# Series … Quiz 1

(1) $\left\{\frac{\ln n}{n^2}\right\}$ converge or diverge?

$\lim_{n\to\infty} \frac{\ln n}{n^2} = 0$, so $\left\{\frac{\ln n}{n^2}\right\}$ Converges to zero

(2) (a) $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}, \cdots = \left\{\frac{n+1}{n^2}\right\}$

$\lim_{n\to\infty} \frac{n+1}{n^2} = 0$, so sequences Converges

(b) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots = \left\{\frac{1}{n!}\right\}$

$\lim_{n\to\infty} \frac{1}{n!} = 0$, so sequence Converges

(3) (a) $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$

$\lim_{n\to\infty} \frac{n^3+3n^2+1}{4n^3-5n+2} = \frac{1}{4} \neq 0$

So series diverges by $n^{th}$ term test

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{1}{n^2} = 0$

so series May or May Not converge ($n^{th}$ term test inconclusive)

(c) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

$\lim_{n\to\infty} \frac{n!}{2\cdot n!+1} = \frac{1}{2} \neq 0$

So series diverges by nth term test

(d) $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!}$

$= \sum_{n=1}^{\infty} \frac{(n+2)(n+1)n!}{10n!}$

$= \sum_{n=1}^{\infty} \frac{n^2+3n+2}{10}$

$\lim_{n\to\infty} \frac{1}{10}(n^2+3n+2) = \infty$

So series diverges by $n^{th}$ term test

(4) $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$

(a) Telescoping series

$= \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \cdots$

$= \frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6} \approx 0.833}$

(b) $S_{500} = 0.8293532299 < 0.833$

* from calculator

sum(seq(1/(x+1) − 1/(x+3), X, 1, 500)

List/Math/5 List/OPS/5

5 (a) $3 + \frac{15}{4} + \frac{75}{16} + \frac{375}{64} + \cdots$

$= 3\left(1 + \frac{5}{4} + \frac{25}{16} + \frac{125}{64} + \cdots\right)$

$= 3\left(\left(\frac{5}{4}\right)^0 + \left(\frac{5}{4}\right)^1 + \left(\frac{5}{4}\right)^2 + \left(\frac{5}{4}\right)^3 + \cdots\right)$

$= 3\sum_{n=0}^{\infty}\left(\frac{5}{4}\right)^n$, $|r| = \frac{5}{4} > 1$

So Series is Divergent Geometric Series

5 (b) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$

$= \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots$

$= \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$, $|r| = \frac{1}{2} < 1$

So convergent Geometric Series

Converges to $\frac{1/2}{1 - 1/2} = \boxed{1}$

5(c). $\sum_{n=1}^{\infty} n^{-2/3} = \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

$p = \frac{2}{3} < 1$, so series is a

<u>divergent p-series</u>

5(d) $\sum_{n=1}^{\infty} \frac{3n}{2n^2+3}$ for all $n > K$, $\exists K \in \mathbb{Z}^+$, this series is Decreasing, Continuous, and Positive

$3\int_1^{\infty} \frac{n}{2n^2+3}\,dn$

$= \frac{3}{4} \ln|2n^2+3| \Big|_1^{\infty}$

$= \frac{3}{4}\left[\ln(\infty) - \ln 5\right] = \infty \Rightarrow$ Diverges

So the series <u>diverges</u> too!

(5) (e) $\sum_{n=1}^{\infty} \frac{e^n}{n}$, compare to $\sum_{n=1}^{\infty} \frac{1}{n}$,

the divergent harmonic series.

Since $\frac{1}{n} \leq \frac{e^n}{n}$ $\forall n \geq 1$

$\sum_{n=1}^{\infty} \frac{e^n}{n}$ <u>diverges</u> too!

(5) (f) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{7^n+1}$, compare to $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^n = \sum_{n=1}^{\infty} \frac{3^n}{7^n}$

a convergent geometric series.

Since $\dfrac{3^n}{7^n+1} \le \dfrac{3^n}{7^n} \ \forall n \ge 1$,

$\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{7^n+1}$ converges too!

(5) (g) $\displaystyle\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$,

compare to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$, the divergent series

$$\lim_{n\to\infty} \left(\frac{3n+6}{7n^2-5n+1} \cdot \frac{n}{1}\right)$$ same as dividing by $\frac{1}{n}$

$$= \lim_{n\to\infty} \frac{3n^2+6n}{7n^2-5n+1} = \frac{3}{7}$$

where $\frac{3}{7}$ is finite & positive, so

$\displaystyle\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$ diverges too!

(5) (h) $\sum_{n=1}^{\infty} \frac{n+5}{3n(4^n)}$, compare to $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$

$= \sum_{n=1}^{\infty} \frac{1}{4^n}$, a convergent geom. series.

$$\lim_{n\to\infty} \left(\frac{n+5}{3n(4^n)}\right)\left(\frac{4^n}{1}\right)$$

$$= \lim_{n\to\infty} \frac{\cancel{(4^n)}(n+5)}{\cancel{(4^n)}(3n)} = \frac{1}{3} > 0$$

So $\sum_{n=1}^{\infty} \frac{n+5}{3n(4^n)}$ <u>converges</u> too!

5) (i) $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

same as dividing by $\frac{n^3}{n!}$

$$\lim_{n\to\infty}\left|\frac{(n+1)^3}{(n+1)!}\cdot\frac{n!}{n^3}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(n+1)^3\cdot \cancel{n!}}{n^3\cdot(n+1)\cdot\cancel{n!}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{n^3+\cdots}{n^4+n^3}\right| = 0 < 1$$

So $\sum_{n=1}^{\infty} \frac{n^3}{n!}$ converges

(5) (j) $\sum_{n=1}^{\infty} \frac{2}{n^2}$

$$\lim_{n\to\infty} \left| \frac{2}{(n+1)^2} \cdot \frac{n^2}{2} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2n^2}{2n^2+\cdots} \right| = 1$$

so Ratio Test is <u>inconclusive</u>

* use Limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$ to show it converges.

5 (k) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

*Alternating series

$\lim_{n\to\infty} \frac{1}{n} = 0$ and

$\{\frac{1}{n}\}$ is decreasing

So $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges

(conditionally convergent)

5 (l) $\sum_{n=1}^{\infty} \frac{(-1)^{n}(n+3)}{2n}$

*Alternating Series

$\lim_{n\to\infty} \frac{n+3}{2n} = 1 \neq 0$

So diverges by $n^{th}$ term test

(Alt. series test is inconclusive)

(5) (m) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, a convergent p-series

$\frac{\sin n}{n^2} \le \frac{1}{n^2}$ since $\sin n \le 1 \ \forall n \in \mathbb{R}$,

so $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges too!

(5)(n) $\sum_{n=1}^{\infty} \frac{(-1)^n(4^n)}{n!}$ *converges by A.S.T. since $\frac{4^n}{n!}$ is decreasing $\forall n \geq k, \exists k \in \mathbb{Z}^+$

or by $\underbrace{\text{Ratio Test}}_{\text{(more fun!)}}$

$$\lim_{n\to\infty} \left| \frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n} \right|$$

*dont need alternators on Ratio test because of abs. values.

$$\lim_{n\to\infty} \left| \frac{\cancel{4^n} \cdot 4 \cdot \cancel{n!}}{\cancel{4^n} \cdot (n+1) \cdot \cancel{n!}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{4}{n+1} \right| = 0 < 1$$ so converges