Series … Quiz 3

(21) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

(a) for Abs convergence, $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{2^n}\right| = \sum_{n=1}^{\infty} \frac{1}{2^n}$ must converge.

$\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ which is a convergent geometric series with $|r| = \frac{1}{2} < 1$

so $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$ converges absolutely (with or without the help of the alternator)

(b) $S_6 = -\frac{1}{2^1} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \frac{1}{2^5} + \frac{1}{2^6} = -0.328125 = -\frac{21}{64}$

$\approx \boxed{-0.328}$

(c) for error to be less than 0.001, the magnitude of the 1st unused term in the partial sum must be < 0.001

trial & error: $\frac{1}{2^6} = 0.0156 \not< 0.001$, $\frac{1}{2^7} = 0.0078 \not< 0.001$, $\frac{1}{2^8} = 0.0039 \not< 0.001$

$\frac{1}{2^9} = 0.00195 \not< 0.001$, $\boxed{\frac{1}{2^{10}} = 0.00097 < 0.001}$

so S_9 approximates S to within 0.001, so 9 terms are needed.

(22) $\sum_{n=1}^{\infty} a_n$ is conditionally convergent, so $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges

(a) $\sum_{n=1}^{\infty} a_n^2$ could converge if $a_n^2 < a_n$

ex) $a_n = \left(\frac{2}{3}\right)^n$

$(a_n)^2 = \left(\frac{4}{9}\right)^n$

(b) $\sum_{n=1}^{\infty} |a_n|$ MUST diverge by def of Abs convergence

(c) $\sum_{n=1}^{\infty} (-1)^{2n} a_n = \sum_{n=1}^{\infty} \left((-1)^2\right)^n a_n = \sum_{n=1}^{\infty} a_n$ which converges

(d) $\sum_{n=1}^{\infty} (-a_n) = -\sum_{n=1}^{\infty} a_n$ which converges

$\Bigg\}$ So only (b) MUST diverge

(23) (a) $\sum_{n=1}^{\infty} \frac{(-1)^n(n-1)}{n\sqrt{n}}$

$= \sum_{n=1}^{\infty} (-1)^n \left(\frac{n-1}{n^{3/2}}\right)$

* Converges conditionally by Alt. series test
* $\sum_{n=1}^{\infty} \left(\frac{n-1}{n^{3/2}}\right)$ Diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$, a divergent p-series

(b) $\sum_{n=0}^{\infty} (-1)^n e^{-n}$

* $\sum_{n=0}^{\infty} \frac{1}{e^n}$ Converges by Ratio Test

So $\sum_{n=0}^{\infty} (-1)^n e^{-n}$ converges absolutely

(c) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$

* converges conditionally by Alt. series test
* $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ diverges by comparison to the harmonic series $\frac{\ln n}{n} > \frac{1}{n} \; \forall n \geq 2$

(23) (d) $\sum_{n=1}^{\infty} \left(-\frac{\pi}{e}\right)^n$

$= \sum_{n=1}^{\infty} (-1)^n \cdot \left(\frac{\pi}{e}\right)^n$

* $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$ converges by the Geom Series Test,

So $\sum_{n=1}^{\infty} (-1)^n \left(\frac{\pi}{e}\right)^n$

Converges absolutely

(c) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{\sqrt{n}}{n+3} \right)$

* converges conditionally by Alt. Series Test

* $\sum_{n=1}^{\infty} \frac{n^{1/2}}{n+3}$ diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$, a divergent p-series.

(24) $S = \sum_{n=1}^{\infty} \frac{4}{n^2}$ (convergent p-series)

$= 4\sum_{n=1}^{\infty} \frac{1}{n^2}$

$S_2 = 4\left[\frac{1}{1} + \frac{1}{4}\right] = 4 + 1 = 5$

$R_2 = |a_3| = \frac{4}{3^2} = \frac{4}{9}$

$S \in \left[5 - \frac{4}{9}, 5 + \frac{4}{9}\right]$

$S \in \left[\frac{41}{9}, \frac{49}{9}\right]$

So $\boxed{\frac{41}{9} \leq S \leq \frac{49}{9}}$ $\boxed{A}$

(25) Which converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ → diverges by n^{th} term test since $\lim_{n \to \infty} \frac{n}{n+2} = 1 \neq 0$

✓ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ → conditionally convergent harmonic series by Alt. Series Tst.

III. $\sum_{n=1}^{\infty} \frac{1}{n}$ → divergent harmonic series

So only II converges $\boxed{B}$

Series … Quiz 3

(26) $\lim_{b\to\infty} \int_1^b \frac{1}{x^p}\,dx$ is finite so $\int_1^\infty \frac{1}{x^p}dx$ converges,

so $p>1$. which MUST be true?

✓(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges → TRUE by IntegraTest. $\boxed{\text{So answer is } A}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges → False, see (A)

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ Converges → Not always true. False if $n \leq 3$

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges → Not always true. False if $n \leq 2$

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ Diverges, false. by comparison, if $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges, $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ must too!

(27) (Review Question) do

$$\lim_{x\to 1} \frac{\int_1^x e^{t^2}\,dt}{x^2-1}$$

by L'Hôp: $\lim_{x\to 1} \frac{e^{x^2}}{2x} = \boxed{\frac{e}{2}}\ \boxed{C}$

2nd Fun. thm of Calculus

(28) For what k, $k>1$ will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

* $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ will only converge (conditionally) if $(-1)^{kn}$ alternates, so k must be an odd integer >1

$\boxed{k = 3, 5, 7, 9, 11, \ldots}$

* $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ will only converge by geometric series test if $\frac{k}{4} < 1$, $k < 4$, (but >1)

$\boxed{k = 3, 2}$

The only value that satisfies both scenarios is $\boxed{k=3}$ $\boxed{D}$