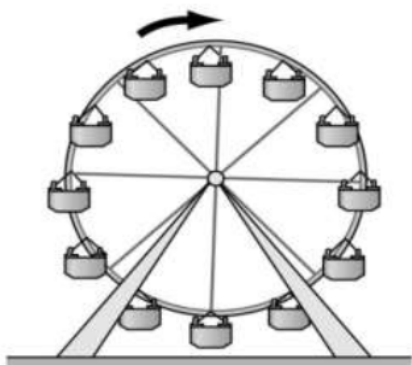


PARAMETRIC EQUATIONS

- OBJECTIVES:** 1) Sketch curves that are represented by sets of parametric equations.
2) Eliminate the parameter to rewrite parametric equations as singular rectangular equations.

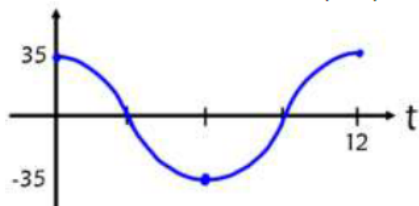
What is a **parametric equation**? A system of equations with more than one dependent variable. Often, parametric equations are used to represent the position of a moving point.

Imagine you are riding a Ferris wheel that has a 35' radius and whose lowest point is 5' off of the ground. It completes one clockwise rotation every 12 seconds. Your x and y position both depend on time. We call time the **parameter**.

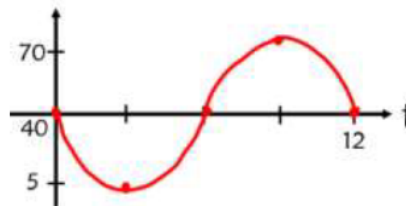


t	x	y
0	35	40
3	0	5
6	-35	40
9	0	75
12	35	40

x position: $x(t) = 35 \cos\left(\frac{\pi}{6}t\right)$



y position: $y(t) = -35 \sin\left(\frac{\pi}{6}t\right) + 40$



So $x(t) = 35 \cos\left(\frac{\pi}{6}t\right)$ and $y(t) = -35 \sin\left(\frac{\pi}{6}t\right) + 40$ are **parametric equations** that describe your position on the Ferris wheel at any time t .

PARAMETRIC EQUATIONS

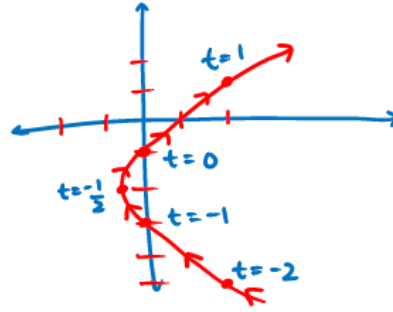
- If f and g are continuous functions of t on an interval I , then the set of ordered pairs (x, y) such that $x = f(t)$ and $y = g(t)$ is a **plane curve**.
- The equations $x = f(t)$ and $y = g(t)$ are **parametric equations** for the curve.
- The variable t is the **parameter**.
- Parametric equations have a definite direction of motion, called the **orientation** of the curve.

GRAPHING USING A TABLE

1) Sketch the parametric curve for the following set of parametric equations: $x = t^2 + t$ and $y = 2t - 1$

Select values of t , plug them into the parametric equations and plot the points:

t	x	y
-2	2	-5
-1	0	-3
$-\frac{1}{2}$	$-\frac{1}{4}$	-2
0	0	-1
1	2	1



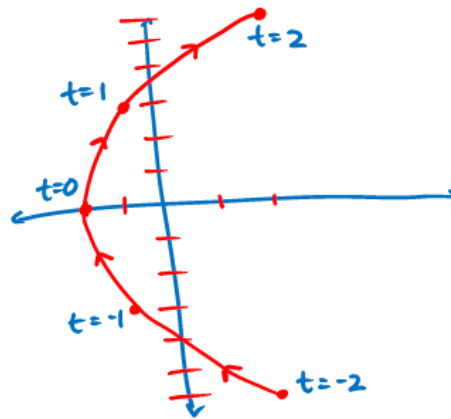
2) Sketch the parametric curve for the following set of parametric equations:

$$x = t^2 - 2$$

$$y = 3t$$

$$-2 \leq t \leq 2$$

t	x	y
-2	2	-6
-1	-1	-3
0	-2	0
1	-1	3
2	2	6



GRAPHING BY ELIMINATING THE PARAMETER

3) Sketch the parametric curve for the following set of parametric equations:

$$x = t^2 - 2$$

$$y = 3t$$

$$-2 \leq t \leq 2$$

$$x+2 = t^2$$
$$\pm\sqrt{x+2} = t$$

$$\text{So, } y = \pm 3\sqrt{x+2}$$

$$\text{OR } \frac{y}{3} = t$$

$$\text{So, } x = \left(\frac{y}{3}\right)^2 - 2$$

Then graph
w/in the interval

$$-2 \leq t \leq 2.$$

$$x = \frac{y^2}{9} - 2$$

Eliminating the parameter is not always easy, and in some cases, it's not even possible. In those cases, you would need to set up a table to graph.

GRAPHING ON YOUR CALCULATOR

- 4) Sketch the parametric curve for the following set of parametric equations:

$$x = t^2 - 2 \qquad y = 3t \qquad -2 \leq t \leq 2$$

```

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^iθL
FULL HORIZ G-T
SET CLOCK 01/01/04 18:09
    
```

```

WINDOW
Tmin=-2
Tmax=2
Tstep=1
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
    
```

- 5) Sketch the parametric curve for the set of parametric equations. Clearly indicate the direction of motion.

$$x = 5 \cos t \qquad y = 2 \sin t \qquad 0 \leq t \leq 2\pi$$

$$\frac{x}{5} = \cos t$$

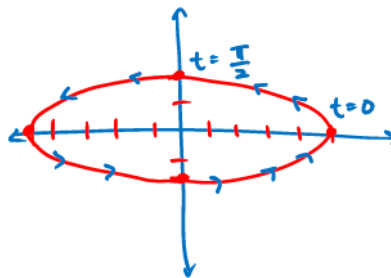
$$\frac{y}{2} = \sin t$$

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

Ellipse!



Direction of motion:

$$\text{@ } t=0 \quad x = 5 \cos 0 \quad y = 2 \sin 0 \\ x = 5 \quad y = 0$$

$$\text{@ } t = \frac{\pi}{2}$$

$$x = 5 \cos \frac{\pi}{2} \quad y = 2 \sin \frac{\pi}{2}$$

$$x = 0 \quad y = 2$$

∴ Counter clockwise

- 6) Sketch the parametric curve for the set of parametric equations. Clearly indicate the direction of motion.

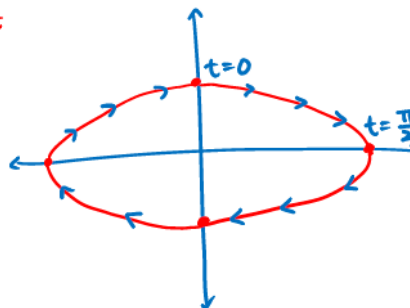
$$x = 6 \sin t \qquad y = 3 \cos t \qquad 0 \leq t \leq 2\pi$$

$$\frac{x}{6} = \sin t \qquad \frac{y}{3} = \cos t$$

$$\sin^2 t + \cos^2 t = 1$$

$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$



$$\text{@ } t=0$$

$$x = 6 \sin 0 \quad y = 3 \cos 0$$

$$x = 0 \quad y = 3$$

$$\text{@ } t = \frac{\pi}{2}$$

$$x = 6 \sin \frac{\pi}{2} \quad y = 3 \cos \frac{\pi}{2}$$

$$x = 6 \quad y = 0$$

Orientation: clockwise