. What is the ratio of computers to households? (Lesson 7-1)

**61. PROOF** **Write a flow proof.** (Lesson 6-6)




**Given:** $E$ and $C$ are midpoints of $\overline{AD}$ and $\overline{DB}$, $\overline{AD} \cong \overline{DB}$, $\angle A \cong \angle 1$.
**Prove:** $ABCE$ is an isosceles trapezoid.

**62. COORDINATE GEOMETRY** Determine the coordinates of the intersection of the diagonals of $\square JKLM$ with vertices $J(2, 5)$, $K(6, 6)$, $L(4, 0)$, and $M(0, -1)$. (Lesson 6-2)

**State the assumption you would make to start an indirect proof of each statement.** (Lesson 5-4)

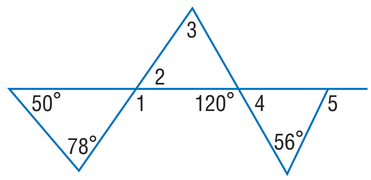
**63.** If $3x > 12$, then $x > 4$.

**64.** $\overline{PQ} \cong \overline{ST}$

**65.** The angle bisector of the vertex angle of an isosceles triangle is also an altitude of the triangle.

**66.** If a rational number is any number that can be expressed as $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$, then 6 is a rational number.

**Find the measures of each numbered angle.** (Lesson 4-2)




**67.** $m\angle 1$

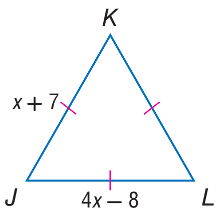
**68.** $m\angle 2$

**69.** $m\angle 3$

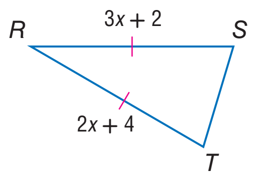
## Skills Review

**ALGEBRA** **Find $x$ and the unknown side measures of each triangle.**

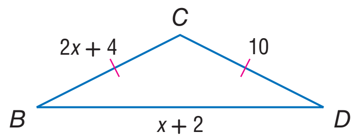
**70.**




**71.**




**72.**



LESSON 7-3

# Similar Triangles

## Then

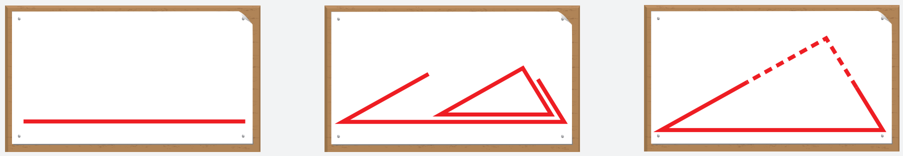
- You used the AAS, SSS, and SAS Congruence Theorems to prove triangles congruent.

## Now

- 1 Identify similar triangles using the AA Similarity Postulate and the SSS and SAS Similarity Theorems.
- 2 Use similar triangles to solve problems.

## Why?

- Julian wants to draw a similar version of his skate club's logo on a poster. He first draws a line at the bottom of the poster. Next, he uses a cutout of the original triangle to copy the two bottom angles. Finally, he extends the noncommon sides of the two angles.



**Common Core State Standards**

**Content Standards**

**G.SRT.4** Prove theorems about triangles.

**G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Mathematical Practices**

4 Model with mathematics.

7 Look for and make use of structure.

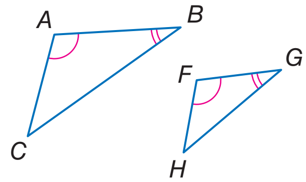
## 1 Identify Similar Triangles

The example suggests that two triangles are similar if two pairs of corresponding angles are congruent.

### Postulate 7.1 Angle-Angle (AA) Similarity

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

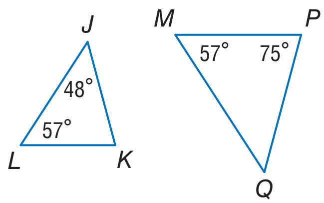
**Example** If $\angle A \cong \angle F$ and $\angle B \cong \angle G$, then $\triangle ABC \sim \triangle FGH$.



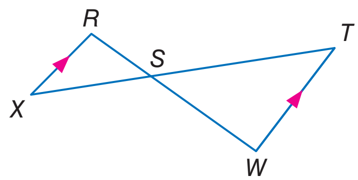

### Example 1 Use the AA Similarity Postulate

**Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.**

**a.**




**b.**

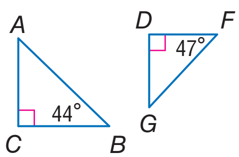



**a.** Since $m\angle L = m\angle M$, $\angle L \cong \angle M$. By the Triangle Sum Theorem, $57 + 48 + m\angle K = 180$, so $m\angle K = 75$. Since $m\angle P = 75$, $\angle K \cong \angle P$. So, $\triangle LJK \sim \triangle MQP$ by AA Similarity.

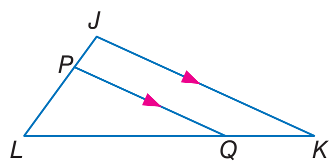
**b.** $\angle RSX \cong \angle WST$ by the Vertical Angles Theorem. Since $\overline{RX} \parallel \overline{TW}$, $\angle R \cong \angle W$. So, $\triangle RSX \sim \triangle WST$ by AA Similarity.

#### GuidedPractice

**1A.**




**1B.**

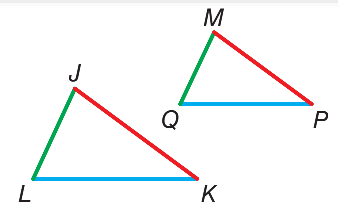


You can use the AA Similarity Postulate to prove the following two theorems.

## Theorems Triangle Similarity

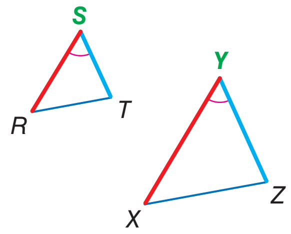
**7.2 Side-Side-Side (SSS) Similarity**
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

**Example** If $\frac{JK}{MP} = \frac{KL}{PQ} = \frac{LJ}{QM}$, then $\triangle JKL \sim \triangle MPQ$.




**7.3 Side-Angle-Side (SAS) Similarity**
If the lengths of two sides of one triangle are proportional to the lengths of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

**Example** If $\frac{RS}{XY} = \frac{ST}{YZ}$ and $\angle S \cong \angle Y$, then $\triangle RST \sim \triangle XYZ$.

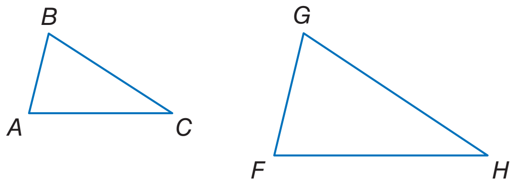



You will prove Theorem 7.3 in Exercise 25.

## Proof Theorem 7.2

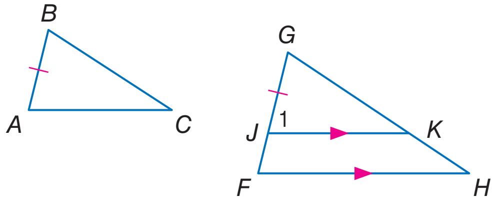
**Given:** $\frac{AB}{FG} = \frac{BC}{GH} = \frac{AC}{FH}$

**Prove:** $\triangle ABC \sim \triangle FGH$




**Paragraph Proof:**

Locate $J$ on $\overline{FG}$ so that $JG = AB$. Draw $\overline{JK}$ so that $\overline{JK} \parallel \overline{FH}$. Label $\angle GJK$ as $\angle 1$.




Since $\angle G \cong \angle G$ by the Reflexive Property and $\angle 1 \cong \angle F$ by the Corresponding Angles Postulate, $\triangle GJK \sim \triangle GFH$ by the AA Similarity Postulate.

By the definition of similar polygons, $\frac{JG}{FG} = \frac{GK}{GH} = \frac{JK}{FH}$. By substitution, $\frac{AB}{FG} = \frac{GK}{GH} = \frac{JK}{FH}$.

Since we are also given that $\frac{AB}{FG} = \frac{BC}{GH} = \frac{AC}{FH}$, we can say that $\frac{GK}{GH} = \frac{BC}{GH}$ and $\frac{JK}{FH} = \frac{AC}{FH}$. This means that $GK = BC$ and $JK = AC$, so $\overline{GK} \cong \overline{BC}$ and $\overline{JK} \cong \overline{AC}$.

By SSS, $\triangle ABC \cong \triangle JGK$.

By CPCTC, $\angle B \cong \angle G$ and $\angle A \cong \angle 1$. Since $\angle 1 \cong \angle F$, $\angle A \cong \angle F$ by the Transitive Property. By AA Similarity, $\triangle ABC \sim \triangle FGH$.

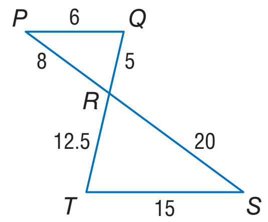
**StudyTip**

**Corresponding Sides** To determine which sides of two triangles correspond, begin by comparing the longest sides, then the next longest sides, and finish by comparing the shortest sides.

## Example 2 Use the SSS and SAS Similarity Theorems

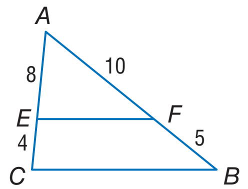
**Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.**

**a.**




$\frac{PR}{SR} = \frac{8}{20}$ or $\frac{2}{5}$, $\frac{PQ}{ST} = \frac{6}{15}$ or $\frac{2}{5}$, and $\frac{QR}{TR} = \frac{5}{12.5} = \frac{50}{125}$ or $\frac{2}{5}$. So, $\triangle PQR \sim \triangle STR$ by the SSS Similarity Theorem.

**b.**



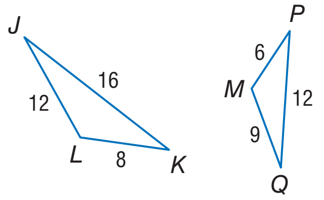

By the Reflexive Property, $\angle A \cong \angle A$.

$\frac{AF}{AB} = \frac{10}{10+5} = \frac{10}{15}$ or $\frac{2}{3}$ and $\frac{AE}{AC} = \frac{8}{8+4} = \frac{8}{12}$ or $\frac{2}{3}$.

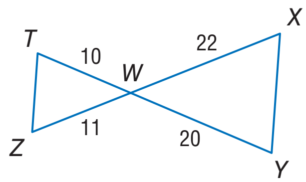
Since the lengths of the sides that include $\angle A$ are proportional, $\triangle AEF \sim \triangle ACB$ by the SAS Similarity Theorem.

### GuidedPractice

**2A.**




**2B.**

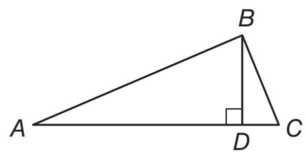



> **StudyTip**
>
> **Draw Diagrams** It is helpful to redraw similar triangles so that the corresponding side lengths have the same orientation.

You can decide what is sufficient to prove that two triangles are similar.

## Standardized Test Example 3 Sufficient Conditions

**In the figure, $\angle ADB$ is a right angle. Which of the following would *not* be sufficient to prove that $\triangle ADB \sim \triangle CDB$?**




**A** $\frac{AD}{BD} = \frac{BD}{CD}$

**B** $\frac{AB}{BC} = \frac{BD}{CD}$

**C** $\angle ABD \cong \angle C$

**D** $\frac{AD}{BD} = \frac{BD}{CD} = \frac{AB}{BC}$

### Read the Test Item

You are given that $\angle ADB$ is a right angle and asked to identify which additional information would not be enough to prove that $\triangle ADB \sim \triangle CDB$.

### Solve the Test Item

Since $\angle ADB$ is a right angle, $\angle CDB$ is also a right angle. Since all right angles are congruent, $\angle ADB \cong \angle CDB$. Check each answer choice until you find one that does not supply a sufficient additional condition to prove that $\triangle ADB \sim \triangle CDB$.

**Choice A:** If $\frac{AD}{BD} = \frac{BD}{CD}$ and $\angle ADB \cong \angle CDB$, then $\triangle ADB \sim \triangle CDB$ by SAS Similarity.

**Choice B:** If $\frac{AB}{BC} = \frac{BD}{CD}$ and $\angle ADB \cong \angle CDB$, then we cannot conclude that $\triangle ADB \sim \triangle CDB$ because the included angle of side $\overline{AB}$ and $\overline{BD}$ is not $\angle ADB$. So the answer is B.

> **Test-TakingTip**
>
> **Identifying Nonexamples** Sometimes test questions require you to find a nonexample, as in this case. You must check each option until you find a valid nonexample. If you would like to check your answer, confirm that each additional option is correct.

## GuidedPractice

**3.** If $\triangle JKL$ and $\triangle FGH$ are two triangles such that $\angle J \cong \angle F$, which of the following would be sufficient to prove that the triangles are similar?

**F** $\frac{KL}{GH} = \frac{JL}{FH}$  **G** $\frac{JL}{JK} = \frac{FH}{FG}$  **H** $\frac{JK}{FG} = \frac{KL}{GH}$  **J** $\frac{JL}{JK} = \frac{GH}{FG}$

## 2 Use Similar Triangles

Like the congruence of triangles, similarity of triangles is reflexive, symmetric, and transitive.

### Theorem 7.4 Properties of Similarity

**Reflexive Property of Similarity** $\triangle ABC \sim \triangle ABC$

**Symmetric Property of Similarity** If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.

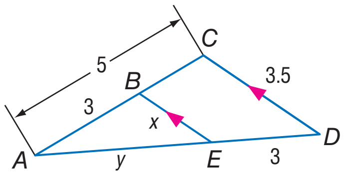
**Transitive Property of Similarity** If $\triangle ABC \sim \triangle DEF$, and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.

You will prove Theorem 7.4 in Exercise 26.

PT

### Example 4 Parts of Similar Triangles

**Find *BE* and *AD*.**




Since $\overline{BE} \parallel \overline{CD}$, $\angle ABE \cong \angle BCD$, and $\angle AEB \cong \angle EDC$ because they are corresponding angles. By AA Similarity, $\triangle ABE \sim \triangle ACD$.

| | |
|---|---|
| $\frac{AB}{AC} = \frac{BE}{CD}$ | Definition of Similar Polygons |
| $\frac{3}{5} = \frac{x}{3.5}$ | $AC = 5$, $CD = 3.5$, $AB = 3$, $BE = x$ |
| $3.5 \cdot 3 = 5 \cdot x$ | Cross Products Property |
| $2.1 = x$ | *BE* is 2.1. |
| $\frac{AC}{AB} = \frac{AD}{AE}$ | Definition of Similar Polygons |
| $\frac{5}{3} = \frac{y+3}{y}$ | $AC = 5$, $AB = 3$, $AD = y + 3$, $AE = y$ |
| $5 \cdot y = 3(y + 3)$ | Cross Products Property |
| $5y = 3y + 9$ | Distributive Property |
| $2y = 9$ | Subtract $3y$ from each side. |
| $y = 4.5$ | *AD* is $y + 3$ or 7.5. |

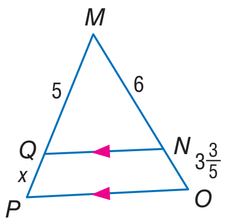
**StudyTip**

**Proportions** An additional proportion that is true for Example 4 is $\frac{AC}{CD} = \frac{AB}{BE}$.

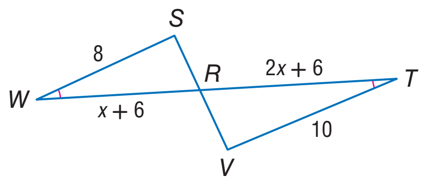
## GuidedPractice

**Find each measure.**

**4A.** *QP* and *MP*



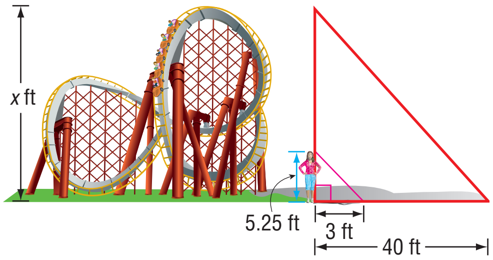

**4B.** *WR* and *RT*



## Real-World Example 5 Indirect Measurement

**ROLLER COASTERS** **Hallie is estimating the height of the Superman roller coaster in Mitchellville, Maryland. She is 5 feet 3 inches tall and her shadow is 3 feet long. If the length of the shadow of the roller coaster is 40 feet, how tall is the roller coaster?**

**Understand** Make a sketch of the situation. 5 feet 3 inches is equivalent to 5.25 feet.




**Plan** In shadow problems, you can assume that the angles formed by the Sun's rays with any two objects are congruent and that the two objects form the sides of two right triangles.

Since two pairs of angles are congruent, the right triangles are similar by the AA Similarity Postulate. So, the following proportion can be written.

$$\frac{\text{Hallie's height}}{\text{coaster's height}} = \frac{\text{Hallie's shadow length}}{\text{coaster's shadow length}}$$

**Solve** Substitute the known values and let $x$ = roller coaster's height.

$\frac{5.25}{x} = \frac{3}{40}$ Substitution

$3 \cdot x = 40(5.25)$ Cross Products Property

$3x = 210$ Simplify.

$x = 70$ Divide each side by 3.

The roller coaster is 70 feet tall.

**Check** The roller coaster's shadow length is $\frac{40 \text{ ft}}{3 \text{ ft}}$ or about 13.3 times Hallie's shadow length. Check to see that the roller coaster's height is about 13.3 times Hallie's height. $\frac{70 \text{ ft}}{5.25 \text{ ft}} \approx 13.3$ ✓

### Problem-Solving Tip

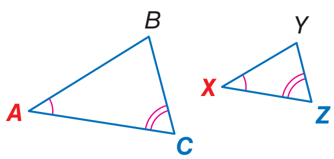
**Reasonable Answers** When you have solved a problem, check your answer for reasonableness. In this example, Hallie's shadow is a little more than half her height. The coaster's shadow is also a little more than half of the height you calculated. Therefore, the answer is reasonable.

### Guided Practice

5. **BUILDINGS** Adam is standing next to the Palmetto Building in Columbia, South Carolina. He is 6 feet tall and the length of his shadow is 9 feet. If the length of the shadow of the building is 322.5 feet, how tall is the building?

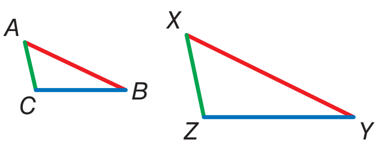
## Concept Summary Triangle Similarity

**AA Similarity Postulate**



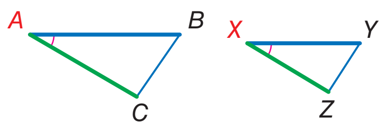

If $\angle A \cong \angle X$ and $\angle C \cong \angle Z$, then $\triangle ABC \sim \triangle XYZ$.

**SSS Similarity Theorem**




If $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

**SAS Similarity Theorem**



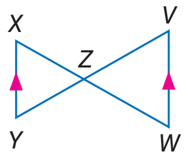

If $\angle A \cong \angle X$ and $\frac{AB}{XY} = \frac{CA}{ZX}$, then $\triangle ABC \sim \triangle XYZ$.

## Check Your Understanding

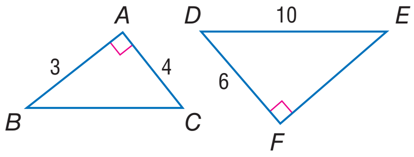
= Step-by-Step Solutions begin on page R14.

**Examples 1–2** **Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.**

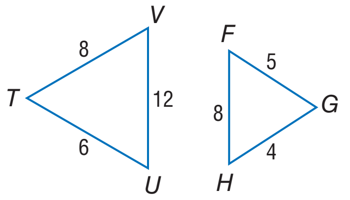
**1.**



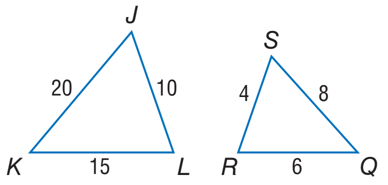

**2.**




**3.**




**4.**




**Example 3**

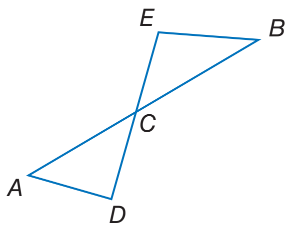
**5. MULTIPLE CHOICE** In the figure, $\overline{AB}$ intersects $\overline{DE}$ at point *C*. Which additional information would be enough to prove that $\triangle ADC \sim \triangle BEC$?

**A** $\angle DAC$ and $\angle ECB$ are congruent.

**B** $\overline{AC}$ and $\overline{BC}$ are congruent.

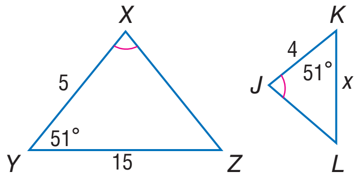
**C** $\overline{AD}$ and $\overline{EB}$ are parallel.

**D** $\angle CBE$ is a right angle.

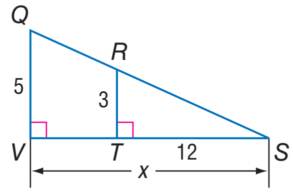



**Example 4** **CCSS STRUCTURE** **Identify the similar triangles. Find each measure.**

**6.** *KL*




**7.** *VS*




**Example 5**

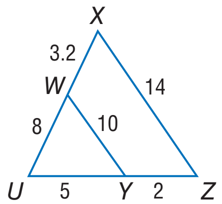
**8. COMMUNICATION** A cell phone tower casts a 100-foot shadow. At the same time, a 4-foot 6-inch post near the tower casts a shadow of 3 feet 4 inches. Find the height of the tower.

## Practice and Problem Solving

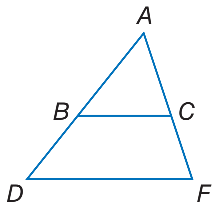
Extra Practice is on page R7.

**Examples 1–3** **Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.**

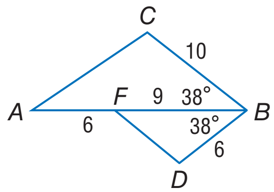
**9.**




**10.**

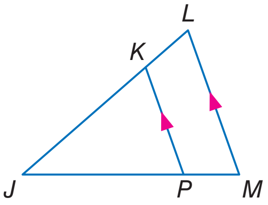



**11**

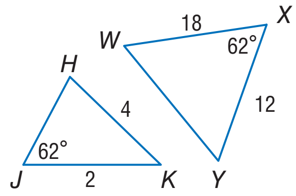


**Examples 1–3** **Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.**

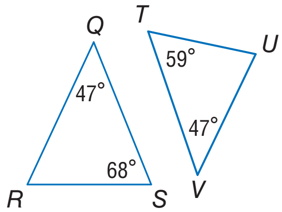
**12.**



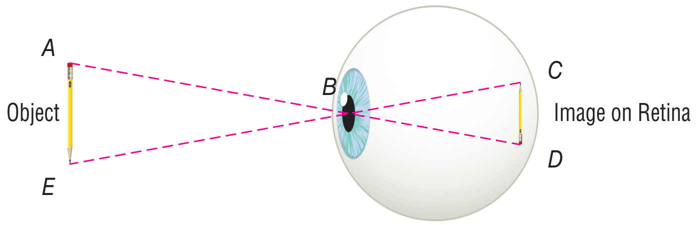

**13.**




**14.**

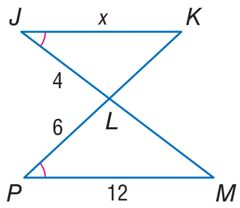



**15.** CCSS **MODELING** When we look at an object, it is projected on the retina through the pupil. The distances from the pupil to the top and bottom of the object are congruent and the distances from the pupil to the top and bottom of the image on the retina are congruent. Are the triangles formed between the object and the pupil and the object and the image similar? Explain your reasoning.

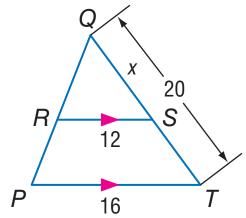



**Example 4** **ALGEBRA** **Identify the similar triangles. Then find each measure.**

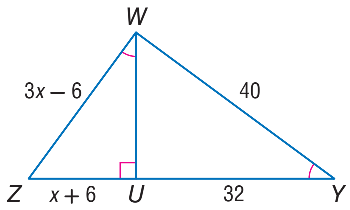
**16.** *JK*



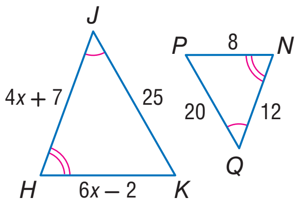

**17** *ST*



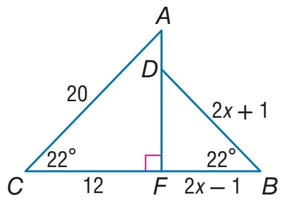

**18.** *WZ*, *UZ*



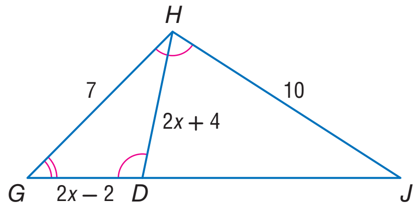

**19.** *HJ*, *HK*




**20.** *DB*, *CB*



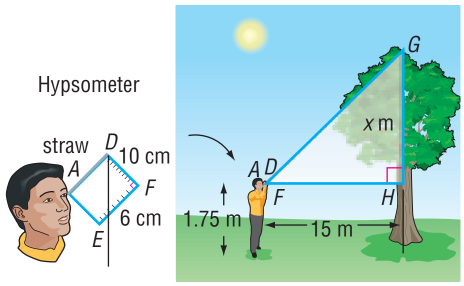

**21.** *GD*, *DH*




**Example 5** **22.** **STATUES** Mei is standing next to a statue in the park. If Mei is 5 feet tall, her shadow is 3 feet long, and the statue's shadow is $10\frac{1}{2}$ feet long, how tall is the statue?

**23.** **SPORTS** When Alonzo, who is 5′11″ tall, stands next to a basketball goal, his shadow is 2′ long, and the basketball goal's shadow is 4′4″ long. About how tall is the basketball goal?

**24.** **FORESTRY** A hypsometer, as shown, can be used to estimate the height of a tree. Bartolo looks through the straw to the top of the tree and obtains the readings given. Find the height of the tree.




**PROOF** **Write a two-column proof.**

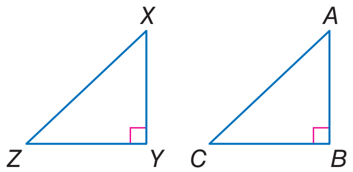
**25.** Theorem 7.3

**26.** Theorem 7.4

**PROOF** **Write a two-column proof.**

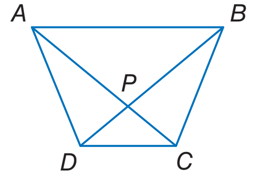
**27.** **Given:** $\triangle XYZ$ and $\triangle ABC$ are right triangles; $\frac{XY}{AB} = \frac{YZ}{BC}$.

**Prove:** $\triangle YXZ \sim \triangle BAC$

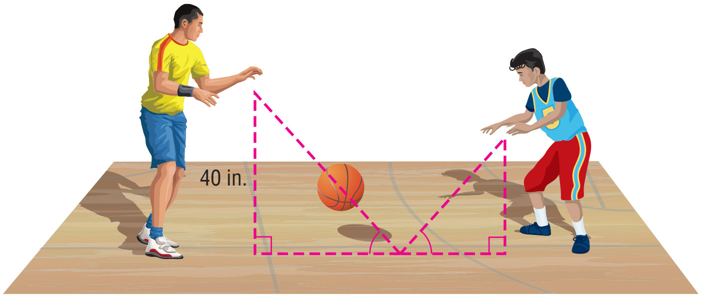



**28.** **Given:** $ABCD$ is a trapezoid.

**Prove:** $\frac{DP}{PB} = \frac{CP}{PA}$




**29.** **CCSS MODELING** When Luis's dad threw a bounce pass to him, the angles formed by the basketball's path were congruent. The ball landed $\frac{2}{3}$ of the way between them before it bounced back up. If Luis's dad released the ball 40 inches above the floor, at what height did Luis catch the ball?

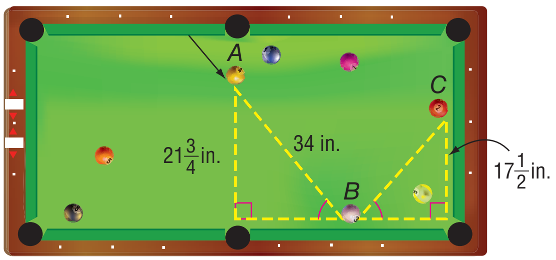



**COORDINATE GEOMETRY** **$\triangle XYZ$ and $\triangle WYV$ have vertices $X(-1, -9)$, $Y(5, 3)$, $Z(-1, 6)$, $W(1, -5)$, and $V(1, 5)$.**

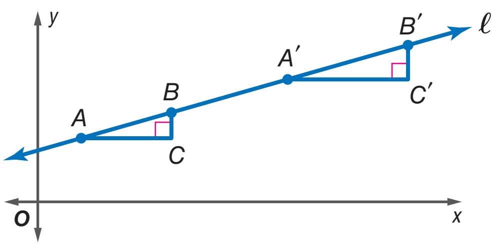
**30.** Graph the triangles, and prove that $\triangle XYZ \sim \triangle WYV$.

**31** Find the ratio of the perimeters of the two triangles.

**32.** **BILLIARDS** When a ball is deflected off a smooth surface, the angles formed by the path are congruent. Booker hit the orange ball and it followed the path from $A$ to $B$ to $C$ as shown below. What was the total distance traveled by the ball from the time Booker hit it until it came to rest at the end of the table?




**33.** **PROOF** Use similar triangles to show that the slope of the line through any two points on that line is constant. That is, if points $A$, $B$, $A'$ and $B'$ are on line $\ell$, use similar triangles to show that the slope of the line from $A$ to $B$ is equal to the slope of the line from $A'$ to $B'$.

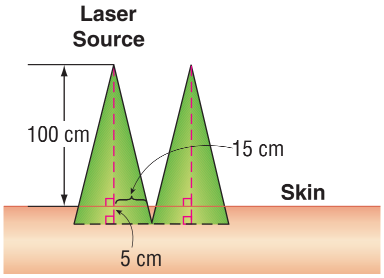


**34. CHANGING DIMENSIONS** Assume that $\triangle ABC \sim \triangle JKL$.

**a.** If the lengths of the sides of $\triangle JKL$ are half the length of the sides of $\triangle ABC$, and the area of $\triangle ABC$ is 40 square inches, what is the area of $\triangle JKL$? How is the area related to the scale factor of $\triangle ABC$ to $\triangle JKL$?

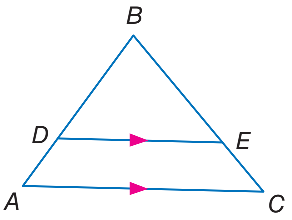
**b.** If the lengths of the sides of $\triangle ABC$ are three times the length of the sides of $\triangle JKL$, and the area of $\triangle ABC$ is 63 square inches, what is the area of $\triangle JKL$? How is the area related to the scale factor of $\triangle ABC$ to $\triangle JKL$?

**35. MEDICINE** Certain medical treatments involve laser beams that contact and penetrate the skin, forming similar triangles. Refer to the diagram at the right. How far apart should the laser sources be placed to ensure that the areas treated by each source do not overlap?




**36. MULTIPLE REPRESENTATIONS** In this problem, you will explore proportional parts of triangles.

**a. Geometric** Draw $\triangle ABC$ with $\overline{DE}$ parallel to $\overline{AC}$ as shown at the right.



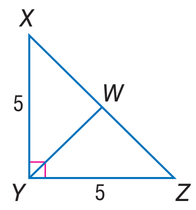

**b. Tabular** Measure and record the lengths $AD$, $DB$, $CD$, and $EB$ and the ratios $\frac{AD}{DB}$ and $\frac{CE}{EB}$ in a table.

**c. Verbal** Make a conjecture about the segments created by a line parallel to one side of a triangle and intersecting the other two sides.

## H.O.T. Problems Use Higher-Order Thinking Skills

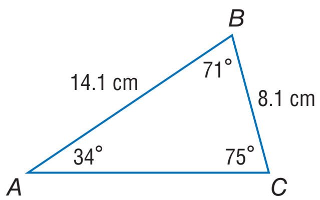
**37. WRITING IN MATH** Compare and contrast the AA Similarity Postulate, the SSS Similarity Theorem, and the SAS similarity theorem.

**38. CHALLENGE** $\overline{YW}$ is an altitude of $\triangle XYZ$. Find $YW$.




**39. CCSS REASONING** A pair of similar triangles has angle measures of 50°, 85°, and 45°. The sides of one triangle measure 3, 3.25, and 4.23 units, and the sides of the second triangle measure $x - 0.46$, $x$, and $x + 1.81$ units. Find the value of $x$.

**40. OPEN ENDED** Draw a triangle that is similar to $\triangle ABC$ shown. Explain how you know that it is similar.




**41. WRITING IN MATH** How can you choose an appropriate scale?

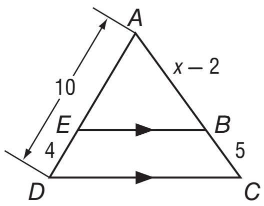
## Standardized Test Practice

**42. PROBABILITY** $\frac{x!}{(x-3)!} =$

**A** 3.0 **C** $x^2 - 3x + 2$

**B** 0.33 **D** $x^3 - 3x^2 + 2x$

**43. EXTENDED RESPONSE** In the figure below, $\overline{EB} \parallel \overline{DC}$.




**a.** Write a proportion that could be used to find $x$.

**b.** Find the value of $x$ and the measure of $\overline{AB}$.

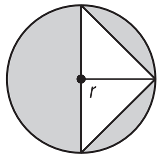
**44. ALGEBRA** Which polynomial represents the area of the shaded region?

**F** $\pi r^2$

**G** $\pi r^2 + r^2$

**H** $\pi r^2 + r$

**J** $\pi r^2 - r^2$




**45. SAT/ACT** The volume of a certain rectangular solid is $16x$ cubic units. If the dimensions of the solid are integers $x$, $y$, and $z$ units, what is the greatest possible value of $z$?

**A** 32 **D** 4

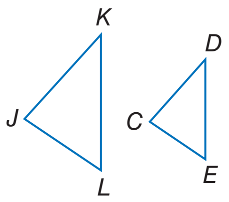
**B** 16 **E** 2

**C** 8

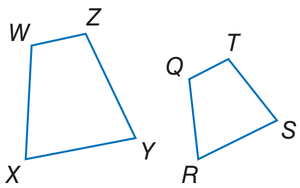
## Spiral Review

**List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.** (Lesson 7-2)

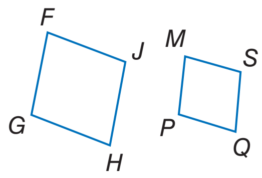
**46.** $\triangle JKL \sim \triangle CDE$




**47.** $WXYZ \sim QRST$



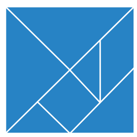

**48.** $FGHJ \sim MPQS$




**Solve each proportion.** (Lesson 7-1)

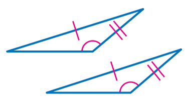
**49.** $\frac{3}{4} = \frac{x}{16}$ **50.** $\frac{x}{10} = \frac{22}{50}$ **51.** $\frac{20.2}{88} = \frac{12}{x}$ **52.** $\frac{x-2}{2} = \frac{3}{8}$

**53. TANGRAMS** A tangram set consists of seven pieces: a small square, two small congruent right triangles, two large congruent right triangles, a medium-sized right triangle, and a quadrilateral. How can you determine the shape of the quadrilateral? Explain. (Lesson 6-3)

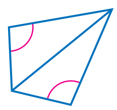


**Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*.** (Lesson 4-4)

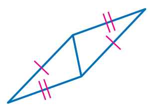
**54.**



**55.**



**56.**

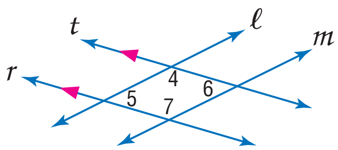


## Skills Review

**Write a two-column proof.**

**57. Given:** $r \parallel t$; $\angle 5 \cong \angle 6$

**Prove:** $\ell \parallel m$



EXTEND 7-3

# Geometry Lab: Proofs of Perpendicular and Parallel Lines

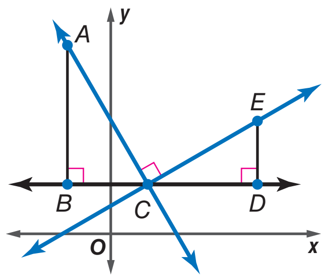
You have learned that two straight lines that are neither horizontal nor vertical are perpendicular if and only if the product of their slopes is –1. In this activity, you will use similar triangles to prove the first half of this theorem: if two straight lines are perpendicular, then the product of their slopes is –1.

**CCSS Common Core State Standards**
**Content Standards**
**G.GPE.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
**Mathematical Practices** 3

## Activity 1 Perpendicular Lines

**Given:** Slope of $\overleftrightarrow{AC} = m_1$, slope of $\overleftrightarrow{CE} = m_2$, and $\overleftrightarrow{AC} \perp \overleftrightarrow{CE}$.
**Prove:** $m_1 m_2 = -1$

**Step 1** On a coordinate plane, construct $\overleftrightarrow{AC} \perp \overleftrightarrow{CE}$ and transversal $\overleftrightarrow{BD}$ parallel to the $x$-axis through $C$. Then construct right $\triangle ABC$ such that $\overline{AC}$ is the hypotenuse and right $\triangle EDC$ such that $\overline{CE}$ is the hypotenuse. The legs of both triangles should be parallel to the $x$-and $y$-axes, as shown.




**Step 2** Find the slopes of $\overleftrightarrow{AC}$ and $\overleftrightarrow{CE}$.

**Slope of $\overleftrightarrow{AC}$**

$m_1 = \frac{\text{rise}}{\text{run}}$ Slope Formula

$= \frac{-AB}{BC}$ or $-\frac{AB}{BC}$ rise = $-AB$, run = $BC$

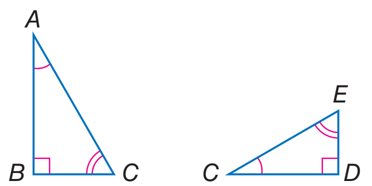
**Slope of $\overleftrightarrow{CE}$**

$m_2 = \frac{\text{rise}}{\text{run}}$ Slope Formula

$= \frac{DE}{CD}$ rise = $DE$, run = $CD$

**Step 3** Show that $\triangle ABC \sim \triangle CDE$.

Since $\triangle ACB$ is a right triangle with right angle $B$, $\angle BAC$ is complementary to $\angle ACB$. It is given that $\overleftrightarrow{AC} \perp \overleftrightarrow{CE}$, so we know that $\triangle ACE$ is a right angle. By construction, $\angle BCD$ is a straight angle. So, $\angle ECD$ is complementary to $\angle ACB$. Since angles complementary to the same angle are congruent, $\angle BAC \cong \angle ECD$. Since right angles are congruent, $\angle B \cong \angle D$. Therefore, by AA Similarity, $\triangle ABC \sim \triangle CDE$.




**Step 4** Use the fact that $\triangle ABC \sim \triangle CDE$ to show that $m_1 m_2 = -1$.

Since $m_1 = -\frac{AB}{BC}$ and $m_2 = \frac{DE}{CD}$, $m_1 m_2 = \left(-\frac{AB}{BC}\right)\left(\frac{DE}{CD}\right)$. Since two similar polygons have proportional sides, $\frac{AB}{BC} = \frac{CD}{DE}$. Therefore, by substitution, $m_1 m_2 = \left(-\frac{CD}{DE}\right)\left(\frac{DE}{CD}\right)$ or $-1$.

## Model

1. **PROOF** Use the diagram from Activity 1 to prove the second half of the theorem.

**Given:** Slope of $\overleftrightarrow{CE} = m_1$, slope of $\overleftrightarrow{AC} = m_2$, and $m_1 m_2 = -1$. $\triangle ABC$ is a right triangle with right angle $B$. $\triangle CDE$ is a right triangle with right angle $D$.

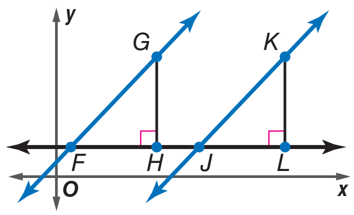
**Prove:** $\overleftrightarrow{CE} \perp \overleftrightarrow{AC}$

You can also use similar triangles to prove statements about parallel lines.

### Activity 2 Parallel Lines

**Given:** Slope of $\overleftrightarrow{FG} = m_1$, slope of $\overleftrightarrow{JK} = m_2$, and $m_1 = m_2$. $\triangle FHG$ is a right triangle with right angle $H$. $\triangle JLK$ is a right triangle with right angle $L$.

**Prove:** $\overleftrightarrow{FG} \parallel \overleftrightarrow{JK}$

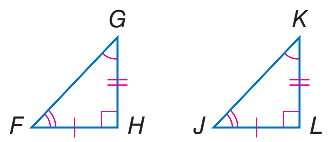



**Step 1** On a coordinate plane, construct $\overleftrightarrow{FG}$ and $\overleftrightarrow{JK}$, right $\triangle FHG$, and right $\triangle JLK$. Then draw horizontal transversal $\overleftrightarrow{FL}$, as shown.

**Step 2** Find the slopes of $\overleftrightarrow{FG}$ and $\overleftrightarrow{JK}$.

| Slope of $\overleftrightarrow{FG}$ | | Slope of $\overleftrightarrow{JK}$ | |
|---|---|---|---|
| $m_1 = \frac{\text{rise}}{\text{run}}$ | Slope Formula | $m_2 = \frac{\text{rise}}{\text{run}}$ | Slope Formula |
| $= \frac{GH}{HF}$ | rise = $GH$, run = $HF$ | $= \frac{KL}{LJ}$ | rise = $KL$, run = $LJ$ |

**Step 3** Show that $\triangle FHG \sim \triangle JLK$.




It is given that $m_1 = m_2$. By substitution, $\frac{GH}{HF} = \frac{KL}{LJ}$. This ratio can be rewritten as $\frac{GH}{KL} = \frac{HF}{LJ}$. Since $\angle H$ and $\angle L$ are right angles, $\angle H \cong \angle L$. Therefore, by SAS similarity, $\triangle FHG \sim \triangle JLK$.

**Step 4** Use the fact that $\triangle FHG \sim \triangle JLK$ to prove that $\overleftrightarrow{FG} \parallel \overleftrightarrow{JK}$.

Corresponding angles in similar triangles are congruent, so $\angle GFH \cong \angle KJL$. From the definition of congruent angles, $m\angle GFH = m\angle KJL$ (or $\angle GFH \cong \angle KJL$). By definition, $\angle KJH$ and $\angle KJL$ form a linear pair. Since linear pairs are supplementary, $m\angle KJH + m\angle KJL = 180$. So, by substitution, $m\angle KJH + m\angle GFH = 180$. By definition, $\angle KJH$ and $\angle GFH$ are supplementary. Since $\angle KJH$ and $\angle GFH$ are supplementary and are consecutive interior angles, $\overleftrightarrow{FG} \parallel \overleftrightarrow{JK}$.

## Model

2. **PROOF** Use the diagram from Activity 2 to prove the following statement.

**Given:** Slope of $\overleftrightarrow{FG} = m_1$, slope of $\overleftrightarrow{JK} = m_2$, and $\overleftrightarrow{FG} \parallel \overleftrightarrow{JK}$.

**Prove:** $m_1 = m_2$

# LESSON 7-4 Parallel Lines and Proportional Parts

**Then**

- You used proportions to solve problems between similar triangles.

**Now**

1. Use proportional parts within triangles.
2. Use proportional parts with parallel lines.

**Why?**

- Photographers have many techniques at their disposal that can be used to add interest to a photograph. One such technique is the use of a vanishing point perspective, in which an image with parallel lines, such as train tracks, is photographed so that the lines appear to converge at a point on the horizon.

**NewVocabulary**
midsegment of a triangle

**Common Core State Standards**

**Content Standards**

G.SRT.4 Prove theorems about triangles.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Mathematical Practices**

1 Make sense of problems and persevere in solving them.

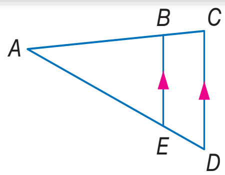
3 Construct viable arguments and critique the reasoning of others.

## 1 Proportional Parts Within Triangles

When a triangle contains a line that is parallel to one of its sides, the two triangles formed can be proved similar using the Angle-Angle Similarity Postulate. Since the triangles are similar, their sides are proportional.

### Theorem 7.5 Triangle Proportionality Theorem

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.



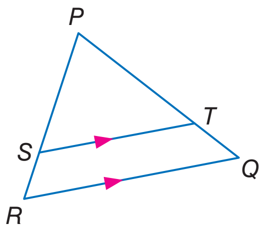

**Example** If $\overline{BE} \parallel \overline{CD}$, then $\frac{AB}{BC} = \frac{AE}{ED}$.

You will prove Theorem 7.5 in Exercise 30.

### Example 1 Find the Length of a Side

**In $\triangle PQR$, $\overline{ST} \parallel \overline{RQ}$. If $PT = 7.5$, $TQ = 3$, and $SR = 2.5$, find $PS$.**

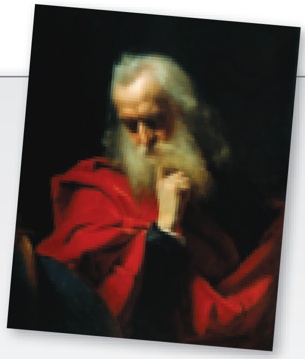
Use the Triangle Proportionality Theorem.




| | |
|---|---|
| $\frac{PS}{SR} = \frac{PT}{TQ}$ | Triangle Proportionality Theorem |
| $\frac{PS}{2.5} = \frac{7.5}{3}$ | Substitute. |
| $PS \cdot 3 = (2.5)(7.5)$ | Cross Products Property |
| $3PS = 18.75$ | Multiply. |
| $PS = 6.25$ | Divide each side by 3. |

**GuidedPractice**

**1.** If $PS = 12.5$, $SR = 5$, and $PT = 15$, find $TQ$.



## Math History Link

**Galileo Galilei** (1564–1642) Galileo was born in Pisa, Italy. He studied philosophy, astronomy, and mathematics. Galileo made essential contributions to all three disciplines. Refer to Exercise 39.

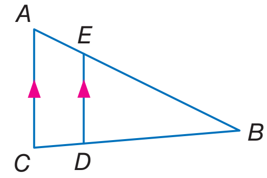
**Source:** *Encyclopaedia Britannica*

The converse of Theorem 7.5 is also true and can be proved using the proportional parts of a triangle.

### Theorem 7.6 Converse of Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle.

**Example** If $\frac{AE}{EB} = \frac{CD}{DB}$, then $\overline{AC} \parallel \overline{ED}$.

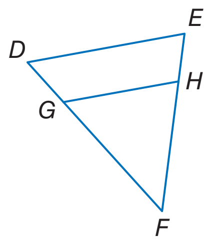



You will prove Theorem 7.6 in Exercise 31.

PT

## Example 2 Determine if Lines are Parallel

**In $\triangle DEF$, $EH = 3$, $HF = 9$, and $DG$ is one-third the length of $\overline{GF}$. Is $\overline{DE} \parallel \overline{GH}$?**




Using the converse of the Triangle Proportionality Theorem, in order to show that $\overline{DE} \parallel \overline{GH}$, we must show that $\frac{DG}{GF} = \frac{EH}{HF}$.

Find and simplify each ratio. Let $DG = x$.
Since $DG$ is one-third of $GF$, $GF = 3x$.

$$\frac{DG}{GF} = \frac{x}{3x} \text{ or } \frac{1}{3} \qquad \frac{EH}{HF} = \frac{3}{9} \text{ or } \frac{1}{3}$$

Since $\frac{1}{3} = \frac{1}{3}$, the sides are proportional, so $\overline{DE} \parallel \overline{GH}$.

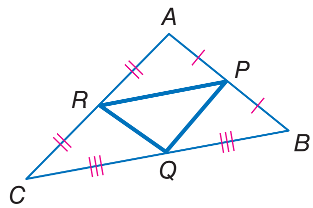
### Guided Practice

**2.** $DG$ is half the length of $\overline{GF}$, $EH = 6$, and $HF = 10$. Is $\overline{DE} \parallel \overline{GH}$?

## Study Tip

**Midsegment Triangle** The three midsegments of a triangle form the *midsegment triangle.*

A **midsegment of a triangle** is a segment with endpoints that are the midpoints of two sides of the triangle. Every triangle has three midsegments. The midsegments of $\triangle ABC$ are $\overline{RP}$, $\overline{PQ}$, $\overline{RQ}$.

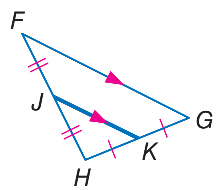



A special case of the Triangle Proportionality Theorem is the Triangle Midsegment Theorem.

### Theorem 7.7 Triangle Midsegment Theorem

A midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.

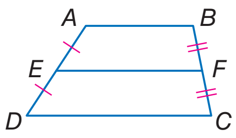
**Example** If $J$ and $K$ are midpoints of $\overline{FH}$ and $\overline{HG}$, respectively, then $\overline{JK} \parallel \overline{FG}$ and $JK = \frac{1}{2}FG$.




You will prove Theorem 7.7 in Exercise 32.

## StudyTip

**Midsegment** The Triangle Midsegment Theorem is similar to the Trapezoid Midsegment Theorem, which states that the midsegment of a trapezoid is parallel to the bases and its length is one half the sum of the measures of the bases. (Lesson 6-6)

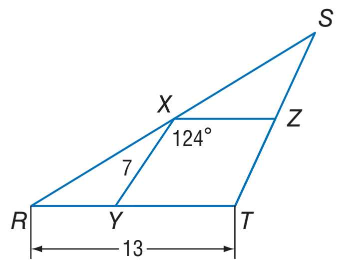



$\overline{EF} \parallel \overline{AB} \parallel \overline{DC}$

$EF = \frac{1}{2}(AB + DC)$

## Example 3 Use the Triangle Midsegment Theorem

**In the figure, $\overline{XY}$ and $\overline{XZ}$ are midsegments of $\triangle RST$. Find each measure.**




**a.** $XZ$

$XZ = \frac{1}{2}RT$ Triangle Midsegment Theorem

$XZ = \frac{1}{2}(13)$ Substitution

$XZ = 6.5$ Simplify.

**b.** $ST$

$XY = \frac{1}{2}ST$ Triangle Midsegment Theorem

$7 = \frac{1}{2}ST$ Substitution

$14 = ST$ Multiply each side by 2.

**c.** $m\angle RYX$

By the Triangle Midsegment Theorem, $\overline{XZ} \parallel \overline{RT}$.

$\angle RYX \cong \angle YXZ$ Alternate Interior Angles Theorem

$m\angle RYX = m\angle YXZ$ Definition of congruence

$m\angle RYX = 124$ Substitution

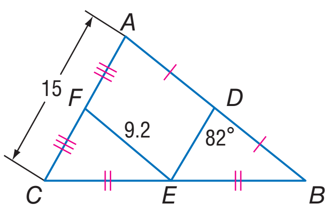
### GuidedPractice

**Find each measure.**

**3A.** $DE$

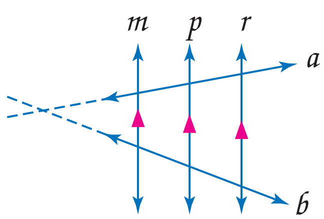
**3B.** $DB$

**3C.** $m\angle FED$




## 2 Proportional Parts with Parallel Lines

Another special case of the Triangle Proportionality Theorem involves three or more parallel lines cut by two transversals. Notice that if transversals $a$ and $b$ are extended, they form triangles with the parallel lines.




## StudyTip

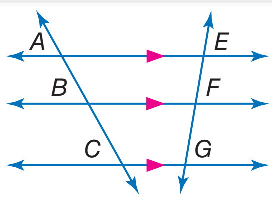
**Other Proportions** Two other proportions can be written for the example in Corollary 7.1.

$\frac{AB}{EF} = \frac{BC}{FG}$ and $\frac{AC}{BC} = \frac{EG}{FG}$

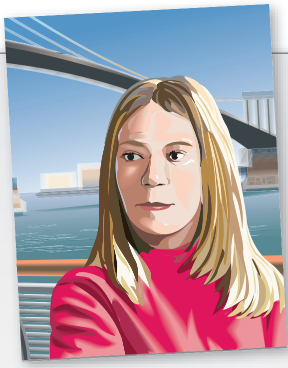
### Corollary 7.1 Proportional Parts of Parallel Lines

If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

**Example** If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, then $\frac{AB}{BC} = \frac{EF}{FG}$.




You will prove Corollary 7.1 in Exercise 28.



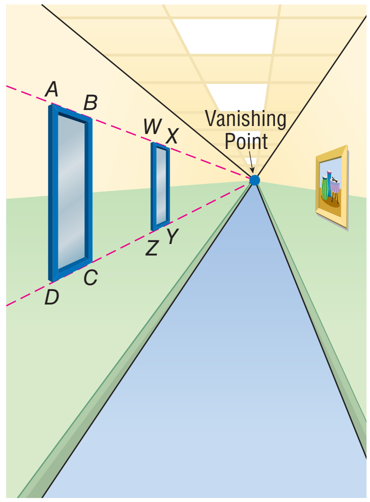
## Real-WorldLink

To make a two-dimensional drawing appear three-dimensional, an artist provides several perceptual cues.

- *size* - faraway items look smaller
- *clarity* - closer objects appear more in focus
- *detail* - nearby objects have texture, while distant ones are roughly outlined

**Source:** Center for Media Literacy

## Real-World Example 4 Use Proportional Segments of Transversals




**ART** **Megan is drawing a hallway in one-point perspective. She uses the guidelines shown to draw two windows on the left wall. If segments $\overline{AD}$, $\overline{BC}$, $\overline{WZ}$, and $\overline{XY}$ are all parallel, $AB = 8$ centimeters, $DC = 9$ centimeters, and $ZY = 5$ centimeters, find $WX$.**

By Corollary 7.1, if $\overline{AD} \parallel \overline{BC} \parallel \overline{WZ} \parallel \overline{XY}$,

then $\frac{AB}{WX} = \frac{DC}{ZY}$.

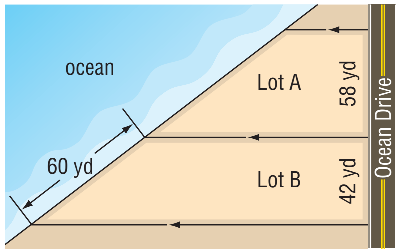
| | |
|---|---|
| $\frac{AB}{WX} = \frac{DC}{ZY}$ | Corollary 7.1 |
| $\frac{8}{WX} = \frac{9}{5}$ | Substitute. |
| $WX \cdot 9 = 8 \cdot 5$ | Cross Products Property |
| $9WX = 40$ | Simplify. |
| $WX = \frac{40}{9}$ | Divide each side by 4. |

The distance between $W$ and $X$ should be $\frac{40}{9}$ or about 4.4 centimeters.

**CHECK** The ratio of $DC$ to $ZY$ is 9 to 5, which is about 10 to 5 or 2 to 1. The ratio of $AB$ to $WX$ is 8 to 4.4 or about 8 to 4 or 2 to 1 as well, so the answer is reasonable. ✓

### GuidedPractice

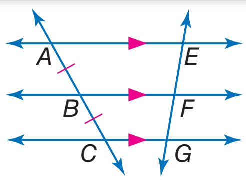
**4. REAL ESTATE** *Frontage* is the measurement of a property's boundary that runs along the side of a particular feature such as a street, lake, ocean, or river. Find the ocean frontage for Lot A to the nearest tenth of a yard.




If the scale factor of the proportional segments is 1, they separate the transversals into congruent parts.

### Corollary 7.2 Congruent Parts of Parallel Lines

If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



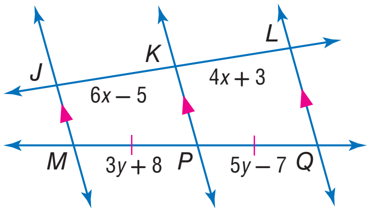

**Example** If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, and $\overline{AB} \cong \overline{BC}$,

then $\overline{EF} \cong \overline{FG}$.

You will prove Corollary 7.2 in Exercise 29.

## Real-World Example 5 Use Congruent Segments of Transversals

**ALGEBRA** **Find $x$ and $y$.**

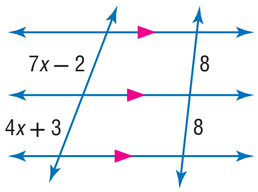



Since $\overleftrightarrow{JM} \parallel \overleftrightarrow{KP} \parallel \overleftrightarrow{LQ}$ and $\overline{MP} \cong \overline{PQ}$, then $\overline{JK} \cong \overline{KL}$ by Corollary 7.2.

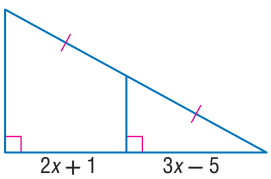
| | |
|---|---|
| $JK = KL$ | Definition of congruence |
| $6x - 5 = 4x + 3$ | Substitution |
| $2x - 5 = 3$ | Subtract $4x$ from each side. |
| $2x = 8$ | Add 5 to each side. |
| $x = 4$ | Divide each side by 2. |
| $MP = PQ$ | Definition of congruence |
| $3y + 8 = 5y - 7$ | Substitution |
| $8 = 2y - 7$ | Subtract $3y$ from each side. |
| $15 = 2y$ | Add 7 to each side. |
| $7.5 = y$ | Divide each side by 2. |

### GuidedPractice

**5A.**




**5B.**

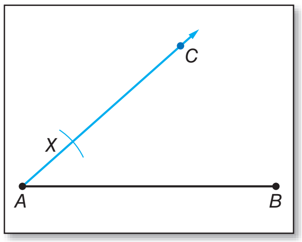



It is possible to separate a segment into two congruent parts by constructing the perpendicular bisector of a segment. However, a segment cannot be separated into three congruent parts by constructing perpendicular bisectors. To do this, you must use parallel lines and Corollary 7.2.

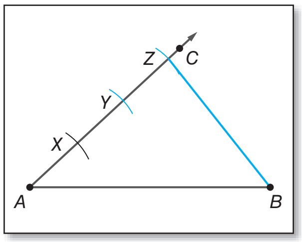
## Construction Trisect a Segment

Draw a segment $\overline{AB}$. Then use Corollary 7.2 to trisect $\overline{AB}$.

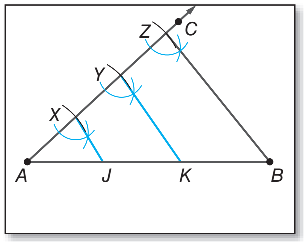

**Step 1** Draw $\overline{AC}$. Then with the compass at $A$, mark off an arc that intersects $\overline{AC}$ at $X$.




**Step 2** Use the same compass setting to mark off $Y$ and $Z$ such that $\overline{AX} \cong \overline{XY} \cong \overline{YZ}$. Then draw $\overline{ZB}$.




**Step 3** Construct lines through $Y$ and $X$ that are parallel to $\overline{ZB}$. Label the intersection points on $\overline{AB}$ as $J$ and $K$.




**Conclusion**: Since parallel lines cut off congruent segments on transversals, $\overline{AJ} \cong \overline{JK} \cong \overline{KB}$.

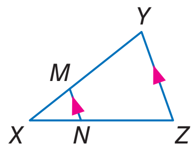
## Check Your Understanding

= Step-by-Step Solutions begin on page R14.

**Example 1**

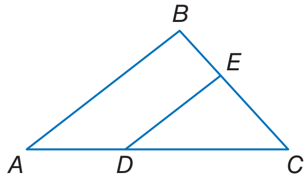
**1.** If $XM = 4$, $XN = 6$, and $NZ = 9$, find $XY$.

**2.** If $XN = 6$, $XM = 2$, and $XY = 10$, find $NZ$.

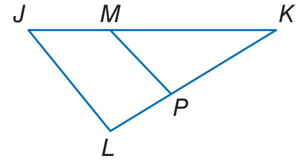



**Example 2**

**3.** In $\triangle ABC$, $BC = 15$, $BE = 6$, $DC = 12$, and $AD = 8$. Determine whether $\overline{DE} \parallel \overline{AB}$. Justify your answer.

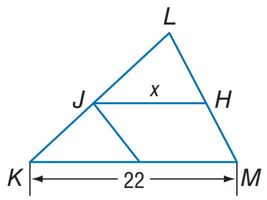



**4.** In $\triangle JKL$, $JK = 15$, $JM = 5$, $LK = 13$, and $PK = 9$. Determine whether $\overline{JL} \parallel \overline{MP}$. Justify your answer.

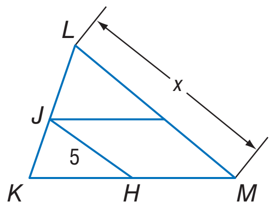



**Example 3** $\overline{JH}$ **is a midsegment of $\triangle KLM$. Find the value of $x$.**

**5.**

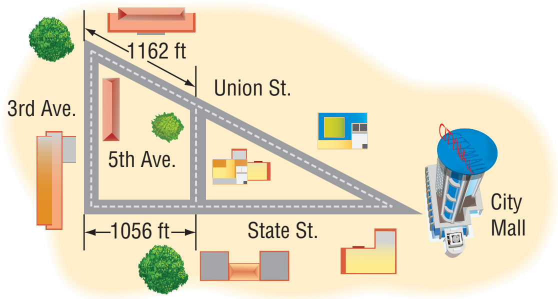



**6.**



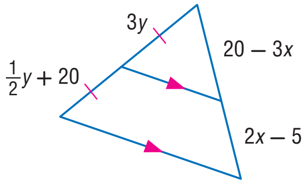

**Example 4**

**7.** **MAPS** Refer to the map at the right. 3rd Avenue and 5th Avenue are parallel. If the distance from 3rd Avenue to City Mall along State Street is 3201 feet, find the distance between 5th Avenue and City Mall along Union Street. Round to the nearest tenth.

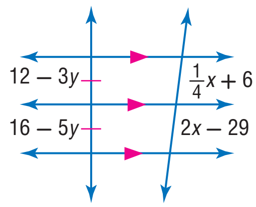



**Example 5** **ALGEBRA** **Find $x$ and $y$.**

**8.**




**9.**




## Practice and Problem Solving

Extra Practice is on page R7.

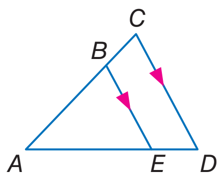
**Example 1**

**10.** If $AB = 6$, $BC = 4$, and $AE = 9$, find $ED$.

**11.** If $AB = 12$, $AC = 16$, and $ED = 5$, find $AE$.

**12.** If $AC = 14$, $BC = 8$, and $AD = 21$, find $ED$.

**13.** If $AD = 27$, $AB = 8$, and $AE = 12$, find $BC$.



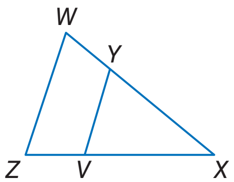
**Example 2** **Determine whether $\overline{VY} \parallel \overline{ZW}$. Justify your answer.**

**14.** $ZX = 18$, $ZV = 6$, $WX = 24$, and $YX = 16$

**15.** $VX = 7.5$, $ZX = 24$, $WY = 27.5$, and $WX = 40$

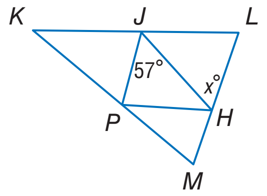
**16.** $ZV = 8$, $VX = 2$, and $YX = \frac{1}{2}WY$

**17.** $WX = 31$, $YX = 21$, and $ZX = 4ZV$

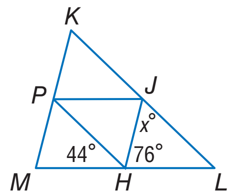



**Example 3** **$\overline{JH}$, $\overline{JP}$, and $\overline{PH}$ are midsegments of $\triangle KLM$. Find the value of $x$.**

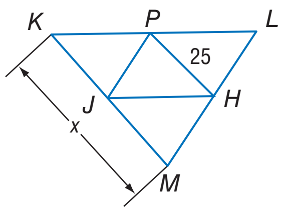
**18.**



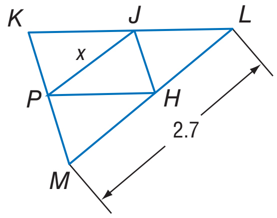

**19.**



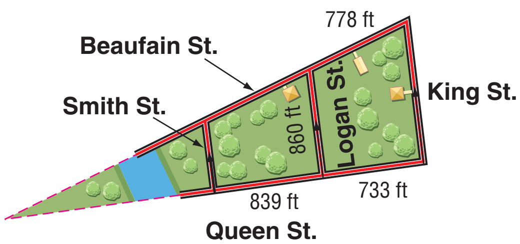

**20.**



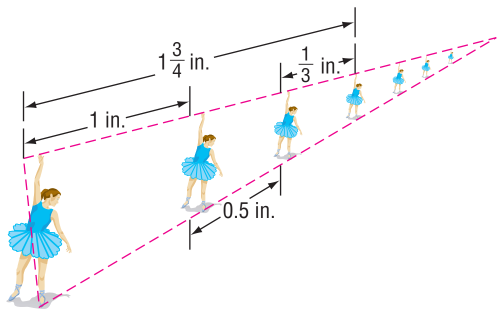

**21.**




**Example 4** **22.** CCSS **MODELING** In Charleston, South Carolina, Logan Street is parallel to both King Street and Smith Street between Beaufain Street and Queen Street. What is the distance from Smith to Logan along Beaufain? Round to the nearest foot.

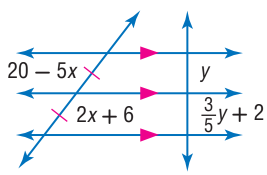



**23.** **ART** Tonisha drew the line of dancers shown below for her perspective project in art class. Each of the dancers is parallel. Find the lower distance between the first two dancers.

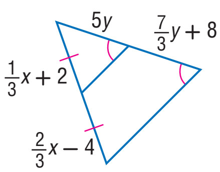



**Example 5** **ALGEBRA** **Find $x$ and $y$.**

**24.**

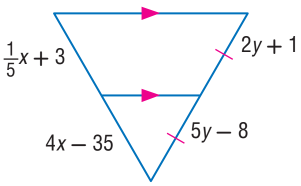



**25.**

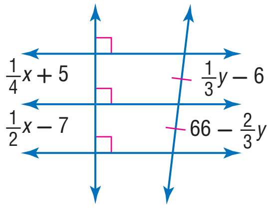


**ALGEBRA** **Find $x$ and $y$.**

**26.**




**27.**




**CCSS ARGUMENTS** **Write a paragraph proof.**

**28.** Corollary 7.1

**29.** Corollary 7.2

**30.** Theorem 7.5

**CCSS ARGUMENTS** **Write a two-column proof.**

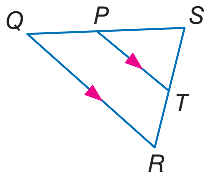
**31.** Theorem 7.6

**32.** Theorem 7.7

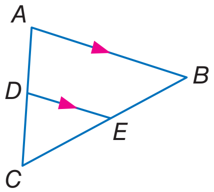
**Refer to $\triangle QRS$.**

**33.** If $ST = 8$, $TR = 4$, and $PT = 6$, find $QR$.

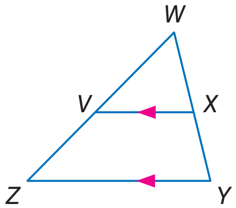
**34.** If $SP = 4$, $PT = 6$, and $QR = 12$, find $SQ$.



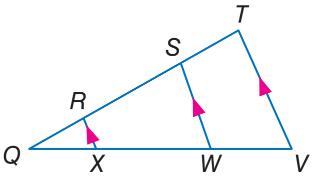

**35** If $CE = t - 2$, $EB = t + 1$, $CD = 2$, and $CA = 10$, find $t$ and $CE$.



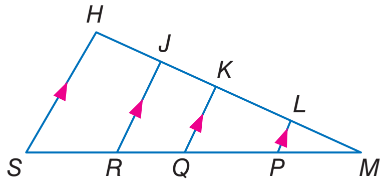

**36.** If $WX = 7$, $WY = a$, $WV = 6$, and $VZ = a - 9$, find $WY$.



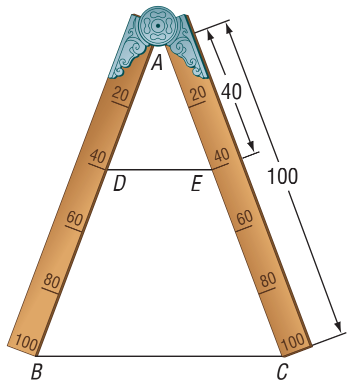

**37.** If $QR = 2$, $XW = 12$, $QW = 15$, and $ST = 5$, find $RS$ and $WV$.




**38.** If $LK = 4$, $MP = 3$, $PQ = 6$, $KJ = 2$, $RS = 6$, and $LP = 2$, find $ML$, $QR$, $QK$, and $JH$.



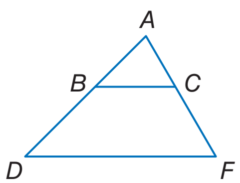

**39.** **MATH HISTORY** The sector compass was a tool perfected by Galileo in the sixteenth century for measurement. To draw a segment two-fifths the length of a given segment, align the ends of the arms with the given segment. Then draw a segment at the 40 mark. Write a justification that explains why the sector compass works for proportional measurement.




**Determine the value of $x$ so that $\overline{BC} \parallel \overline{DF}$.**

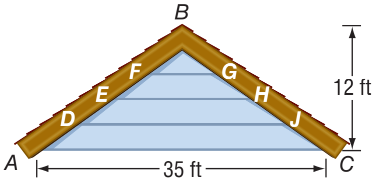
**40.** $AB = x + 5$, $BD = 12$, $AC = 3x + 1$, and $CF = 15$

**41.** $AC = 15$, $BD = 3x - 2$, $CF = 3x + 2$, and $AB = 12$



**42.** **COORDINATE GEOMETRY** $\triangle ABC$ has vertices $A(-8, 7)$, $B(0, 1)$, and $C(7, 5)$. Draw $\triangle ABC$. Determine the coordinates of the midsegment of $\triangle ABC$ that is parallel to $\overline{BC}$. Justify your answer.

**43** **HOUSES** Refer to the diagram of the gable at the right. Each piece of siding is a uniform width. Find the lengths of $\overline{FG}$, $\overline{EH}$, and $\overline{DJ}$.




**CONSTRUCTIONS** **Construct each segment as directed.**

**44.** a segment separated into five congruent segments

**45.** a segment separated into two segments in which their lengths have a ratio of 1 to 3

**46.** a segment 3 inches long, separated into four congruent segments

**47.** **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle bisectors and proportions.

**a. Geometric** Draw three triangles, one acute, one right, and one obtuse. Label one triangle *ABC* and draw angle bisector $\overrightarrow{BD}$. Label the second *MNP* with angle bisector $\overrightarrow{NQ}$ and the third *WXY* with angle bisector $\overrightarrow{XZ}$.

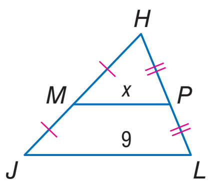
**b. Tabular** Copy and complete the table at the right with the appropriate values.

**c. Verbal** Make a conjecture about the segments of a triangle created by an angle bisector.

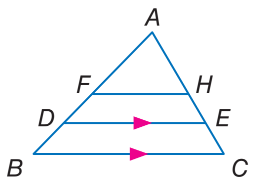
| Triangle | Length | | Ratio | |
|---|---|---|---|---|
| *ABC* | *AD* | | $\frac{AD}{CD}$ | |
| | *CD* | | | |
| | *AB* | | $\frac{AB}{CB}$ | |
| | *CB* | | | |
| *MNP* | *MQ* | | $\frac{MQ}{PQ}$ | |
| | *PQ* | | | |
| | *MN* | | $\frac{MN}{PN}$ | |
| | *PN* | | | |
| *WXY* | *WZ* | | $\frac{WZ}{YZ}$ | |
| | *YZ* | | | |
| | *WX* | | $\frac{WX}{YX}$ | |
| | *YX* | | | |

## H.O.T. Problems Use Higher-Order Thinking Skills

**48.** CCSS **CRITIQUE** Jacob and Sebastian are finding the value of $x$ in $\triangle JHL$. Jacob says that $MP$ is one half of $JL$, so $x$ is 4.5. Sebastian says that $JL$ is one half of $MP$, so $x$ is 18. Is either of them correct? Explain.



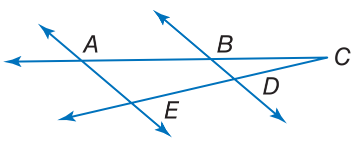

**49.** **REASONING** In $\triangle ABC$, $AF = FB$ and $AH = HC$. If $D$ is $\frac{3}{4}$ of the way from $A$ to $B$ and $E$ is $\frac{3}{4}$ of the way from $A$ to $C$, is $DE$ *always*, *sometimes*, or *never* $\frac{3}{4}$ of $BC$? Explain.




**50.** **CHALLENGE** Write a two-column proof.

**Given:** $AB = 4$, $BC = 4$, and $CD = DE$

**Prove:** $\overline{BD} \parallel \overline{AE}$

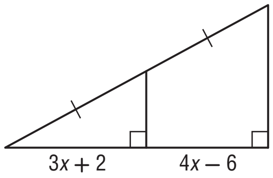



**51.** **OPEN ENDED** Draw three segments, $a$, $b$, and $c$, of all different lengths. Draw a fourth segment, $d$, such that $\frac{a}{b} = \frac{c}{d}$.

**52.** **WRITING IN MATH** Compare the Triangle Proportionality Theorem and the Triangle Midsegment Theorem.

## Standardized Test Practice

**53. SHORT RESPONSE** What is the value of $x$?




**54.** If the vertices of triangle $JKL$ are (0, 0), (0, 10) and (10, 10), then the area of triangle $JKL$ is

**A** 20 units$^2$ **C** 40 units$^2$
**B** 30 units$^2$ **D** 50 units$^2$

**55. ALGEBRA** A breakfast cereal contains wheat, rice, and oats in the ratio 2 : 4 : 1. If the manufacturer makes a mixture using 110 pounds of wheat, how many pounds of rice will be used?

**F** 120 lb **H** 240 lb
**G** 220 lb **J** 440 lb

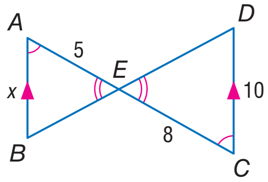
**56. SAT/ACT** If the area of a circle is 16 square meters, what is its radius in meters?

**A** $\frac{4\sqrt{\pi}}{\pi}$ **D** $12\pi$
**B** $\frac{8}{\pi}$ **E** $16\pi$
**C** $\frac{16}{\pi}$

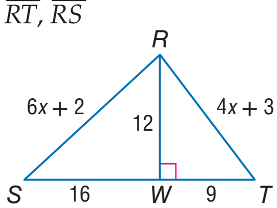
## Spiral Review

**ALGEBRA** **Identify the similar triangles. Then find the measure(s) of the indicated segment(s).** (Lesson 7-3)

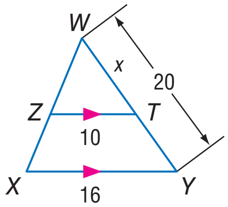
**57.** $\overline{AB}$



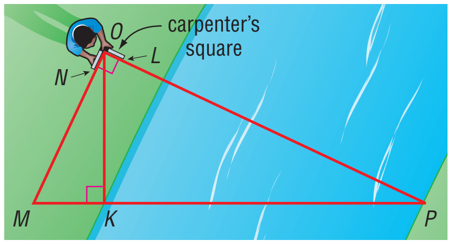

**58.** $\overline{RT}$, $\overline{RS}$




**59.** $\overline{TY}$




**60. SURVEYING** Mr. Turner uses a carpenter's square to find the distance across a stream. The carpenter's square models right angle $NOL$. He puts the square on top of a pole that is high enough to sight along $\overline{OL}$ to point $P$ across the river. Then he sights along $\overline{ON}$ to point $M$. If $MK$ is 1.5 feet and $OK$ is 4.5 feet, find the distance $KP$ across the stream. (Lesson 7-2)




**COORDINATE GEOMETRY** **For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.** (Lesson 6-6)

**61.** $Q(-12, 1)$, $R(-9, 4)$, $S(-4, 3)$, $T(-11, -4)$

**62.** $A(-3, 3)$, $B(-4, -1)$, $C(5, -1)$, $D(2, 3)$

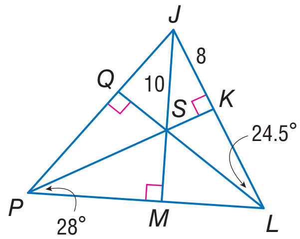
**Point $S$ is the incenter of $\triangle JPL$. Find each measure.** (Lesson 5-1)

**63.** $SQ$

**64.** $QJ$

**65.** $m\angle MPQ$

**66.** $m\angle SJP$




## Skills Review

**Solve each proportion.**

**67.** $\frac{1}{3} = \frac{x}{2}$

**68.** $\frac{3}{4} = \frac{5}{x}$

**69.** $\frac{2.3}{4} = \frac{x}{3.7}$

**70.** $\frac{x-2}{2} = \frac{4}{5}$

**71.** $\frac{x}{12-x} = \frac{8}{3}$

# CHAPTER 7 Mid-Chapter Quiz
## Lessons 7-1 through 7-4

Solve each proportion. (Lesson 7-1)

1. $\frac{2}{5} = \frac{x}{25}$

2. $\frac{10}{3} = \frac{7}{x}$

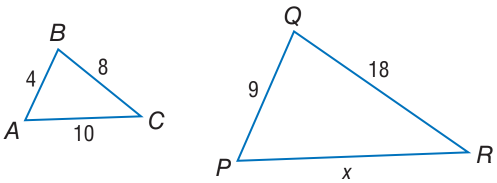
3. $\frac{y+4}{11} = \frac{y-2}{9}$

4. $\frac{z-1}{3} = \frac{8}{z+1}$

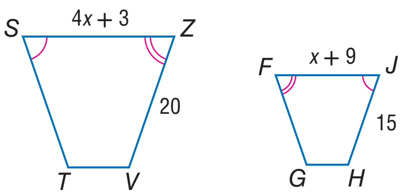
5. **BASEBALL** A pitcher's earned run average, or ERA, is the product of 9 and the ratio of earned runs the pitcher has allowed to the number of innings pitched. During the 2007 season, Johan Santana of the Minnesota Twins allowed 81 earned runs in 219 innings pitched. Find his ERA to the nearest hundredth. (Lesson 7-1)

Each pair of polygons is similar. Find the value of $x$. (Lesson 7-2)

6.




7.




8. **MULTIPLE CHOICE** Two similar polygons have a scale factor of 3:5. The perimeter of the larger polygon is 120 feet. Find the perimeter of the smaller polygon. (Lesson 7-2)

**A** 68 ft

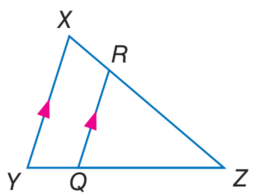
**B** 72 ft

**C** 192 ft

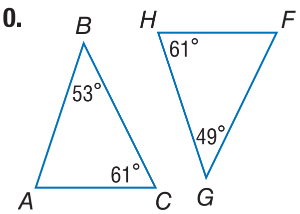
**D** 200 ft

Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning. (Lesson 7-3)

9.

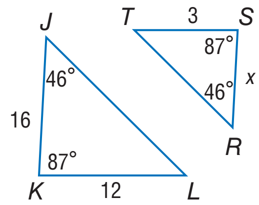



10.

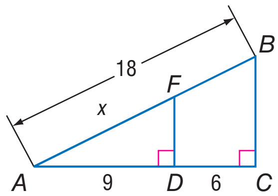



**ALGEBRA** Identify the similar triangles. Find each measure. (Lesson 7-3)

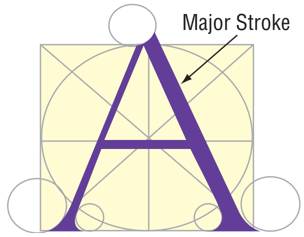
11. $SR$




12. $AF$




13. **HISTORY** In the fifteenth century, mathematicians and artists tried to construct the perfect letter. A square was used as a frame to design the letter "A," as shown below. The thickness of the major stroke of the letter was $\frac{1}{12}$ the height of the letter. (Lesson 7-4)

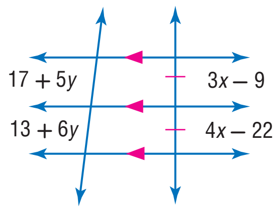



a. Explain why the bar through the middle of the A is half the length of the space between the outside bottom corners of the sides of the letter.

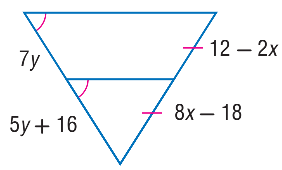
b. If the letter were 3 centimeters tall, how wide would the major stroke be?

**ALGEBRA** Find $x$ and $y$. (Lesson 7-4)

14.




15.



# LESSON 7-5 Parts of Similar Triangles

## Then

- You learned that corresponding sides of similar polygons are proportional.

## Now

1. Recognize and use proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles.
2. Use the Triangle Bisector Theorem.

## Why?

- The "Rule of Thumb" uses the average ratio of a person's arm length to the distance between his or her eyes and the altitudes of similar triangles to estimate the distance between a person and an object of approximately known width.

**CCSS Common Core State Standards**

**Content Standards**

**G.SRT.4** Prove theorems about triangles.

**G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Mathematical Practices**

1 Make sense of problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

## 1 Special Segments of Similar Triangles

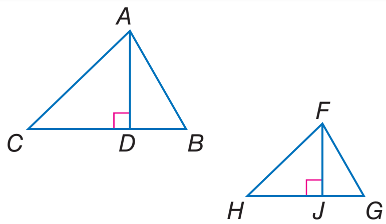
You learned in Lesson 7-2 that the corresponding side lengths of similar polygons, such as triangles, are proportional. This concept can be extended to other segments in triangles.

### Theorems Special Segments of Similar Triangles

**7.8** If two triangles are similar, the lengths of corresponding altitudes are proportional to the lengths of corresponding sides.

**Abbreviation** ~△s *have corr. altitudes proportional to corr. sides.*

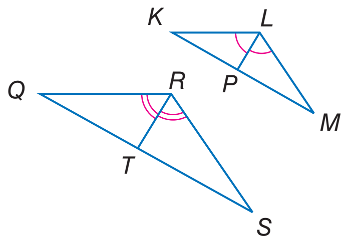
**Example** If $\triangle ABC \sim \triangle FGH$, then $\frac{AD}{FJ} = \frac{AB}{FG}$.




**7.9** If two triangles are similar, the lengths of corresponding angle bisectors are proportional to the lengths of corresponding sides.

**Abbreviation** ~△s *have corr. ∠ bisectors proportional to corr. sides.*

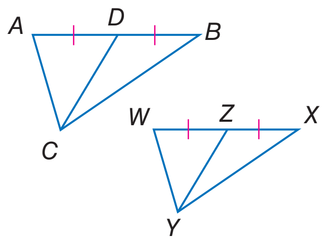
**Example** If $\triangle KLM \sim \triangle QRS$, then $\frac{LP}{RT} = \frac{LM}{RS}$.




**7.10** If two triangles are similar, the lengths of corresponding medians are proportional to the lengths of corresponding sides.

**Abbreviation** ~△s *have corr. medians proportional to corr. sides.*

**Example** If $\triangle ABC \sim \triangle WXY$, then $\frac{CD}{YZ} = \frac{AB}{WX}$.




You will prove Theorems 7.9 and 7.10 in Exercises 18 and 19, respectively.

**Real-WorldCareer**

**Athletic Trainer** Athletic trainers help prevent and treat sports injuries. They ensure that protective equipment is used properly and that people understand safe practices that prevent injury. An athletic trainer must have a bachelor's degree to be certified. Most also have master's degrees. Refer to Exercise 29.

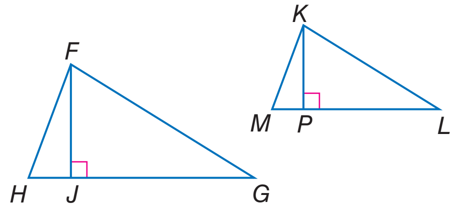
**StudyTip**

**Use Scale Factor** Example 1 could also have been solved by first finding the scale factor between $\triangle ABC$ and $\triangle FDG$. The ratio of the angle bisector in $\triangle ABC$ to the angle bisector in $\triangle FDG$ would then be equal to this scale factor.

## Proof Theorem 7.8

**Given:** $\triangle FGH \sim \triangle KLM$
$\overline{FJ}$ and $\overline{KP}$ are altitudes.

**Prove:** $\frac{FJ}{KP} = \frac{HF}{MK}$




**Paragraph Proof:**
Since $\triangle FGH \sim \triangle KLM$, $\angle H \cong \angle M$. $\angle FJH \cong \angle KPM$ because they are both right angles created by the altitudes drawn to the opposite side and all right angles are congruent.

Thus $\triangle HFJ \sim \triangle MKP$ by AA Similarity. So $\frac{FJ}{KP} = \frac{HF}{MK}$ by the definition of similar polygons.

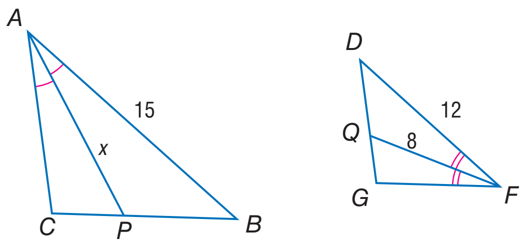
Since the corresponding altitudes are chosen at random, we need not prove Theorem 7.8 for every pair of altitudes.

You can use special segments in similar triangles to find missing measures.

PT

## Example 1 Use Special Segments in Similar Triangles

**In the figure, $\triangle ABC \sim \triangle FDG$. Find the value of $x$.**



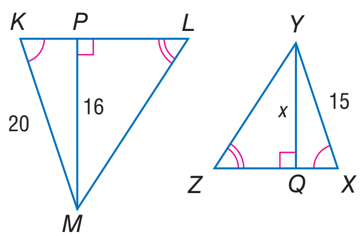

$\overline{AP}$ and $\overline{FQ}$ are corresponding angle bisectors and $\overline{AB}$ and $\overline{FD}$ are corresponding sides of similar triangles $ABC$ and $FDG$.

| | |
|---|---|
| $\frac{AP}{FQ} = \frac{AB}{FD}$ | $\sim\triangle$s have corr. $\angle$ bisectors proportional to the corr. sides. |
| $\frac{x}{8} = \frac{15}{12}$ | Substitution |
| $8 \cdot 15 = x \cdot 12$ | Cross Products Property |
| $120 = 12x$ | Simplify. |
| $10 = x$ | Divide each side by 12. |

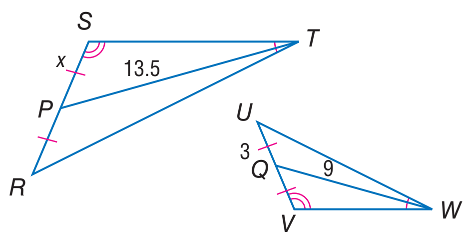
### GuidedPractice

**Find the value of $x$.**

**1A.**




**1B.**



You can use special segments in similar triangles to solve real-world problems.

**Real-World Link**

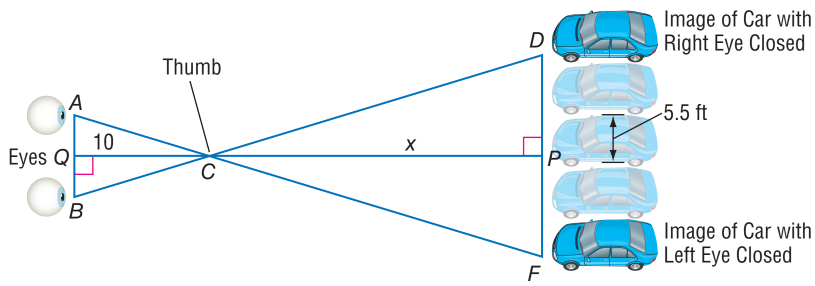
Hold your outstretched hand horizontal at arm's length with your palm facing you; for each hand width the sun is above the horizon, there is one remaining hour of sunlight.

**Source:** Sail Island Channels

## Real-World Example 2 Use Similar Triangles to Solve Problems

**ESTIMATING DISTANCES Liliana holds her arm straight out in front of her with her elbow straight and her thumb pointing up. Closing one eye, she aligns one edge of her thumb with a car she is sighting. Next she switches eyes without moving her head or her arm. The car appears to jump 4 car widths. If Liliana's arm is about 10 times longer than the distance between her eyes, and the car is about 5.5 feet wide, estimate the distance from Liliana's thumb to the car.**

**Understand** Make a diagram of the situation labeling the given distances and the distance you need to find as $x$. Also, label the vertices of the triangles formed.




**Note:** Not drawn to scale.

We assume that if Liliana's thumb is straight out in front of her, then $\overline{PC}$ is an altitude of $\triangle ABC$. Likewise, $\overline{QC}$ is the corresponding altitude. We assume that $\overline{AB} \parallel \overline{DF}$.

**Plan** Since $\overline{AB} \parallel \overline{DF}$, $\angle BAC \cong \angle DFC$ and $\angle CBA \cong \angle CDF$ by the Alternate Interior Angles Theorem. Therefore $\triangle ABC \sim \triangle FDC$ by AA Similarity. Write a proportion and solve for $x$.

**Solve**

$\frac{PC}{QC} = \frac{AB}{DF}$ Theorem 7.8

$\frac{10}{x} = \frac{1}{5.5 \cdot 4}$ Substitution

$\frac{10}{x} = \frac{1}{22}$ Simplify.

$10 \cdot 22 = x \cdot 1$ Cross Products Property

$220 = x$ Simplify.

So the estimated distance to the car is 220 feet.

**Check** The ratio of Liliana's arm length to the width between her eyes is 10 to 1. The ratio of the distance to the car to the distance the image of the car jumped is 22 to 220 or 10 to 1. ✓

### Guided Practice

**2.** Suppose Liliana stands at the back of her classroom and sights a clock on the wall at the front of the room. If the clock is 30 centimeters wide and appears to move 3 clock widths when she switches eyes, estimate the distance from Liliana's thumb to the clock.

## 2 Triangle Angle Bisector Theorem
An angle bisector of a triangle also divides the side opposite the angle proportionally.

**StudyTip**

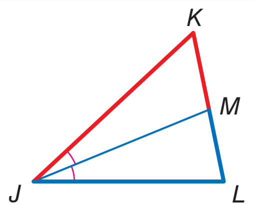
**Proportions** Another proportion that could be written using the Triangle Angle Bisector Theorem is $\frac{KM}{KJ} = \frac{LM}{LJ}$.

### Theorem 7.11 Triangle Angle Bisector

An angle bisector in a triangle separates the opposite side into two segments that are proportional to the lengths of the other two sides.

**Example** If $\overline{JM}$ is an angle bisector of $\triangle JKL$,

then $\frac{KM}{LM} = \frac{KJ}{LJ}$. ← segments with vertex $K$ ← segments with vertex $L$

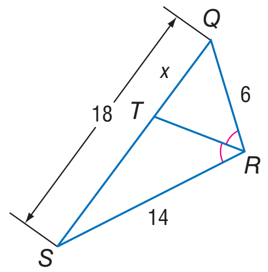



You will prove Theorem 7.11 in Exercise 25.

### Example 3 Use the Triangle Angle Bisector Theorem

**Find $x$.**

Since $\overline{RT}$ is an angle bisector of $\triangle QRS$, you can use the Triangle Angle Bisector Theorem to write a proportion.

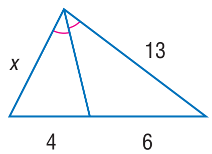



| | |
|---|---|
| $\frac{QT}{ST} = \frac{QR}{SR}$ | Triangle Angle Bisector Theorem |
| $\frac{x}{18 - x} = \frac{6}{14}$ | Substitution |
| $(18 - x)(6) = x \cdot 14$ | Cross Products Property |
| $108 - 6x = 14x$ | Simplify. |
| $108 = 20x$ | Add $6x$ to each side. |
| $5.4 = x$ | Divide each side by 20. |

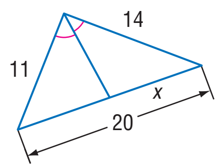
**GuidedPractice**

**Find the value of $x$.**

**3A.**




**3B.**

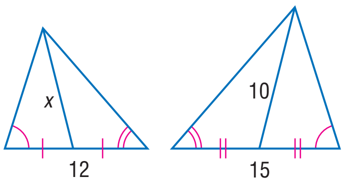



## Check Your Understanding

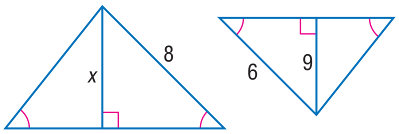
= Step-by-Step Solutions begin on page R14.

**Example 1** **Find $x$.**

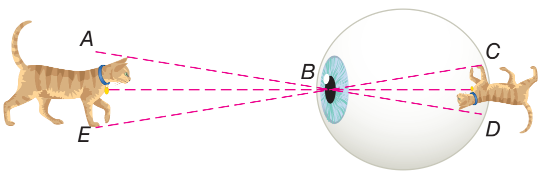
**1**




**2.**

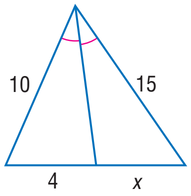



**Example 2** **3. VISION** A cat that is 10 inches tall forms a retinal image that is 7 millimeters tall. If $\triangle ABE \sim \triangle DBC$ and the distance from the pupil to the retina is 25 millimeters, how far away from your pupil is the cat?

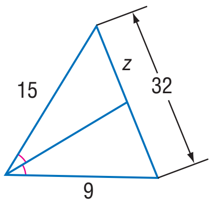


**Example 3** Find the value of each variable.

**4.**




**5.**

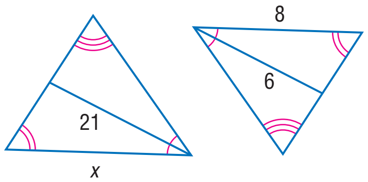



## Practice and Problem Solving

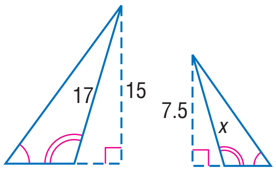
Extra Practice is on page R7.

**Example 1** Find *x*.

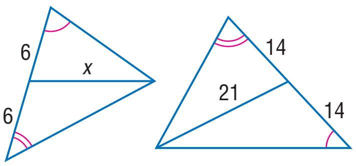
**6.**



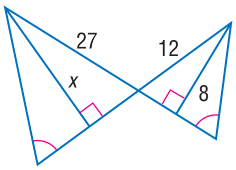

**7.**




**8.**

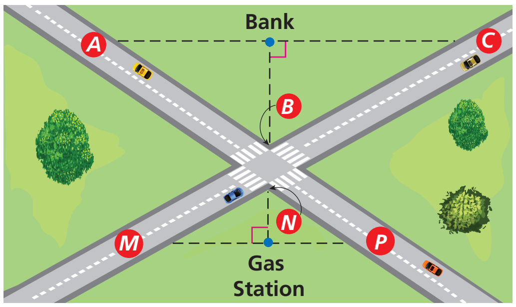



**9.**



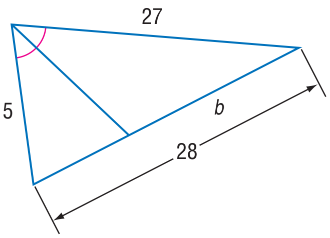

**Example 2**

**10.** **ROADWAYS** The intersection of the two roads shown forms two similar triangles. If *AC* is 382 feet, *MP* is 248 feet, and the gas station is 50 feet from the intersection, how far from the intersection is the bank?

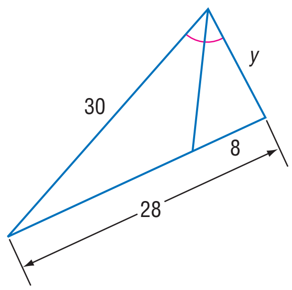



**Example 3** **CCSS SENSE-MAKING** Find the value of each variable.

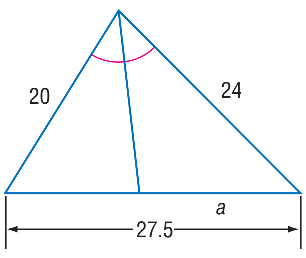
**11.**



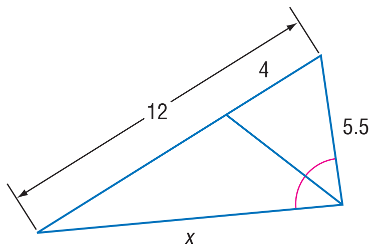

**12.**



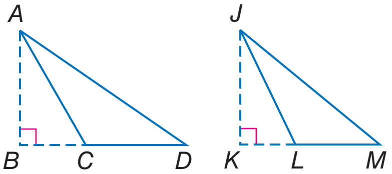

**13.**



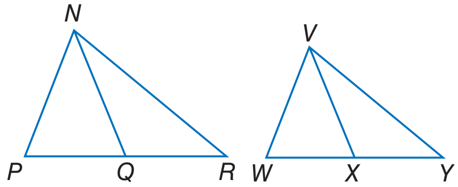

**14.**



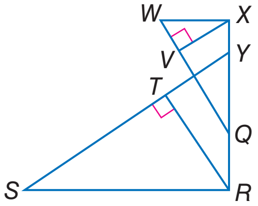
**15.** **ALGEBRA** If $\overline{AB}$ and $\overline{JK}$ are altitudes, $\triangle DAC \sim \triangle MJL$, $AB = 9$, $AD = 4x - 8$, $JK = 21$, and $JM = 5x + 3$, find $x$.




**16.** **ALGEBRA** If $\overline{NQ}$ and $\overline{VX}$ are medians, $\triangle PNR \sim \triangle WVY$, $NQ = 8$, $PR = 12$, $WY = 7x - 1$, and $VX = 4x + 2$, find $x$.




**17.** If $\triangle SRY \sim \triangle WXQ$, $\overline{RT}$ is an altitude of $\triangle SRY$, $\overline{XV}$ is an altitude of $\triangle WXQ$, $RT = 5$, $RQ = 4$, $QY = 6$, and $YX = 2$, find $XV$.

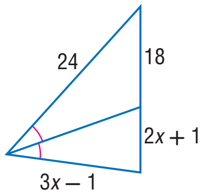



**18.** **PROOF** Write a paragraph proof of Theorem 7.9.

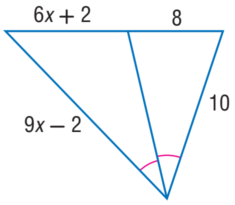
**19.** **PROOF** Write a two-column proof of Theorem 7.10.

**ALGEBRA** **Find $x$.**

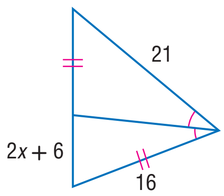
**20.**



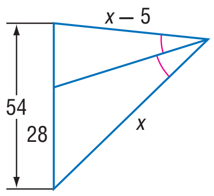

**21.**



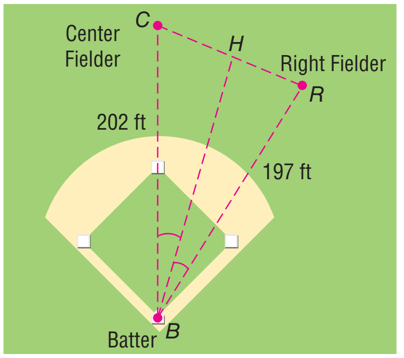

**22.**




**23.**




**24.** **SPORTS** Consider the triangle formed by the path between a batter, center fielder, and right fielder as shown. If the batter gets a hit that bisects the triangle at $\angle B$, is the center fielder or the right fielder closer to the ball? Explain your reasoning.

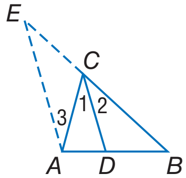



**CCSS** **ARGUMENTS** **Write a two-column proof.**

**25.** Theorem 7.11

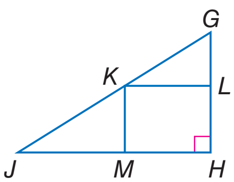
**Given:** $\overline{CD}$ bisects $\angle ACB$.
By construction, $\overline{AE} \parallel \overline{CD}$.

**Prove:** $\dfrac{AD}{DB} = \dfrac{AC}{BC}$




**26.** **Given:** $\angle H$ is a right angle.
$L$, $K$, and $M$ are midpoints.

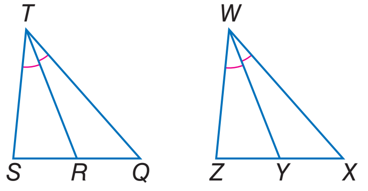
**Prove:** $\angle LKM$ is a right angle.



**PROOF** **Write a two-column proof.**

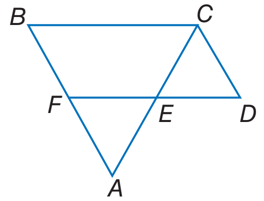
**27.** **Given:** $\triangle QTS \sim \triangle XWZ$, $\overline{TR}$ and $\overline{WY}$ are angle bisectors.

**Prove:** $\frac{TR}{WY} = \frac{QT}{XW}$

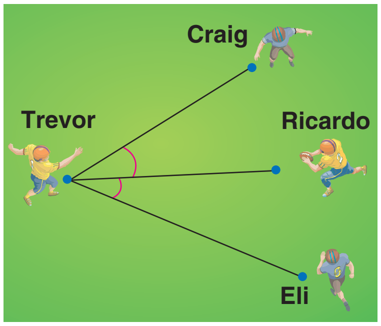



**28.** **Given:** $\overline{FD} \parallel \overline{BC}$, $\overline{BF} \parallel \overline{CD}$, $\overline{AC}$ bisects $\angle C$.

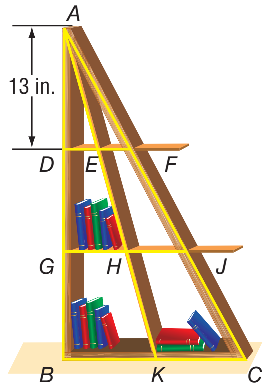
**Prove:** $\frac{DE}{EC} = \frac{BA}{AC}$




**29.** **SPORTS** During football practice, Trevor threw a pass to Ricardo as shown below. If Eli is farther from Trevor when he completes the pass to Ricardo and Craig and Eli move at the same speed, who will reach Ricardo to tackle him first?

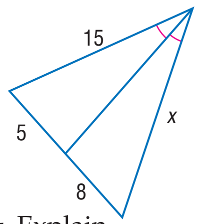



**30.** **SHELVING** In the bookshelf shown, the distance between each shelf is 13 inches and $\overline{AK}$ is a median of $\triangle ABC$. If $EF$ is $3\frac{1}{3}$ inches, what is $BK$?




## H.O.T. Problems Use Higher-Order Thinking Skills

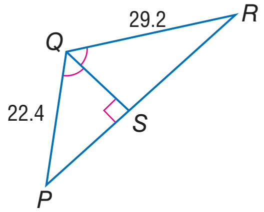
**31.** **ERROR ANALYSIS** Chun and Traci are determining the value of $x$ in the figure. Chun says to find $x$, solve the proportion $\frac{5}{8} = \frac{15}{x}$, but Traci says to find $x$, the proportion $\frac{5}{x} = \frac{8}{15}$ should be solved. Is either of them correct? Explain.




**32.** **CCSS ARGUMENTS** Find a counterexample to the following statement. Explain.

*If the measure of an altitude and side of a triangle are proportional to the corresponding altitude and corresponding side of another triangle, then the triangles are similar.*

**33.** **CHALLENGE** The perimeter of $\triangle PQR$ is 94 units. $\overline{QS}$ bisects $\angle PQR$. Find $PS$ and $RS$.




**34.** **OPEN ENDED** Draw two triangles so that the measures of corresponding medians and a corresponding side are proportional, but the triangles are not similar.

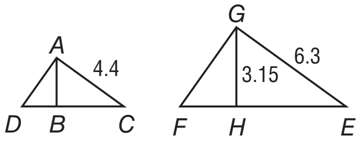
**35.** **WRITING IN MATH** Compare and contrast Theorem 7.9 and the Triangle Angle Bisector Theorem.

## Standardized Test Practice

**36. ALGEBRA** Which shows 0.00234 written in scientific notation?

**A** $2.34 \times 10^5$
**B** $2.34 \times 10^3$
**C** $2.34 \times 10^{-2}$
**D** $2.34 \times 10^{-3}$

**37. SHORT RESPONSE** In the figures below, $\overline{AB} \perp \overline{DC}$ and $\overline{GH} \perp \overline{FE}$.




If $\triangle ACD \sim \triangle GEF$, find $AB$.

**38.** Quadrilateral *HJKL* is a parallelogram. If the diagonals are perpendicular, which statement must be true?

**F** Quadrilateral *HJKL* is a square.
**G** Quadrilateral *HJKL* is a rectangle.
**H** Quadrilateral *HJKL* is a rhombus.
**J** Quadrilateral *HJKL* is an isosceles trapezoid.

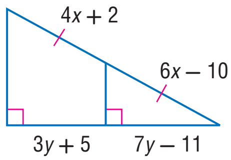
**39. SAT/ACT** The sum of three numbers is 180. Two of the numbers are the same, and each of them is one third of the greatest number. What is the least number?

**A** 15
**B** 30
**C** 36
**D** 45
**E** 60

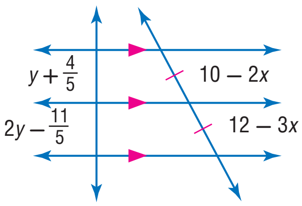
## Spiral Review

**ALGEBRA** **Find $x$ and $y$.** (Lesson 7-4)

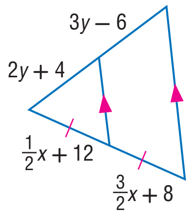
**40.**




**41.**

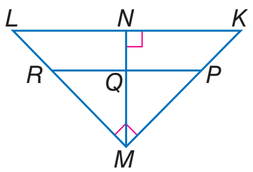



**42.**

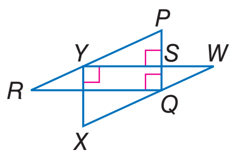



**Find the indicated measure(s).** (Lesson 7-3)

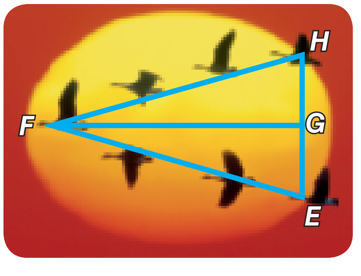
**43.** If $\overline{PR} \parallel \overline{KL}$, $KN = 9$, $LN = 16$, and $PM = 2(KP)$, find $KP$, $KM$, $MR$, $ML$, $MN$, and $PR$.




**44.** If $\overline{PR} \parallel \overline{WX}$, $WX = 10$, $XY = 6$, $WY = 8$, $RY = 5$, and $PS = 3$, find $PY$, $SY$, and $PQ$.




**45. GEESE** A flock of geese flies in formation. Prove that $\triangle EFG \cong \triangle HFG$ if $\overline{EF} \cong \overline{HF}$ and that $G$ is the midpoint of $\overline{EH}$. (Lesson 4-4)




Chase Swift/CORBIS

## Skills Review

**Find the distance between each pair of points.**

**46.** $E(-3, -2)$, $F(5, 8)$

**47.** $A(2, 3)$, $B(5, 7)$

**48.** $C(-2, 0)$, $D(6, 4)$

**49.** $W(7, 3)$, $Z(-4, -1)$

**50.** $J(-4, -5)$, $K(2, 9)$

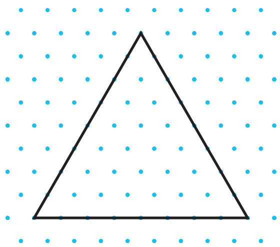
**51.** $R(-6, 10)$, $S(8, -2)$

# Geometry Lab
# Fractals

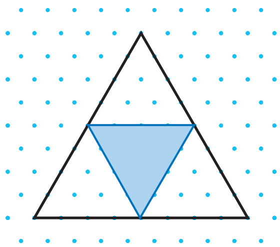
A **fractal** is a geometric figure that is created using iteration. **Iteration** is a process of repeating the same operation over and over again. Fractals are **self-similar**, which means that the smaller details of the shape have the same geometric characteristics as the original form.

## Activity 1

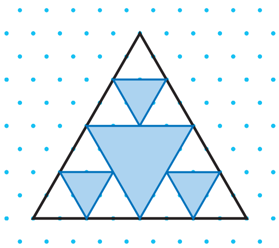
**Stage 0** Draw an equilateral triangle on isometric dot paper in which each side is 8 units long.



**Stage 1** Connect the midpoints of the sides to form another triangle. Shade the center triangle.



**Stage 2** Repeat the process using the three unshaded triangles. Connect the midpoints of the sides to form three other triangles.



If you repeat this process indefinitely, the figure that results is called the Sierpinski Triangle.

### Analyze the Results

**1.** If you continue the process, how many unshaded triangles will you have at Stage 3?

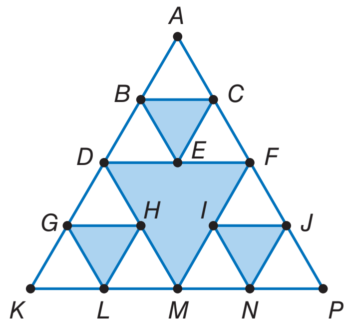
**2.** What is the perimeter of an unshaded triangle in Stage 4?

**3.** If you continue the process indefinitely, what will happen to the perimeters of the unshaded triangles?

**4.** **CHALLENGE** Complete the proof below.

**Given:** $\triangle KAP$ is equilateral. $D$, $F$, $M$, $B$, $C$, and $E$ are midpoints of $\overline{KA}$, $\overline{AP}$, $\overline{PK}$, $\overline{DA}$, $\overline{AF}$, and $\overline{FD}$, respectively.

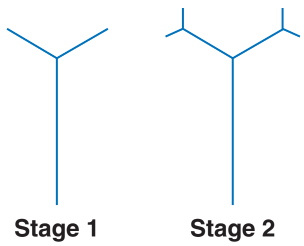
**Prove:** $\triangle BAC \sim \triangle KAP$




**5.** A *fractal tree* can be drawn by making two new branches from the endpoint of each original branch, each one-third as long as the previous branch.

**a.** Draw Stages 3 and 4 of a fractal tree. How many total branches do you have in Stages 1 through 4? (Do not count the stems.)

**b.** Write an expression to predict the number of branches at each stage.




(continued on the next page)

# Geometry Lab
# Fractals *Continued*

Not all iterative processes involve manipulation of geometric shapes. Some iterative processes can be translated into formulas or algebraic equations, similar to the expression you wrote in Exercise 5 on the previous page.

## Activity 2

*Pascal's Triangle* is a numerical pattern in which each row begins and ends with 1 and all other terms in the row are the sum of the two numbers above it. Find a formula in terms of the row number for any row in Pascal's Triangle.

**Step 1** Draw rows 1 through 5 in Pascal's Triangle.

**Step 2** Find the sum of values in each row.

**Step 3** Find a pattern using the **row number** that can be used to determine the sum of any row.

| Row | Pascal's Triangle | Sum | Pattern |
|---|---|---|---|
| 1 | **1** | 1 | $2^0 = 2^{1-1}$ |
| 2 | **1 1** | 2 | $2^1 = 2^{2-1}$ |
| 3 | **1 2 1** | 4 | $2^2 = 2^{3-1}$ |
| 4 | **1 3 3 1** | 8 | $2^3 = 2^{4-1}$ |
| 5 | **1 4 6 4 1** | 16 | $2^4 = 2^{5-1}$ |

### Analyze the Results

**6.** Write a formula for the sum $S$ of any row $n$ in the Pascal Triangle.

**7.** What is the sum of the values in the eighth row of Pascal's Triangle?

## Exercises

**Write a formula for $F(x)$.**

**8.**

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $F(x)$ | 3 | 7 | 11 | 15 | 19 |

**9.**

| $x$ | 0 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| $F(x)$ | 0 | 20 | 90 | 210 | 380 |

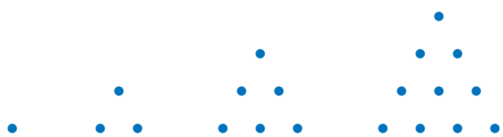
**10.**

| $x$ | 1 | 2 | 4 | 8 | 10 |
|---|---|---|---|---|---|
| $F(x)$ | 1 | 0.5 | 0.25 | 0.125 | 0.1 |

**11.**

| $x$ | 4 | 9 | 16 | 25 | 36 |
|---|---|---|---|---|---|
| $F(x)$ | 5 | 6 | 7 | 8 | 9 |

**12. CHALLENGE** The figural pattern below represents a sequence of figural numbers called *triangular numbers*. How many dots will be in the 8th term in the sequence? Is it possible to write a formula that can be used to determine the number of dots in the $n$th triangular number in the series? If so, write the formula. If not, explain why not.



1 2 3 4

LESSON 7-6

# Similarity Transformations

## Then

- You identified congruence transformations.

## Now

1. Identify similarity transformations.
2. Verify similarity after a similarity transformation.

## Why?

- Adriana uses a copier to enlarge a movie ticket to use as the background for a page in her movie ticket scrapbook. She places the ticket on the glass of the copier. Then she must decide what percentage to input in order to create an image that is three times as big as her original ticket.


### NewVocabulary

- dilation
- similarity transformation
- center of dilation
- scale factor of a dilation
- enlargement
- reduction

### Common Core State Standards

**Content Standards**

**G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Mathematical Practices**

6 Attend to precision.

4 Model with mathematics.

## 1 Identify Similarity Transformations

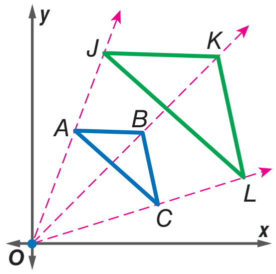
Recall from Lesson 4-7 that a *transformation* is an operation that maps an original figure, the *preimage*, onto a new figure called the *image*.

A **dilation** is a transformation that enlarges or reduces the original figure proportionally. Since a dilation produces a similar figure, a dilation is a type of **similarity transformation**.

Dilations are performed with respect to a fixed point called the **center of dilation**.

The **scale factor of a dilation** describes the extent of the dilation. The scale factor is the ratio of a length on the image to a corresponding length on the preimage.

The letter $k$ usually represents the scale factor of a dilation. The value of $k$ determines whether the dilation is an enlargement or a reduction.



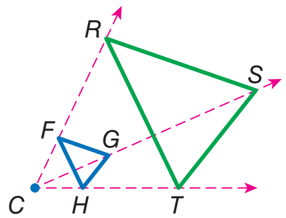

$\triangle JKL$ is a dilation of $\triangle ABC$.
Center of dilation: (0, 0)
Scale factor: $\frac{JK}{AB}$

### ConceptSummary Types of Dilations

A dilation with a scale factor greater than 1 produces an **enlargement**, or an image that is larger than the original figure.

**Symbols** If $k > 1$, the dilation is an enlargement.

**Example** $\triangle FGH$ is dilated by a scale factor of 3 to produce $\triangle RST$. Since $3 > 1$, $\triangle RST$ is an enlargement of $\triangle FGH$.

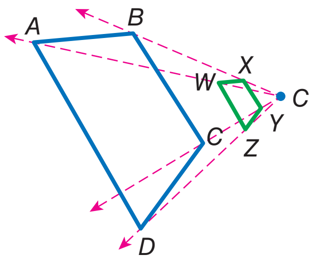



$k < 1$

A dilation with a scale factor between 0 and 1 produces a **reduction**, an image that is smaller than the original figure.

**Symbols** If $0 < k < 1$, the dilation is a reduction.

**Example** $ABCD$ is dilated by a scale factor of $\frac{1}{4}$ to produce $WXYZ$. Since $0 < \frac{1}{4} < 1$, $WXYZ$ is a reduction of $ABCD$.




$0 < k < 1$

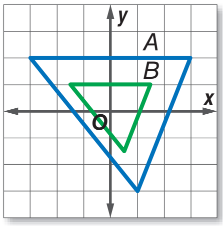
**StudyTip**

**Multiple Representations** The scale factor of a dilation can be represented as a fraction, a decimal, or as a percent. For example, a scale factor of $\frac{2}{5}$ can also be written as 0.4 or as 40%.

## Example 1 Identify a Dilation and Find Its Scale Factor

**Determine whether the dilation from *A* to *B* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.**

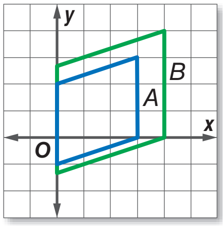
**a.**




*B* is smaller than *A*, so the dilation is a reduction.

The distance between the vertices at (−3, 2) and (3, 2) for *A* is 6 and from the vertices at (−1.5, 1) and (1.5, 1) for *B* is 3. So the scale factor is $\frac{3}{6}$ or $\frac{1}{2}$.

**b.**

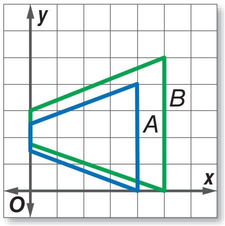



*B* is larger than *A*, so the dilation is an enlargement.

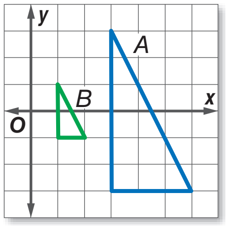
The distance between the vertices at (3, 3) and (3, 0) for *A* is 3 and between the vertices at (4, 4) and (4, 0) for *B* is 4. So the scale factor is $\frac{4}{3}$.

### GuidedPractice

**1A.**




**1B.**




Dilations and their scale factors are used in many real-world situations.

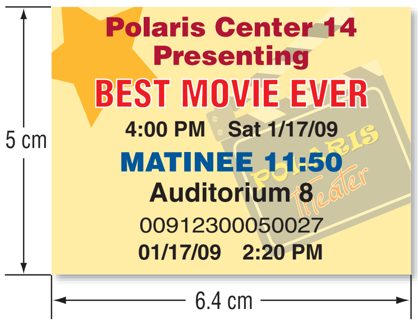
**Real-WorldLink**

Hew Weng Fatt accepted a contest challenge to collect the most movie stubs from a certain popular fantasy movie. He collected 6561 movie stubs in 38 days!

**Source:** *Youth2, Star Publications*

## Real-World Example 2 Find and Use a Scale Factor

**COLLECTING Refer to the beginning of the lesson. By what percent should Adriana enlarge the ticket stub so that the dimensions of its image are 3 times that of her original? What will be the dimensions of the enlarged image?**




Adriana wants to create a dilated image of her ticket stub using the copier. The scale factor of her enlargement is 3. Written as a percent, the scale factor is (3 • 100)% or 300%. Now find the dimension of the enlarged image using the scale factor.

width: 5 cm • 300% = 15 cm    length: 6.4 cm • 300% = 19.2 cm

The enlarged ticket stub image will be 15 centimeters by 19.2 centimeters.

### GuidedPractice

**2.** If the resulting ticket stub image was 1.5 centimeters wide by about 1.9 centimeters long instead, what percent did Adriana mistakenly use to dilate the original image? Explain your reasoning.

## 2 Verify Similarity

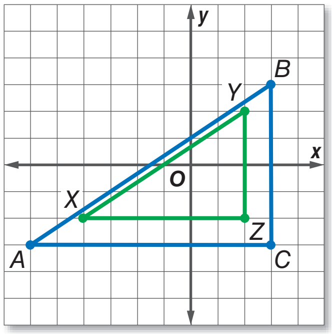
You can verify that a dilation produces a similar figure by comparing corresponding sides and angles. For triangles, you can also use SAS Similarity.

### Example 3 Verify Similarity after a Dilation

**Graph the original figure and its dilated image. Then verify that the dilation is a similarity transformation.**

**a. original: $A(-6, -3)$, $B(3, 3)$, $C(3, -3)$; image: $X(-4, -2)$, $Y(2, 2)$, $Z(2, -2)$**

Graph each figure. Since $\angle C$ and $\angle Z$ are both right angles, $\angle C \cong \angle Z$. Show that the lengths of the sides that include $\angle C$ and $\angle Z$ are proportional.



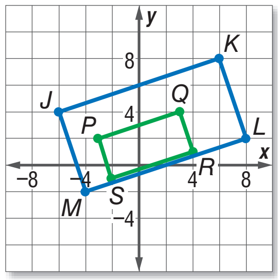

Use the coordinate grid to find the side lengths.

$\frac{XZ}{AC} = \frac{6}{9}$ or $\frac{2}{3}$, and $\frac{YZ}{BC} = \frac{4}{6}$ or $\frac{2}{3}$, so $\frac{XZ}{AC} = \frac{YZ}{BC}$.

Since the lengths of the sides that include $\angle C$ and $\angle Z$ are proportional, $\triangle XYZ \sim \triangle ABC$ by SAS Similarity.

**b. original: $J(-6, 4)$, $K(6, 8)$, $L(8, 2)$, $M(-4, -2)$;**
**image: $P(-3, 2)$, $Q(3, 4)$, $R(4, 1)$, $S(-2, -1)$**

Use the Distance Formula to find the length of each side.




$JK = \sqrt{[6-(-6)]^2 + (8-4)^2} = \sqrt{160}$ or $4\sqrt{10}$

$PQ = \sqrt{[3-(-3)]^2 + (4-2)^2} = \sqrt{40}$ or $2\sqrt{10}$

$KL = \sqrt{(8-6)^2 + (2-8)^2} = \sqrt{40}$ or $2\sqrt{10}$

$QR = \sqrt{(4-3)^2 + (1-4)^2} = \sqrt{10}$

$LM = \sqrt{(-4-8)^2 + (-2-2)^2} = \sqrt{160}$ or $4\sqrt{10}$

$RS = \sqrt{(-2-4)^2 + (-1-1)^2} = \sqrt{40}$ or $2\sqrt{10}$

$MJ = \sqrt{[-6-(-4)]^2 + [4-(-2)]^2} = \sqrt{40}$ or $2\sqrt{10}$

$SP = \sqrt{[-3-(-2)]^2 + [2-(-1)]^2} = \sqrt{10}$

Find and compare the ratios of corresponding sides.

$\frac{PQ}{JK} = \frac{2\sqrt{10}}{4\sqrt{10}}$ or $\frac{1}{2}$ $\quad \frac{QR}{KL} = \frac{\sqrt{10}}{2\sqrt{10}}$ or $\frac{1}{2}$ $\quad \frac{RS}{LM} = \frac{2\sqrt{10}}{4\sqrt{10}}$ or $\frac{1}{2}$ $\quad \frac{SP}{MJ} = \frac{\sqrt{10}}{2\sqrt{10}}$ or $\frac{1}{2}$

$PQRS$ and $JKLM$ are both rectangles. This can be proved by showing that diagonals $\overline{PR} \cong \overline{SQ}$ and $\overline{JL} \cong \overline{KM}$ are congruent using the Distance Formula. Since they are both rectangles, their corresponding angles are congruent.

Since $\frac{PQ}{JK} = \frac{QR}{KL} = \frac{RS}{LM} = \frac{SP}{MJ}$ and corresponding angles are congruent, $PQRS \sim JKLM$.

**Study Tip**

**Center of Dilation** Unless otherwise stated, all dilations on the coordinate plane use the origin as their center of dilation.

### Guided Practice

**3A.** original: $A(2, 3)$, $B(0, 1)$, $C(3, 0)$
image: $D(4, 6)$, $F(0, 2)$, $G(6, 0)$

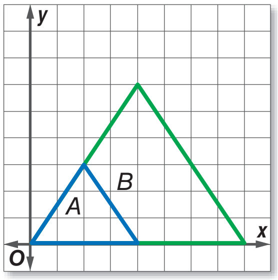
**3B.** original: $H(0, 0)$, $J(6, 0)$, $K(6, 4)$, $L(0, 4)$
image: $W(0, 0)$, $X(3, 0)$, $Y(3, 2)$, $Z(0, 2)$

## Check Your Understanding

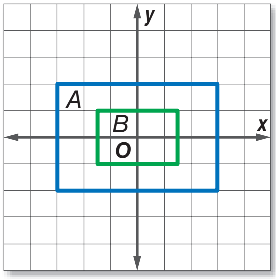
= Step-by-Step Solutions begin on page R14.

**Example 1** **Determine whether the dilation from *A* to *B* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.**

**1.**

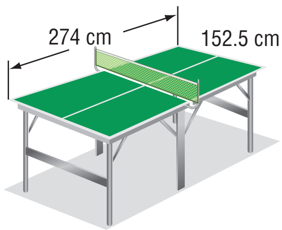



**2.**



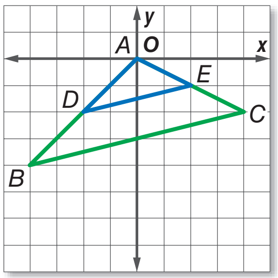

**Example 2**

**3.** **GAMES** The dimensions of a regulation tennis court are 27 feet by 78 feet. The dimensions of a table tennis table are 152.5 centimeters by 274 centimeters. Is a table tennis table a dilation of a tennis court? If so, what is the scale factor? Explain.

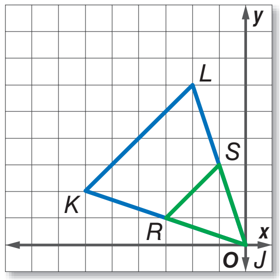



**Example 3** **CCSS ARGUMENTS** **Verify that the dilation is a similarity transformation.**

**4.**




**5.**

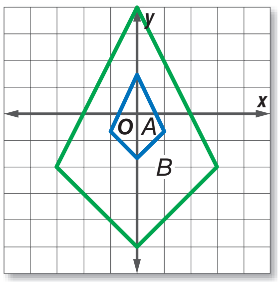



## Practice and Problem Solving

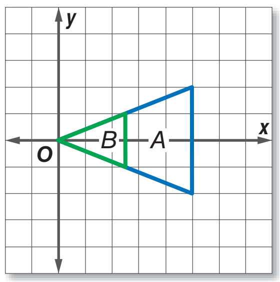
Extra Practice is on page R7.

**Example 1** **Determine whether the dilation from *A* to *B* is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.**

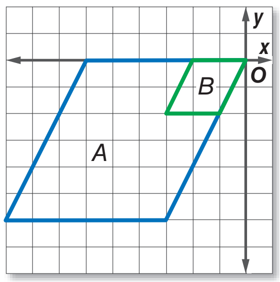
**6.**



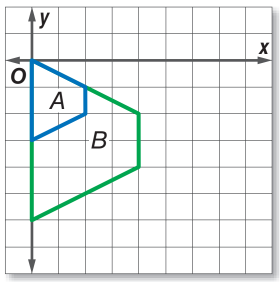

**7.**




**8.**

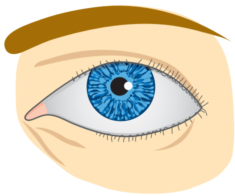


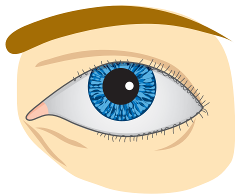

**9.**



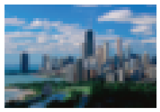
**Determine whether each dilation is an *enlargement* or *reduction*.**

**10.** Before After





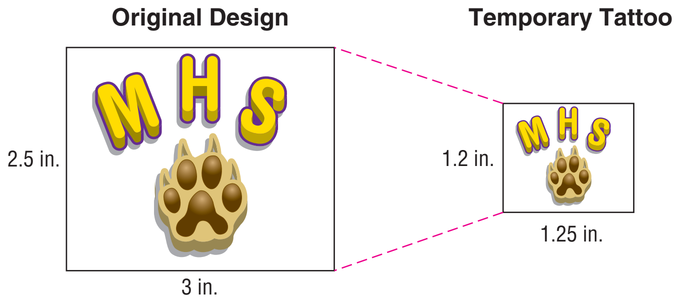
**11.** Painting Postcard



**Example 2**

**12. YEARBOOK** Jordan is putting a photo of the lacrosse team in a full-page layout in the yearbook. The original photo is 4 inches by 6 inches. If the photo in the yearbook is $6\frac{2}{3}$ inches by 10 inches, is the yearbook photo a dilation of the original photo? If so, what is the scale factor? Explain.

**13.** CCSS **MODELING** Candace created a design to be made into temporary tattoos for a homecoming game as shown. Is the temporary tattoo a dilation of the original design? If so, what is the scale factor? Explain.




**Example 3**

**Graph the original figure and its dilated image. Then verify that the dilation is a similarity transformation.**

**14.** $M(1, 4)$, $P(2, 2)$, $Q(5, 5)$; $S(-3, 6)$, $T(0, 0)$, $U(9, 9)$

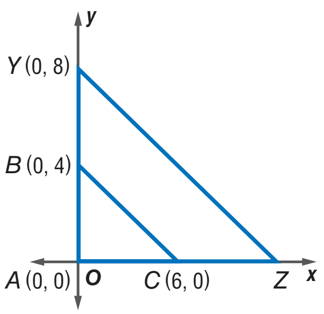
**15.** $A(1, 3)$, $B(-1, 2)$, $C(1, 1)$; $D(-7, -1)$, $E(1, -5)$

**16.** $V(-3, 4)$, $W(-5, 0)$, $X(1, 2)$; $Y(-6, -2)$, $Z(3, 1)$

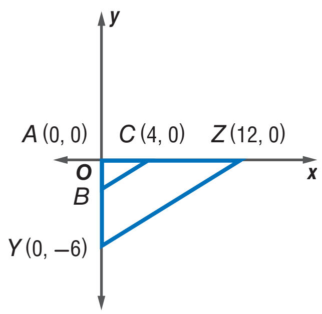
**17.** $J(-6, 8)$, $K(6, 6)$, $L(-2, 4)$; $D(-12, 16)$, $G(12, 12)$, $H(-4, 8)$

**If $\triangle ABC \sim \triangle AYZ$, find the missing coordinate.**

**18.**




**19**



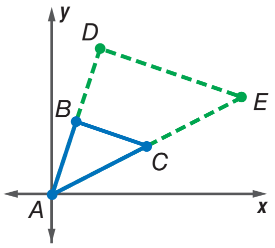
**20. GRAPHIC ART** Aimee painted the sample sign shown using $\frac{1}{2}$ bottle of glass paint. The actual sign she will paint in a shop window is to be 3 feet by $7\frac{1}{2}$ feet.


**a.** Explain why the actual sign is a dilation of her sample.

**b.** How many bottles of paint will Aimee need to complete the actual sign?

**21** **MULTIPLE REPRESENTATIONS** In this problem, you will investigate similarity of triangles on the coordinate plane.

**a. Geometric** Draw a triangle with vertex *A* at the origin. Make sure that the two additional vertices *B* and *C* have whole-number coordinates. Draw a similar triangle that is twice as large as △*ABC* with its vertex also located at the origin. Label the triangle *ADE*.




**b. Geometric** Repeat the process in part **a** two times. Label the second pair of triangles *MNP* and *MQR* and the third pair *TWX* and *TYZ*. Use different scale factors than part **a**.

**c. Tabular** Copy and complete the table below with the appropriate values.

| Coordinates | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| △*ABC* | | △*ADE* | | △*MNP* | | △*MQR* | | △*TWX* | | △*TYZ* | |
| *A* | | *A* | | *M* | | *M* | | *T* | | *T* | |
| *B* | | *D* | | *N* | | *Q* | | *W* | | *Y* | |
| *C* | | *E* | | *P* | | *R* | | *X* | | *Z* | |

**d. Verbal** Make a conjecture about how you could predict the coordinates of a dilated triangle with a scale factor of *n* if the two similar triangles share a corresponding vertex at the origin.

## H.O.T. Problems Use Higher-Order Thinking Skills

**22. CHALLENGE** *MNOP* is a dilation of *ABCD*. How is the scale factor of the dilation related to the similarity ratio of *ABCD* to *MNOP*? Explain your reasoning.

**23. CCSS REASONING** The coordinates of two triangles are provided in the table at the right. Is △*XYZ* a dilation of △*PQR*? Explain.

| △*PQR* | | △*XYZ* | |
|---|---|---|---|
| *P* | (*a*, *b*) | *X* | (3*a*, 2*b*) |
| *Q* | (*c*, *d*) | *Y* | (3*c*, 2*d*) |
| *R* | (*e*, *f*) | *Z* | (3*e*, 2*f*) |

**OPEN ENDED Describe a real-world example of each transformation other than those given in this lesson.**

**24.** enlargement **25.** reduction **26.** congruence transformation

**27. WRITING IN MATH** Explain how you can use scale factor to determine whether a transformation is an enlargement, a reduction, or a congruence transformation.

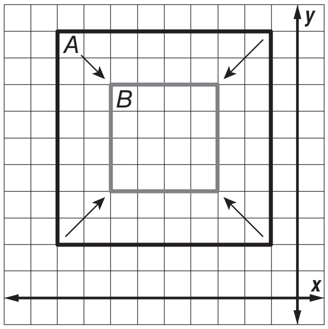
## Standardized Test Practice

**28. ALGEBRA** Which equation describes the line that passes through $(-3, 4)$ and is perpendicular to $3x - y = 6$?

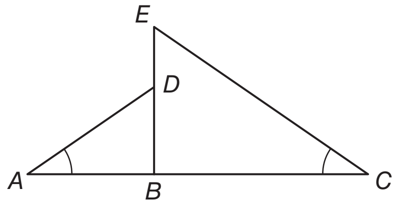
**A** $y = -\frac{1}{3}x + 4$ **C** $y = 3x + 4$

**B** $y = -\frac{1}{3}x + 3$ **D** $y = 3x + 3$

**29. SHORT RESPONSE** What is the scale factor of the dilation shown below?




**30.** In the figure below, $\angle A \cong \angle C$.




Which additional information would *not* be enough to prove that $\triangle ADB \sim \triangle CEB$?

**F** $\frac{AB}{DB} = \frac{CB}{EB}$ **H** $\overline{ED} \cong \overline{DB}$

**G** $\angle ADB \cong \angle CEB$ **J** $\overline{EB} \perp \overline{AC}$

**31. SAT/ACT** $x = \frac{6}{4p+3}$ and $xy = \frac{3}{4p+3}$. $y =$

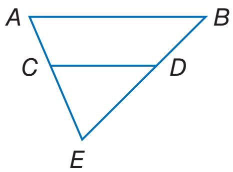
**A** 4 **C** 1 **E** $\frac{1}{2}$

**B** 2 **D** $\frac{3}{4}$

## Spiral Review

**32. LANDSCAPING** Shea is designing two gardens shaped like similar triangles. One garden has a perimeter of 53.5 feet, and the longest side is 25 feet. She wants the second garden to have a perimeter of 32.1 feet. Find the length of the longest side of this garden. (Lesson 7-5)

**Determine whether $\overline{AB} \parallel \overline{CD}$. Justify your answer.** (Lesson 7-4)



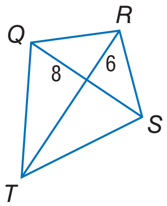

**33.** $AC = 8.4$, $BD = 6.3$, $DE = 4.5$, and $CE = 6$

**34.** $AC = 7$, $BD = 10.5$, $BE = 22.5$, and $AE = 15$

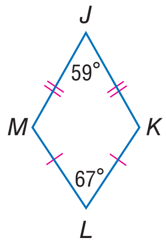
**35.** $AB = 8$, $AE = 9$, $CD = 4$, and $CE = 4$

**If each figure is a kite, find each measure.** (Lesson 6-6)

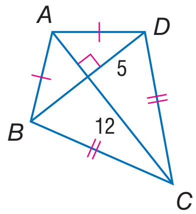
**36.** $QR$




**37.** $m\angle K$




**38.** $BC$




**39. PROOF** Write a coordinate proof for the following statement. (Lesson 4-8)
*If a line segment joins the midpoints of two sides of a triangle, then it is parallel to the third side.*

## Skills Review

**Solve each equation.**

**40.** $145 = 29 \cdot t$

**41.** $216 = d \cdot 27$

**42.** $2r = 67 \cdot 5$

**43.** $100t = \frac{70}{240}$

**44.** $\frac{80}{4} = 14d$

**45.** $\frac{2t + 15}{t} = 92$

# LESSON 7-7 Scale Drawings and Models

**Then**

- You used scale factors to solve problems with similar polygons.

**Now**

1. Interpret scale models.
2. Use scale factors to solve problems.

**Why?**

- In Saint-Luc, Switzerland, Le Chemin des planetes, has constructed a scale model of each planet in the solar system. It is one of the largest complete three-dimensional scale models of the solar system. The diameter of the center of the model of Saturn shown is 121 millimeters; the diameter of the real planet is about 121,000 kilometers.

**NewVocabulary**
scale model
scale drawing
scale

**Common Core State Standards**

**Content Standards**
G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

**Mathematical Practices**
4 Model with mathematics.
7 Look for and make use of structure.

## 1 Scale Models

A **scale model** or a **scale drawing** is an object or drawing with lengths proportional to the object it represents. The **scale** of a model or drawing is the ratio of a length on the model or drawing to the actual length of the object being modeled or drawn.

### Example 1 Use a Scale Drawing

**MAPS The scale on the map shown is 0.4 inch : 40 miles. Find the actual distance from Nashville to Memphis.**


Use a ruler. The distance between Nashville and Memphis is about 1.5 inches.

**Method 1** Write and solve a proportion.

Let $x$ represent the distance between Nashville and Memphis.

| | Scale | | Nashville to Memphis | |
|---|---|---|---|---|
| map → / actual → | $\frac{0.4 \text{ in.}}{40 \text{ mi}}$ | = | $\frac{1.5 \text{ in.}}{x \text{ mi}}$ | ← map / ← actual |

$0.4 \cdot x = 40 \cdot 1.5$ Cross Products Property

$x = 150$ Simplify.

**Method 2** Write and solve an equation.

Let $a$ = actual distance in miles between Nashville and Memphis and $m$ = map distance in inches. Write the scale as $\frac{40 \text{ mi}}{0.4 \text{ in.}}$, which is $40 \div 0.4$ or 100 miles per inch. So for every inch on the map, the actual distance is 100 miles.

$a = 100 \cdot m$ Write an equation.

$= 100 \cdot 1.5$ $m = 1.5$ in.

$= 150$ Solve.

**CHECK** Use dimensional analysis.

$$\text{mi} = \frac{\text{mi}}{\text{in.}} \cdot \text{in.} \Rightarrow \text{mi} = \text{mi} \checkmark$$

The distance between Nashville and Memphis is 150 miles.

**GuidedPractice**

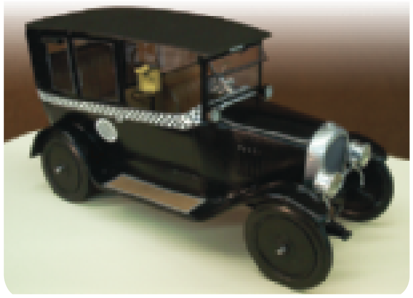
1. **MAPS** Find the actual distance between Nashville and Chattanooga.

Robert F. Bukaty/AP Images

## 2 Use Scale Factors

The scale factor of a drawing or scale model is written as a unitless ratio in simplest form. Scale factors are always written so that the model length in the ratio comes first.

### Example 2 Find the Scale



**SCALE MODEL This is a miniature replica of a 1923 Checker Cab. The length of the model is 6.5 inches. The actual length of the car was 13 feet.**

**a. What is the scale of the model?**

To find the scale, write the ratio of a model length to an actual length.

$$\frac{\text{model length}}{\text{actual length}} = \frac{6.5 \text{ in.}}{13 \text{ ft}} \text{ or } \frac{1 \text{ in.}}{2 \text{ ft}}$$

The scale of the model is 1 in.:2 ft.

**b. How many times as long as the actual car is the model?**

To answer this question, find the scale factor of the model. Multiply by a conversion factor that relates inches to feet to obtain a unitless ratio.

$$\frac{1 \text{ in.}}{2 \text{ ft}} = \frac{1 \text{ in.}}{2 \text{ ft}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{1}{24}$$

The scale factor is 1:24. That is, the model is $\frac{1}{24}$ as long as the actual car.

**StudyTip**

**CCSS Regularity** The scale factor of a model that is smaller than the original object is between 0 and 1 and the scale factor for a model that is larger than the original object is greater than 1.

**GuidedPractice**

**2. SCALE MODEL** Mrs. Alejandro's history class made a scale model of the Alamo that is 3 feet tall. The actual height of the building is 33 feet 6 inches.

**A.** What is the scale of the model?

**B.** How many times as tall as the actual building is the model? How many times as tall as the model is the actual building?

**Real-WorldLink**

The St. Louis Gateway Arch is the tallest national monument in the United States at 630 feet. The span of the base is also 630 feet. The arch weighs 17,246 tons and can sway a maximum of 9 inches in each direction during high winds.

**Source:** Gateway Arch Facts



### Real-World Example 3 Construct a Scale Model

**SCALE MODEL Suppose you want to build a model of the St. Louis Gateway Arch that is no more than 11 inches tall. Choose an appropriate scale and use it to determine the height of the model. Use the information at the left.**

The actual monument is 630 feet tall. Since 630 feet ÷ 11 inches = 57.3 feet per inch, a scale of 1 inch = 60 feet is an appropriate scale. So, for every inch on the model $m$, let the actual measure $a$ be 60 feet. Write this as an equation.

| | |
|---|---|
| $a = 60 \cdot m$ | Write an equation. |
| $630 = 60 \cdot m$ | $a = 630$ |
| $10.5 = m$ | So the height of the model would be 10.5 inches. |

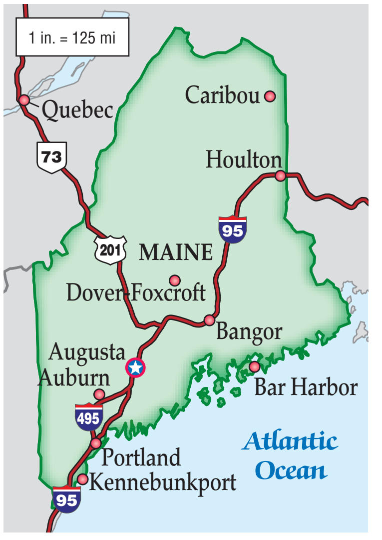
**GuidedPractice**

**3. SCALE DRAWING** Sonya is making a scale drawing of her room on an 8.5-by-11-inch sheet of paper. If her room is 14 feet by 12 feet, find an appropriate scale for the drawing and determine the dimensions of the drawing.

## Check Your Understanding

= Step-by-Step Solutions begin on page R14.

**Example 1** **MAPS** **Use the map of Maine shown and a customary ruler to find the actual distance between each pair of cities. Measure to the nearest sixteenth of an inch.**




1. Bangor and Portland
2. Augusta and Houlton

**Example 2**

3. **SCALE MODELS** Carlos made a scale model of a local bridge. The model spans 6 inches; the actual bridge spans 50 feet.
   a. What is the scale of the model?
   b. What scale factor did Carlos use to build his model?

**Example 3**

4. **SPORTS** A volleyball court is 9 meters wide and 18 meters long. Choose an appropriate scale and construct a scale drawing of the court to fit on a 3-inch by 5-inch index card.

## Practice and Problem Solving

Extra Practice is on page R7.

**Example 1** CCSS **MODELING** **Use the map of Oklahoma shown and a metric ruler to find the actual distance between each pair of cities. Measure to the nearest centimeter.**

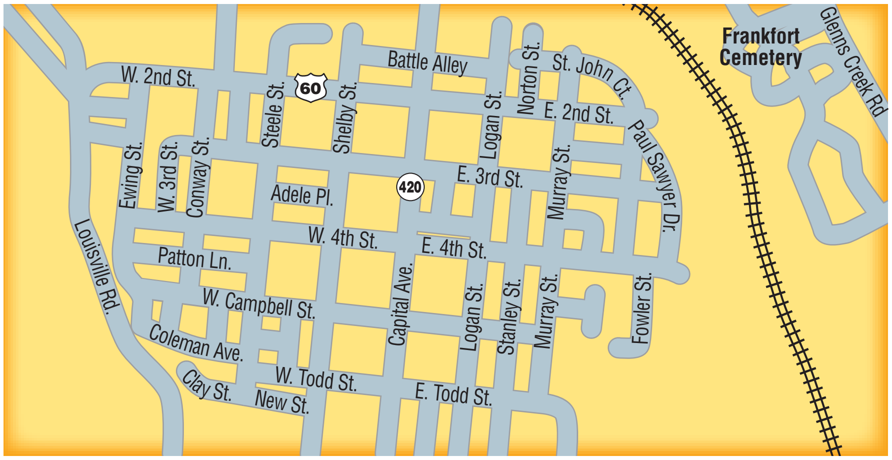

5. Guymon and Oklahoma City
6. Lawton and Tulsa
7. Enid and Tulsa
8. Ponca City and Shawnee

**Example 2**

9. **SCULPTURE** A replica of *The Thinker* is 10 inches tall. A statue of *The Thinker* at the University of Louisville is 10 feet tall.
   a. What is the scale of the replica?
   b. How many times as tall as the actual sculpture is the replica?

Buena Vista Images/Iconica/Getty Images

10. **MAPS** The map below shows a portion of Frankfort, Kentucky.




**a.** If the actual distance from the intersection of Conway Street and 4th Street to the intersection of Murray Street and 4th Street is 0.47 mile, use a customary ruler to estimate the scale of the map.

**b.** What is the approximate scale factor of the map? Interpret its meaning.

**Example 3** **SPORTS** **Choose an appropriate scale and construct a scale drawing of each playing area so that it would fit on an 8.5-by-11-inch sheet of paper.**

11. A baseball diamond is a square 90 feet on each side with about a 128-foot diagonal.

12. A high school basketball court is a rectangle with length 84 feet and width 50 feet.

**CCSS MODELING** **Use the map shown and an inch ruler to answer each question. Measure to the nearest sixteenth of an inch and assume that you can travel along any straight line.**

13. About how long would it take to drive from Valdosta, Georgia, to Daytona Beach, Florida, traveling at 65 miles per hour?

14. How long would it take to drive from Gainesville to Miami, Florida, traveling at 70 miles per hour?


15. **SCALE MODELS** If the distance between Earth and the Sun is actually 150,000,000 kilometers, how far apart are Earth and the Sun when using the 1:93,000,000 scale model?

16. **LITERATURE** In the book, *Alice's Adventures in Wonderland*, Alice's size changes from her normal height of about 50 inches. Suppose Alice came across a door about 15 inches high and her height changed to 10 inches.

**a.** Find the ratio of the height of the door to Alice's height in Wonderland.

**b.** How tall would the door have been in Alice's normal world?

17. **ROCKETS** Peter bought a $\frac{1 \text{ in.}}{12 \text{ ft}}$ scale model of the Mercury-Redstone rocket.

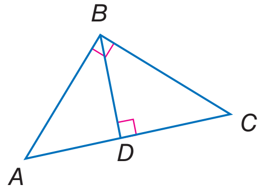
**a.** If the height of the model is 7 inches, what is the approximate height of the rocket?

**b.** If the diameter of the rocket is 70 inches, what is the diameter of the model? Round to the nearest half inch.

**18.** **ARCHITECTURE** A replica of the Statue of Liberty in Austin, Texas, is $16\frac{3}{4}$ feet tall. If the scale factor of the replica to the actual statue is 1:9, how tall is the actual statue in New York Harbor?

**19** **AMUSEMENT PARK** The Eiffel Tower in Paris, France, is 986 feet tall, not including its antenna. A replica of the Eiffel Tower was built as a ride in an amusement park. If the scale factor of the replica to the actual tower is approximately 1:3, how tall is the ride?

**20.** **MULTIPLE REPRESENTATIONS** In this problem, you will explore the altitudes of right triangles.




**a. Geometric** Draw right $\triangle ABC$ with the right angle at vertex $B$. Draw altitude $\overline{BD}$. Draw right $\triangle MNP$, with right angle $N$ and altitude $\overline{NQ}$, and right $\triangle WXY$, with right angle $X$ and altitude $\overline{XZ}$.

**b. Tabular** Measure and record indicated angles in the table below.

| | Angle Measure | | | | | |
|---|---|---|---|---|---|---|
| **△ABC** | △ABC | | △BDC | | △ADB | |
| | ABC | | BDC | | ADB | |
| | A | | CBD | | BAD | |
| | C | | DCB | | DBA | |
| **△MNP** | △MNP | | △NQP | | △MQN | |
| | MNP | | NQP | | MQN | |
| | M | | PNQ | | NMQ | |
| | P | | QPN | | QNM | |
| **△WXY** | △WXY | | △WZX | | △XZY | |
| | WXY | | WZX | | XZY | |
| | W | | XWZ | | YXZ | |
| | Y | | ZXW | | ZYX | |

**c. Verbal** Make a conjecture about the altitude of a right triangle originating at the right angle of the triangle.

## H.O.T. Problems Use Higher-Order Thinking Skills

**21.** **ERROR ANALYSIS** Felix and Tamara are building a replica of their high school. The high school is 75 feet tall and the replica is 1.5 feet tall. Felix says the scale factor of the actual high school to the replica is 50:1, while Tamara says the scale factor is 1:50. Is either of them correct? Explain your reasoning.

**22.** **CHALLENGE** You can produce a scale model of a certain object by extending each dimension by a constant. What must be true of the shape of the object? Explain your reasoning.

**23.** CCSS **SENSE-MAKING** Sofia is making two scale drawings of the lunchroom. In the first drawing, Sofia used a scale of 1 inch = 1 foot, and in the second drawing she used a scale of 1 inch = 6 feet. Which scale will produce a larger drawing? What is the scale factor of the first drawing to the second drawing? Explain.

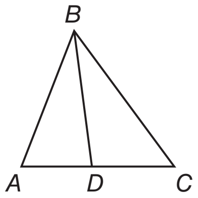
**24.** **OPEN ENDED** Draw a scale model of your classroom using any scale.

**25.** **WRITING IN MATH** Compare and contrast scale and scale factor.

## Standardized Test Practice

**26. SHORT RESPONSE** If $3^x = 27^{(x-4)}$, then what is the value of $x$?

**27.** In $\triangle ABC$, $\overline{BD}$ is a median. If $AD = 3x + 5$ and $CD = 5x - 1$, find $AC$.




**A** 6  
**B** 12  
**C** 14  
**D** 28

**28.** In a triangle, the ratio of the measures of the sides is 4 : 7 : 10, and its longest side is 40 centimeters. Find the perimeter of the triangle in centimeters.

**F** 37 cm  
**G** 43 cm  
**H** 84 cm  
**J** 168 cm

**29. SAT/ACT** If Lydia can type 80 words in two minutes, how long will it take Lydia to type 600 words?

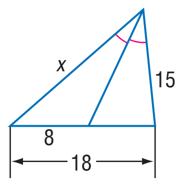
**A** 30 min  
**B** 20 min  
**C** 15 min  
**D** 10 min  
**E** 5 min

## Spiral Review

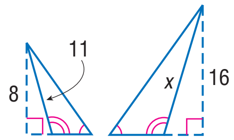
**30. PAINTING** Aaron is painting a portrait of a friend for an art class. Since his friend doesn't have time to model, he uses a photo that is 6 inches by 8 inches. If the canvas is 24 inches by 32 inches, is the painting a dilation of the original photo? If so, what is the scale factor? Explain. (Lesson 7-6)

**Find $x$.** (Lesson 7-5)

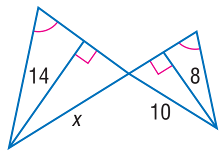
**31.**



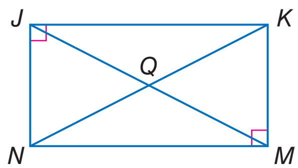

**32.**




**33.**




**ALGEBRA Quadrilateral $JKMN$ is a rectangle.** (Lesson 6-4)

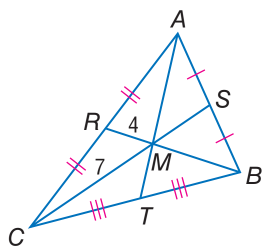



**34.** If $NQ = 2x + 3$ and $QK = 5x - 9$, find $JQ$.

**35.** If $m\angle NJM = 2x - 3$ and $m\angle KJM = x + 5$, find $x$.

**36.** If $NM = 8x - 14$ and $JK = x^2 + 1$, find $JK$.

**In $\triangle ABC$, $MC = 7$, $RM = 4$, and $AT = 16$. Find each measure.** (Lesson 5-2)




**37.** $MS$ **38.** $AM$ **39.** $SC$

**40.** $RB$ **41.** $MB$ **42.** $TM$

**Determine whether $\triangle JKL \cong \triangle XYZ$. Explain.** (Lesson 4-4)

**43.** $J(3, 9)$, $K(4, 6)$, $L(1, 5)$, $X(1, 7)$, $Y(2, 4)$, $Z(-1, 3)$

**44.** $J(-1, -1)$, $K(0, 6)$, $L(2, 3)$, $X(3, 1)$, $Y(5, 3)$, $Z(8, 1)$

## Skills Review

**Simplify each expression.**

**45.** $\sqrt{4 \cdot 16}$ **46.** $\sqrt{3 \cdot 27}$ **47.** $\sqrt{32 \cdot 72}$ **48.** $\sqrt{15 \cdot 16}$ **49.** $\sqrt{33 \cdot 21}$

CHAPTER 7

# Study Guide and Review

## Study Guide

### KeyConcepts

**Proportions** (Lesson 7-1)

- For any numbers $a$ and $c$ and any nonzero numbers $b$ and $d$, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

**Similar Polygons and Triangles** (Lessons 7-2 and 7-3)

- Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.
- Two triangles are similar if:

  AA: Two angles of one triangle are congruent to two angles of the other triangle.

  SSS: The measures of the corresponding sides of the two triangles are proportional.

  SAS: The measures of two sides of one triangle are proportional to the measures of two corresponding sides of another triangle and their included angles are congruent.

**Proportional Parts** (Lessons 7-4 and 7-5)

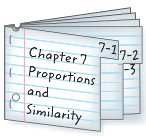
- If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional length.
- A midsegment of a triangle is parallel to one side of the triangle and its length is one-half the length of that side.
- Two triangles are similar when each of the following are proportional in measure: their perimeters, their corresponding altitudes, their corresponding angle bisectors, and their corresponding medians.

**Similarity Transformations and Scale Drawings and Models** (Lessons 7-6 and 7-7)

- A scale model or scale drawing has lengths that are proportional to the corresponding lengths in the object it represents.

### FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.




### KeyVocabulary

| | |
|---|---|
| cross products (p. 462) | reduction (p. 511) |
| dilation (p. 511) | scale (p. 518) |
| enlargement (p. 511) | scale drawing (p. 518) |
| extremes (p. 462) | scale factor (p. 470) |
| means (p. 462) | scale model (p. 518) |
| midsegment of a triangle (p. 491) | similar polygons (p. 469) |
| proportion (p. 462) | similarity transformation (p. 511) |
| ratio (p. 461) | |

### VocabularyCheck

**Choose the letter of the word or phrase that best completes each statement.**

| | |
|---|---|
| **a.** ratio | **h.** SSS Similarity Theorem |
| **b.** proportion | **i.** SAS Similarity Theorem |
| **c.** means | **j.** midsegment |
| **d.** extremes | **k.** dilation |
| **e.** similar | **l.** enlargement |
| **f.** scale factor | **m.** reduction |
| **g.** AA Similarity Post. | |

**1.** A(n) __?__ of a triangle has endpoints that are the midpoints of two sides of the triangle.

**2.** A(n) __?__ is a comparison of two quantities using division.

**3.** If $\angle A \cong \angle X$ and $\angle C \cong \angle Z$, then $\triangle ABC \sim \triangle XYZ$ by the __?__.

**4.** A(n) __?__ is an example of a similarity transformation.

**5.** If $\frac{a}{b} = \frac{c}{d}$, then $a$ and $d$ are the __?__.

**6.** The ratio of the lengths of two corresponding sides of two similar polygons is the __?__.

**7.** A(n) __?__ is an equation stating that two ratios are equivalent.

**8.** A dilation with a scale factor of $\frac{2}{5}$ will result in a(n) __?__.

# Lesson-by-Lesson Review

## 7-1 Ratios and Proportions

**Solve each proportion.**

**9.** $\frac{x+8}{6} = \frac{2x-3}{10}$

**10.** $\frac{3x+9}{x} = \frac{12}{5}$

**11.** $\frac{x}{12} = \frac{50}{6x}$

**12.** $\frac{7}{x} = \frac{14}{9}$

**13.** The ratio of the lengths of the three sides of a triangle is 5 : 8 : 10. If its perimeter is 276 inches, find the length of the longest side of the triangle.

**14.** **CARPENTRY** A board that is 12 feet long must be cut into two pieces that have lengths in a ratio of 3 to 2. Find the lengths of the two pieces.

### Example 1

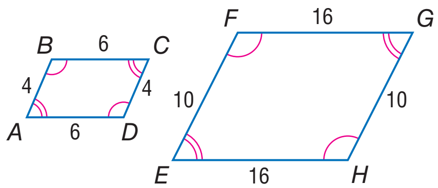
Solve $\frac{2x-3}{4} = \frac{x+9}{3}$.

| | |
|---|---|
| $\frac{2x-3}{4} = \frac{x+9}{3}$ | Original proportion |
| $3(2x-3) = 4(x+9)$ | Cross Products Property |
| $6x - 9 = 4x + 36$ | Simplify. |
| $2x - 9 = 36$ | Subtract. |
| $2x = 45$ | Add 9 to each side. |
| $x = 22.5$ | Divide each side by 2. |

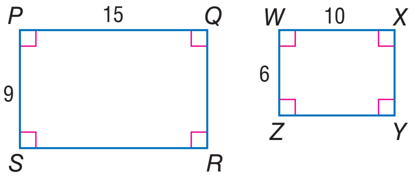
## 7-2 Similar Polygons

**Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.**

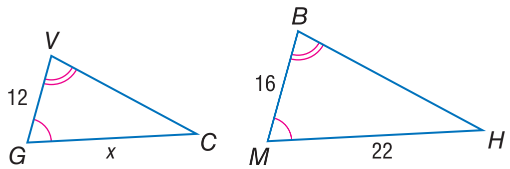
**15.**




**16.**



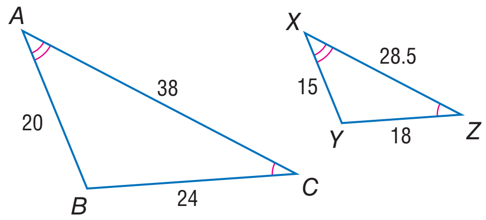

**17.** The two triangles in the figure below are similar. Find the value of *x*.




**18.** **PHOTOS** If the dimensions of a photo are 2 inches by 3 inches and the dimensions of a poster are 8 inches by 12 inches, are the photo and poster similar? Explain.

### Example 2

**Determine whether the pair of triangles is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.**




$\angle A \cong \angle X$ and $\angle C \cong \angle Z$, so by the Third Angle Theorem, $\angle B \cong \angle Y$. All of the corresponding angles are therefore congruent.

Similar polygons must also have proportional side lengths. Check the ratios of corresponding side lengths.

$\frac{AB}{XY} = \frac{20}{15}$ or $\frac{4}{3}$ $\quad \frac{BC}{YZ} = \frac{24}{18}$ or $\frac{4}{3}$ $\quad \frac{AC}{XZ} = \frac{38}{28.5}$ or $\frac{4}{3}$

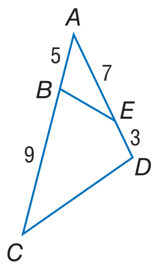
Since corresponding sides are proportional, $\triangle ABC \sim \triangle XYZ$. So, the triangles are similar with a scale factor of $\frac{4}{3}$.

# Study Guide and Review *Continued*

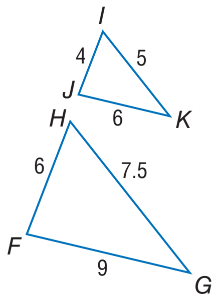
## 7-3 Similar Triangles

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

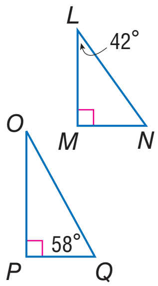
**19.**



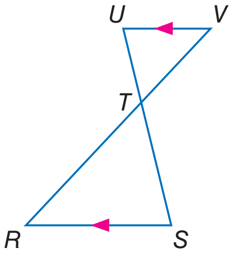

**20.**




**21.**



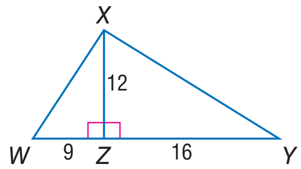

**22.**




**23. TREES** To estimate the height of a tree, Dave stands in the shadow of the tree so that his shadow and the tree's shadow end at the same point. Dave is 6 feet 4 inches tall and his shadow is 15 feet long. If he is standing 66 feet away from the tree, what is the height of the tree?

### Example 3

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.




$\angle WZX \cong \angle XZY$ because they are both right angles. Now compare the ratios of the legs of the right triangles.

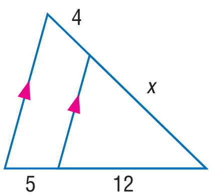
$$\frac{WZ}{XZ} = \frac{9}{12} = \frac{3}{4} \qquad \frac{XZ}{YZ} = \frac{12}{16} = \frac{3}{4}$$

Since two pairs of sides are proportional with the included angles congruent, $\triangle WZX \sim \triangle XZY$ by SAS Similarity.

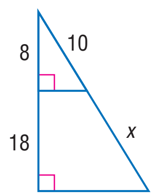
## 7-4 Parallel Lines and Proportional Parts

Find *x*.

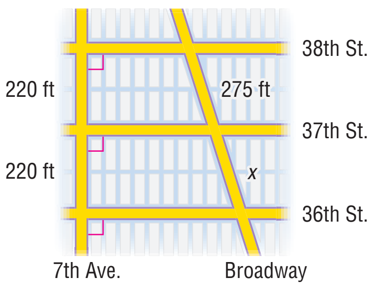
**24.**




**25.**

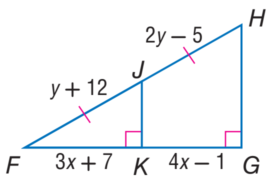



**26. STREETS** Find the distance along Broadway between 37th Street and 36th Street.




### Example 4

**ALGEBRA** Find *x* and *y*.



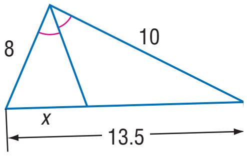

$$FK = KG$$
$$3x + 7 = 4x - 1$$
$$-x = -8$$
$$x = 8$$

| | |
|---|---|
| $FJ = JH$ | Definition of congruence |
| $y + 12 = 2y - 5$ | Substitution |
| $-y = -17$ | Subtract. |
| $y = 17$ | Simplify. |

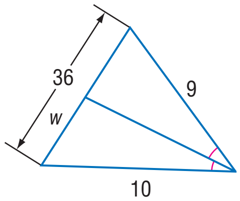
## 7-5 Parts of Similar Triangles

Find the value of each variable.

**27.**



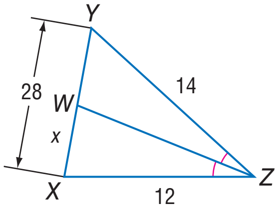

**28.**




**29.** **MAPS** The scale given on a map of the state of Missouri indicates that 3 inches represents 50 miles. The cities of St. Louis, Springfield, and Kansas City form a triangle. If the measurements of the lengths of the sides of this triangle on the map are 15 inches, 10 inches, and 13 inches, find the perimeter of the actual triangle formed by these cities to the nearest mile.

### Example 5

Find $x$.



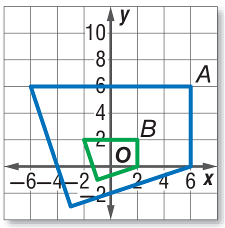

Use the Triangle Angle Bisector Theorem to write a proportion.

| | |
|---|---|
| $\frac{WX}{YW} = \frac{XZ}{YZ}$ | Triangle Angle Bisector Thm. |
| $\frac{x}{28 - x} = \frac{12}{14}$ | Substitution |
| $(28 - x)(12) = x \cdot 14$ | Cross Products Property |
| $336 - 12x = 14x$ | Simplify. |
| $336 = 26x$ | Add. |
| $12.9 = x$ | Simplify. |

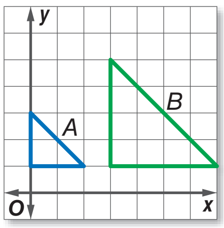
## 7-6 Similarity Transformations

Determine whether the dilation from $A$ to $B$ is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

**30.**



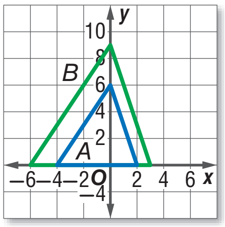

**31.**




**32.** **GRAPHIC DESIGN** Jamie wants to use a photocopier to enlarge her design for the Honors Program at her school. She sets the copier to 250%. If the original drawing was 6 inches by 9 inches, find the dimensions of the enlargement.

### Example 6

Determine whether the dilation from $A$ to $B$ is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.




$B$ is larger than $A$, so the dilation is an enlargement. The distance between the vertices at (−4, 0) and (2, 0) for $A$ is 6 and the distance between the vertices at (−6, 0) and (3, 0) for $B$ is 9. So the scale factor is $\frac{9}{6}$ or $\frac{3}{2}$.

# Study Guide and Review *Continued*

## 7-7 Scale Drawings and Models

**33.** **BUILDING PLANS** In a scale drawing of a school's floor plan, 6 inches represents 100 feet. If the distance from one end of the main hallway to the other is 175 feet, find the corresponding length in the scale drawing.

**34.** **MODEL TRAINS** A popular scale for model trains is the 1 : 48 scale. If the actual train car had a length of 72 feet, find the corresponding length of the model in inches.

**35.** **MAPS** A map of the eastern United States has a scale where 3 inches = 25 miles. If the distance on the map between Columbia, South Carolina, and Charlotte, North Carolina, is 11.5 inches what is the actual distance between the cities?

### Example 7

**In the scale of a map of the Pacific Northwest 1 inch = 20 miles. The distance on the map between Portland, Oregon, and Seattle, Washington, is 8.75 inches. Find the distance between the two cities.**

$\frac{1}{20} = \frac{8.75}{x}$ Write a proportion.

$x = 20(8.75)$ Cross Products Property

$x = 175$ Simplify.

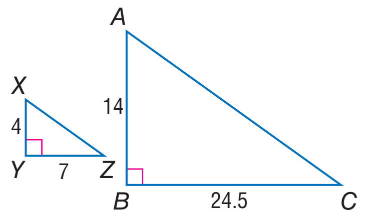
The distance between the two cities is 175 miles.

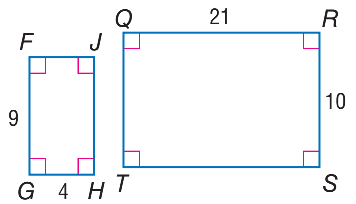
# CHAPTER 7 Practice Test

**Solve each proportion.**

1. $\frac{3}{7} = \frac{12}{x}$
2. $\frac{2x}{5} = \frac{x+3}{3}$
3. $\frac{4x}{15} = \frac{60}{x}$
4. $\frac{5x-4}{4x+7} = \frac{13}{11}$

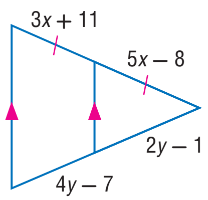
**Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.**

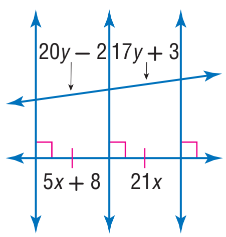
5.



6.



7. **CURRENCY** Jane is traveling to Europe this summer with the French Club. She plans to bring \$300 to spend while she is there. If \$90 in U.S. currency is equivalent to 63 euros, how many euros will she receive when she exchanges her money?

**ALGEBRA Find $x$ and $y$. Round to the nearest tenth if necessary.**

8.


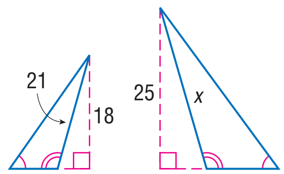

9.


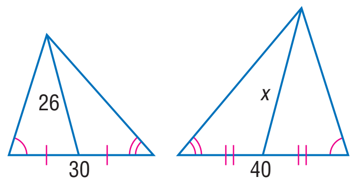

10. **ALGEBRA** Equilateral $\triangle MNP$ has perimeter $12a + 18b$. $\overline{QR}$ is a midsegment. What is $QR$?

11. **ALGEBRA** Right isosceles $\triangle ABC$ has hypotenuse length $h$. $\overline{DE}$ is a midsegment with length $4x$ that is not parallel to the hypotenuse. What is the perimeter of $\triangle ABC$?

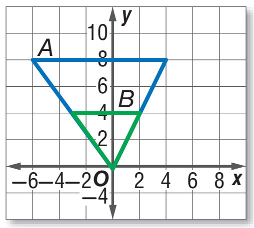
12. **SHORT RESPONSE** Jimmy has a diecast metal car that is a scale model of an actual race car. If the actual length of the car is 10 feet and 6 inches and the model has a length of 7 inches, what is the scale factor of model to actual car?

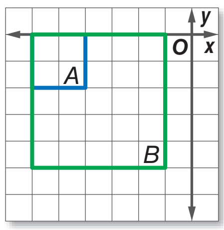
**Find $x$.**

13.


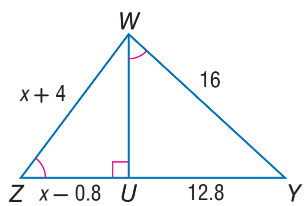

14.



**Determine whether the dilation from $A$ to $B$ is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.**

15.



16.



17. **ALGEBRA** Identify the similar triangles. Find $WZ$ and $UZ$.



CHAPTER 7

# Preparing for Standardized Tests

## Identifying Nonexamples

Multiple choice items sometimes ask you to determine which of the given answer choices is a nonexample. These types of problems require a different approach when solving them.

### Strategies for Identifying Nonexamples

**Step 1**

Read and understand the problem statement.

- **Nonexample:** A nonexample is an answer choice that does not satisfy the conditions of the problem statement.
- **Keywords:** Look for the word *not* (usually bold, all capital letters, or italicized) to indicate that you need to find a nonexample.

**Step 2**

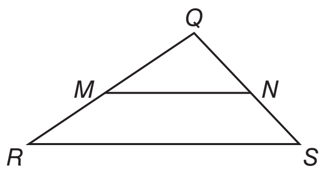
Follow the concepts and steps below to help you identify nonexamples. Identify any answer choices that are clearly incorrect and eliminate them.

- Eliminate any answer choices that are not in the proper format.
- Eliminate any answer choices that do not have the correct units.

### Standardized Test Example

**Read the problem. Identify what you need to know. Then use the information in the problem to solve.**

In the adjacent triangle, you know that $\angle MQN \cong \angle RQS$. Which of the following would *not* be sufficient to prove that $\triangle QMN \sim \triangle QRS$?




**A** $\angle QMN \cong \angle QRS$

**B** $\overline{MN} \parallel \overline{RS}$

**C** $\overline{QN} \cong \overline{NS}$

**D** $\frac{QM}{QR} = \frac{QN}{QS}$

Sean Ellis/Photographer's Choice/Getty Images

The italicized *not* indicates that you need to find a nonexample. Test each answer choice using the principles of triangle similarity to see which one would not prove $\triangle QMN \cong \triangle QRS$.

**Choice A:** $\angle QMN \cong \angle QRS$

If $\angle QMN \cong \angle QRS$, then $\triangle QMN \sim \triangle QRS$ by AA Similarity.

**Choice B:** $\overline{MN} \parallel \overline{RS}$

If $\overline{MN} \parallel \overline{RS}$, then $\angle QMN \cong \angle QRS$, because they are corresponding angles of two parallel lines cut by transversal $\overline{QR}$. Therefore, $\triangle QMN \sim \triangle QRS$ by AA Similarity.

**Choice C:** $\overline{QN} \cong \overline{NS}$

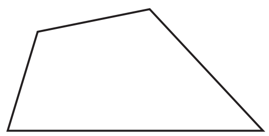
If $\overline{QN} \cong \overline{NS}$, we cannot conclude that $\triangle QMN \sim \triangle QRS$ because we do not know anything about $\overline{QM}$ and $\overline{MR}$. So, answer choice C is a nonexample.

The correct answer is C. You should also check answer choice D to make sure it is a valid example if you have time.

## Exercises

**Read each problem. Identify what you need to know. Then use the information in the problem to solve.**

**1.** The ratio of the measures of the angles of the quadrilateral below is 6:5:4:3. Which of the following is *not* an angle measure of the figure?



**A** 60° **C** 120°

**B** 80° **D** 140°

**2.** Which figure can serve as a counterexample to the conjecture below?

| |
|---|
| If all angles of a quadrilateral are right angles, then the quadrilateral is a square. |

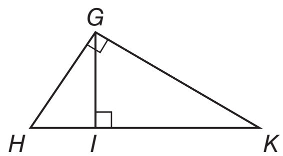
**F** parallelogram

**G** rectangle

**H** rhombus

**J** trapezoid

**3.** Consider the figure below. Which of the following is *not* sufficient to prove that $\triangle GIK \sim \triangle HIG$?




**A** $\angle GKI \cong \angle HGI$

**B** $\frac{HI}{GI} = \frac{GI}{IK}$

**C** $\frac{GH}{GI} = \frac{GK}{IK}$

**D** $\angle IGK \cong \angle IHG$

**4.** Which triangles are *not* necessarily similar?

**F** two right triangles with one angle measuring 30°

**G** two right triangles with one angle measuring 45°

**H** two isosceles triangles

**J** two equilateral triangles

CHAPTER 7

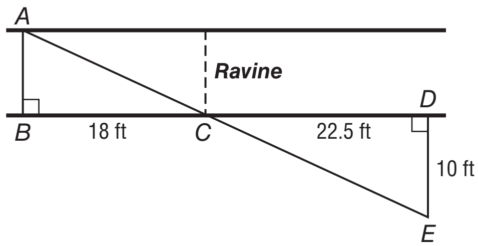
# Standardized Test Practice

## Cumulative, Chapters 1 through 7

### Multiple Choice

**Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.**

**1.** Adrian wants to measure the width of a ravine. He marks distances as shown in the diagram.




Using this information, what is the *approximate* width of the ravine?

**A** 5 ft  **C** 7 ft

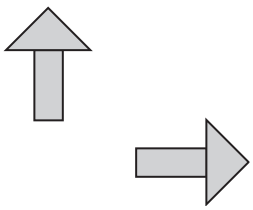
**B** 6 ft  **D** 8 ft

**2.** Kyle and his family are planning a vacation in Cancun, Mexico. Kyle wants to convert 200 US dollars to Mexican pesos for spending money. If 278 Mexican pesos are equivalent to $25, how many pesos will Kyle get for $200?

**F** 2178  **H** 2396

**G** 2224  **J** 2504

**3.** Which of the following terms *best* describes the transformation below?

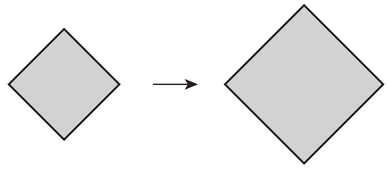


**A** dilation  **C** rotation

**B** reflection  **D** translation

> **Test-Taking Tip**
>
> **Question 2** Set up and solve the proportion for the number of pesos. Use the ratio pesos : dollars.

**4.** Refer to the figures below. Which of the following terms *best* describes the transformation?



**F** congruent

**G** enlargement

**H** reduction

**J** scale

**5.** The ratio of North Carolina residents to Americans is about 295 to 10,000. If there are approximately 300,000,000 Americans, how many of them are North Carolina residents?

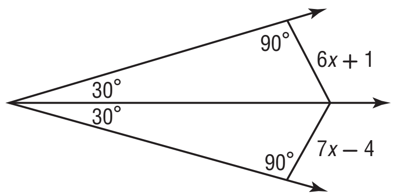
**A** 7,950,000

**B** 8,400,000

**C** 8,850,000

**D** 9,125,000

**6.** Solve for $x$.




**F** 3  **H** 5

**G** 4  **J** 6

**7.** Two similar trapezoids have a scale factor of 3:2. The perimeter of the larger trapezoid is 21 yards. What is the perimeter of the smaller trapezoid?

**A** 14 yd

**B** 17.5 yd

**C** 28 yd

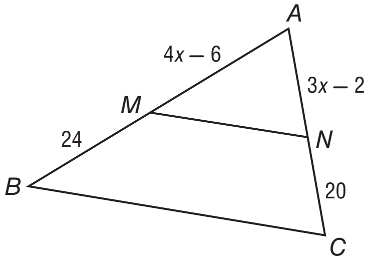
**D** 31.5 yd

## Short Response/Gridded Response

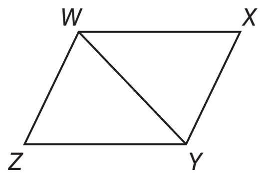
**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**

**8.** **GRIDDED RESPONSE** Colleen surveyed 50 students in her school and found that 35 of them have homework at least four nights a week. If there are 290 students in the school altogether, how many of them would you expect to have homework at least four nights a week?

**9.** **GRIDDED RESPONSE** In the triangle below, $\overline{MN} \parallel \overline{BC}$. Solve for $x$.



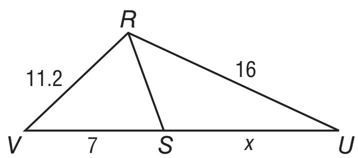

**10.** Quadrilateral $WXYZ$ is a rhombus. If $m\angle XYZ = 110°$, find $m\angle ZWY$.




**11.** What is the contrapositive of the statement below?

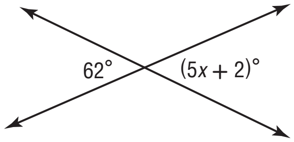
| |
|---|
| If Tom was born in Louisville, then he was born in Kentucky. |

**12.** **GRIDDED RESPONSE** In the triangle below, $\overline{RS}$ bisects $\angle VRU$. Solve for $x$.




**13.** **GRIDDED RESPONSE** The scale of a map is 1 inch = 2.5 miles. What is the distance between two cities that are 3.3 inches apart on the map? Round to the nearest tenth, if necessary.

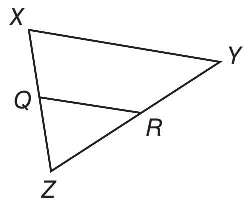
**14.** What is the value of $x$ in the figure?




## Extended Response

**Record your answers on a sheet of paper. Show your work.**

**15.** Refer to triangle $XYZ$ to answer each question.




**a.** Suppose $\overline{QR} \parallel \overline{XY}$. What do you know about the relationship between segments $XQ$, $QZ$, $YR$, and $RZ$?

**b.** If $\overline{QR} \parallel \overline{XY}$, $XQ = 15$, $QZ = 12$, and $YR = 20$, what is the length of $\overline{RZ}$?

**c.** Suppose $\overline{QR} \parallel \overline{XY}$, $\overline{XQ} \cong \overline{QZ}$, and $QR = 9.5$ units. What is the length of $\overline{XY}$?

**Need ExtraHelp?**

| If you missed Question... | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Go to Lesson... | 7-3 | 7-1 | 4-7 | 7-6 | 7-1 | 5-1 | 7-2 | 7-1 | 7-6 | 6-5 | 2-3 | 7-5 | 7-7 | 1-5 | 7-4 |