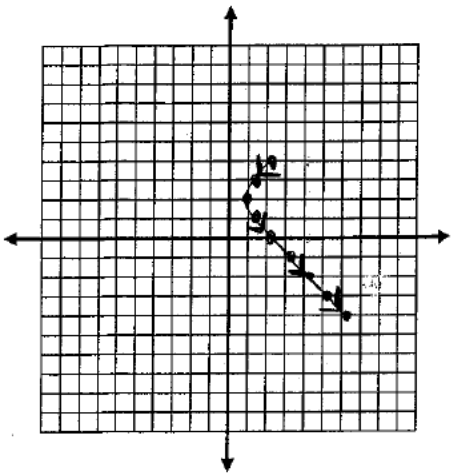
Accelerated Math 3
Parametric Equations Worksheet

1. Fill in the table and sketch the parametric equation for t [-2,6]

$x = \sqrt{t^2 + 1}$

$y = 2 - t$

t	x	y
-2	$\sqrt{5}$	4
-1	$\sqrt{2}$	3
0	1	2
1	$\sqrt{2}$	1
2	$\sqrt{5}$	0
3	$\sqrt{10}$	-1
4	$\sqrt{17}$	-2
5	$\sqrt{26}$	-3
6	$\sqrt{37}$	-4



Problems 2 – 11: Eliminate the parameter to write the parametric equations as a rectangular equation.

2. $x = \frac{1}{t-2}$, $y = 4t + 5$ — $y = 13 + \frac{4}{x}$

3. $x = 6 - t$, $y = \sqrt{3t - 4}$ — $y = \sqrt{14 - 3x}$

4. $x = \frac{1}{2}t + 4$, $y = t^3$ — $y = (2x-8)^3$ OR $y = 8x^3 - 96x^2 + 384x - 512$

5. $x = 3\cos t$, $y = 3\sin t$ — $x^2 + y^2 = 1$

6. $x = 4\sin(2t)$, $y = 2\cos(2t)$ — $\frac{x^2}{16} + \frac{y^2}{4} = 1$

7. $x = \cos t$, $y = 2\sin^2 t$ — $y = -2x^2 + 2$ OR $x^2 = -\frac{1}{2}(y-2)$

8. $x = 4\sec t$, $y = 3\tan t$ — $\frac{x^2}{16} - \frac{y^2}{9} = 1$

9. $x = 4 + 2\cos t$, $y = -1 + 4\sin t$ — $\frac{(x-4)^2}{4} + \frac{(y+1)^2}{16} = 1$

10. $x = -4 + 3\tan t$, $y = 7 - 2\sec t$ — $\frac{(y-7)^2}{4} - \frac{(x+4)^2}{9} = 1$

Problems 11 and 12: Write two new sets of parametric equations for the following rectangular equations.

11. $y = (x+2)^3 - 4$

$x = t$, $y = (t+2)^3 - 4$

$x = t - 2$, $y = t^3 - 4$

12. $x = \sqrt{y^2 - 3}$

$x = \sqrt{t^2 - 3}$, $y = t$

$x = \sqrt{3}\tan t$, $y = \sqrt{3}\sec t$

Problems 13 – 15: Write a new set of parametric equations with the following transformations for $x = t^4 - 3$ and $y = 2t$

$x = 4t^4 - 3$, $y = 2t + 7$

13. Shift right 7

$x = t^4 + 4$, $y = 2t$

14. Shift up 7 and horizontally stretched by a factor of 4

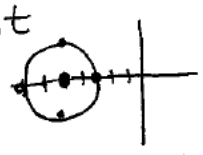
15. Shift left 3 and shift down 5

$x = t^4 - 6$, $y = 2t - 5$

16. For the curve $(x+5)^2 + y^2 = 4$ complete the following:

a) Write a pair of parametric equations for the curve.
b) Give an appropriate window and sketch the graph.
c) How could you graph just the top half of the curve?
d) How could you graph just the left side of the curve?

(a) $x = -5 + 2\cos t$, $y = 2\sin t$



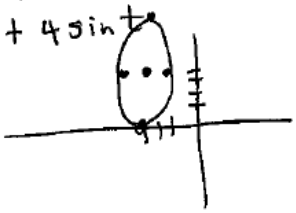
Circle
$r = 2$ Center $(-5, 0)$

17. For the curve $\frac{(x+3)^2}{1} + \frac{(y-4)^2}{16} = 1$ complete the following:

a) Write a pair of parametric equations for the curve.
b) Give an appropriate window and sketch the graph.
c) How could you graph just the right side of the curve?
d) How could you graph just the bottom half of the curve?

ellipse: Center $(-3, 4)$

(a) $x = -3 + \cos t$
$y = 4 + 4\sin t$



18. Graph the parametric equations using the window provided.

$x_1 = 700t$ and $x_2 = 300$ T [0, 0.6] Tstep = 0.005
$y_1 = -4.9t^2$ $y_2 = -4.9t^2$ X [0, 310] X scl = 100
Y [-2, 2] Yscl = 0.5

a) Do the paths intersect or just cross? If the paths intersect, where and when? when $t = \frac{3}{7}$
b) Change x_1 to 900t and then to 500t. What do you observe? Still intersect where $(300, -9)$

19. Do the following sets of parametric equations cross or intersect? Justify your answer.

a) $x_1 = 3 - t$ and $x_2 = t + 19$
$y_1 = t^2 - 60$ $y_2 = t + 12$
cross at $(-4, -11)$ meet at $(11, 4)$ $t = -8$

b) $x_1 = 3 - t$ and $x_2 = 3 - 2t$
$y_1 = 2t + 1$ $y_2 = 2 + 3t$
cross at $(1, 5)$

c) $x_1 = 4t$ and $x_2 = 5t - 6$
$y_1 = \frac{1}{2}t + 5$ $y_2 = t + 2$
meets at $(24, 8)$ $t = 6$

Problems 20 – 22: Use parametric equations to solve each situation.

20. Jack and Mark live 4 miles apart from each other on Baseline Road. At 6pm, their moms agree to let them both leave and ride their bikes toward one another. If Jack can pedal 12 mph and Mark can pedal 8 mph, where and when will the boys meet each other? 2.4 miles from Jack, 6:12 pm

21. Andrew's dog Toby ran out of the house at 4pm and continued to run along Guadalupe Road without stopping at a speed of 4 mph. When Andrew's mom returned home from the grocery store at 4:30pm, she and Andrew hopped in the minivan to retrieve Toby. If Andrew's mom drove at a speed of 20 mph, when and where will they find Toby? 4:37 30 sec pm, $2\frac{1}{2}$ miles from house

22. A dart is thrown from a point of 5 feet above the ground with an initial velocity of 58 ft/sec and angle of elevation of 41°. Assume the only force acting on the dart is gravity.

a) Write a pair of parametric equations to simulate the motion of the dart.
b) Sketch a graph of the motion and give an appropriate window.
c) What is the maximum height reached by the dart?
d) When and where will the dart hit the ground?

$x(t) = 58t\cos 41^\circ$
$y(t) = 58t\sin 41^\circ - 16t^2 + 5$