Series … Quiz 2

⑥ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

$S_5 = \frac{1}{1^2} - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25}$

$= 0.838611111 = \frac{3019}{3600}$

$|R_5| \leq |a_6| = \left|\frac{-1}{36}\right| = \frac{1}{36}$ ← (Abs. value of 1st unused term)

So $S \in \left[\frac{3019}{3600} - \frac{1}{36}, \frac{3019}{3600} + \frac{1}{36}\right]$

or $S \in [0.81083, 0.86638]$

(actual sum $\approx \underset{S_{999}}{\underset{\parallel}{0.822}}$)

⑦ $\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$, convergent geom series

with $|r| = \left|\frac{2}{7}\right| = \frac{2}{7} < 1$.

Series converges to $\frac{1}{1 - 2/7} = \boxed{\frac{7}{5}}$

(8) $\sum_{n=1}^{\infty} \frac{4}{n^3} = 4\left[\sum_{n=1}^{\infty} \frac{1}{n^3}\right]$ (or Limit-Comparison test) convergent p-series with $p = 3 > 1$

(9) $\sum_{n=1}^{\infty} \frac{n^2}{5^n}$, $\lim_{n\to\infty} \left| \frac{(n+1)^2}{5^{n+1}} \cdot \frac{5^n}{n^2} \right|$

$= \lim_{n\to\infty} \left| \frac{(n^2+2n+1)\cancel{5^n}}{n^2 \cdot 5 \cdot \cancel{5^n}} \right| = \frac{1}{5} < 1$

So series converges by Ratio Test

(10) $$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5+5}}$$

Compare with $\sum_{n=1}^{\infty} \frac{1}{n^{5/3}}$, a

Convergent p-series:

since $\frac{1}{\sqrt[3]{n^5+3}} \le \frac{1}{n^{5/3}} \; \forall n \geq 1$,

$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5+5}}$ converges by Direct Comparison

(Limit Comparison works too!)

(11) $$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

$$\lim_{n\to\infty} \frac{n^n}{n!} = \infty$$

So series diverges by n^{th} term test

(12) $\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots$

$= \sum_{n=5}^{\infty} \frac{1}{n}$ which is the

<u>divergent</u> <u>harmonic</u> <u>series</u>

(13) $2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$

$= 2 + \left(\frac{1}{4}\right)^0\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)^1\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right)$

$= 2 + \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$ convergent geometric series with $|r| = \left|\frac{1}{4}\right| = \frac{1}{4} < 1$

converges to $2 + \frac{1}{2}\left[\frac{1}{1-\frac{1}{4}}\right] = 2 + \frac{2}{3} = \boxed{\frac{8}{3}}$

(14) $\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$, compare with

$\sum_{n=1}^{\infty} \frac{1}{n}$, the divergent harmonic series

$$\lim_{n\to\infty}\left(\frac{5n^2-6n+3}{n^3-7n+8}\cdot\frac{n}{1}\right)=5>0$$

So $\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$ <u>diverges</u> by the Limit Comparison Test.

(15) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$, $\cos n\pi = -1, 1, -1, 1$ for $n = 1, 2, 3 \ldots$

* Alternating series

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ and $\frac{1}{\sqrt{n}}$ is decreasing

So $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$ <u>converges</u> by the Alternating Series Test

(16) $\sum_{n=1}^{\infty} \frac{3^n+4}{2^n}$ *Diverges by nth term test OR

compare with $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n = \sum_{n=1}^{\infty} \frac{3^n}{2^n}$, a divergent geom series.

Since $\frac{3^n+4}{2^n} \geq \frac{3^n}{2^n}$, $\sum \frac{3^n+4}{2^n}$ diverges by Direct Comparison Test

(17) $\sum_{n=1}^{\infty} \frac{8n^3-6n^5}{12n^4+9n^5}$

$\lim_{n\to\infty} \frac{8n^3-6n^5}{12n^4+9n^5} = -\frac{6}{9} \neq 0$

So series diverges by n^{th} term test

(18) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{3n+1}{n^5+2}}$ compare

with $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{1}{n^4}} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ a

convergent p-series.

$$\lim_{n\to\infty}\left(\sqrt{\frac{3n+1}{n^5+2}} \cdot \sqrt{\frac{n^4}{1}}\right)$$

$$= \sqrt{\lim_{n\to\infty}\left(\frac{3n^5+n^4}{n^5+2}\right)}$$

$$= \sqrt{3} > 0$$

So $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{3n+1}{n^5+2}}$ <u>converges</u> too!

by Limit Comparison Test

(19) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{3n+4}}$

the series converges by Alt. series test.

* test for $\underline{\text{Abs}}$ convergence (w/o alternator)

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{3n+4}}$ compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$, a divergent p series:

$$\lim_{n\to\infty} \frac{\sqrt[5]{n}}{\sqrt[5]{3n+4}} = \sqrt[5]{\lim_{n\to\infty}\left(\frac{n}{3n+4}\right)} = \sqrt[5]{\frac{1}{3}} > 0$$

So $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{3n+4}}$ diverges

So $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{3n+4}}$ $\underline{\text{converges conditionally}}$

(20) (a) $\displaystyle\sum_{n=0}^{\infty} \frac{3}{2^n}$ (convergent geom series)

$= 3\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$

$S = 3\left[\frac{1}{1-1/2}\right] = \boxed{6}$

(b) $\sum_{n=2}^{\infty} \left(-\frac{3}{2}\right)^{-n}$

$= \sum_{n=2}^{\infty} \left(-\frac{2}{3}\right)^{n}$ (convergent geom/alt series)

$S = \frac{\overset{\text{1st term}}{4/9}}{1-(\underset{r}{-2/3})} = \left(\frac{4}{9}\right)\left(\frac{3}{5}\right) = \boxed{\frac{4}{15}}$

(c) $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$ (telescoping series)

$= \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \cdots$

$= \frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6}}$

partial fraction decomp.

$$(d)\ \sum_{n=1}^{\infty} \frac{3}{(2n-1)(2n+1)} = 3\sum_{n=1}^{\infty}\left(\frac{1/2}{2n-1} - \frac{1/2}{2n+1}\right)$$

$$= \frac{3}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$$

$$= \frac{3}{2}\left[\left(1 - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \left(\cancel{\frac{1}{7}} - \frac{1}{9}\right) + \dots\right.$$

$$= \frac{3}{2}(1) = \boxed{\frac{3}{2}}$$