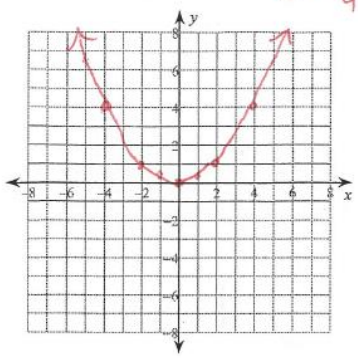
Parametric Equations

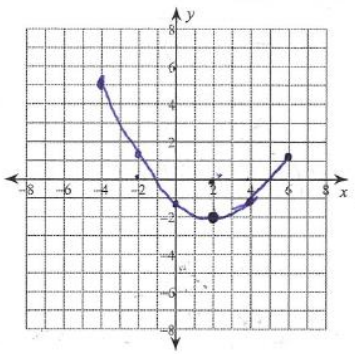
Sketch the curve for each pair of parametric equations.

1) $x = t,\ y = \frac{t^2}{4}$

$y = \frac{1}{4}x^2$



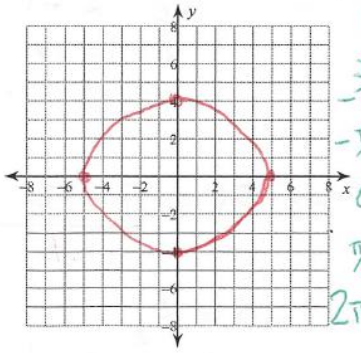
2) $x = -2t + 2,\ y = \frac{4t^2}{5} - 2,\ -2 \le t \le 3$



t	x	y
-2	6	1.2
-1	4	-1.2
0	2	-2
1	0	-2
2	-2	-1.2
3	-4	5.2

3) $x = 5\sin t,\ y = 4\cos t$

OR use a table



t	x	y
$-\pi$	0	-4
$-\pi/2$	-5	0
0	0	4
$\pi/2$	5	0
2π	0	4

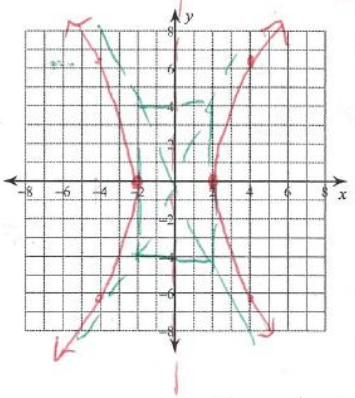
$\sin^2 x + \cos^2 x = 1$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$x = \frac{2}{\cos t}\quad y = \frac{4\sin t}{\cos t}$

4) $x = 2\sec t,\ y = 4\tan t$



t	x	y
$-\pi$	-2	0
$-\pi/2$	under	undef
0	2	0
$\pi/2$	under	undef
$\pi/3$	$2/\frac{1}{2} = 4$	$\frac{4\sqrt{3}}{2} = 4\sqrt{3}$ $\frac{1}{2}$ 6.7

$\frac{\sin^2 x}{\cos} + \frac{\cos^2 x}{\cos} = \frac{1}{\cos}$

$\tan^2 x + 1 = \sec^2 x$

$\left(\frac{y}{4}\right)^2 + 1 = \left(\frac{x}{2}\right)^2$ box

$\frac{x^2}{4} - \frac{y^2}{16} = 1$

Write each pair of parametric equations in rectangular form.

5) $x = -\frac{t^2}{3},\ y = t$ Solve for t!

$x = -\frac{y^2}{3}$

$y^2 = -3x$ | $y = \pm\sqrt{-3x}$

6) $x = t,\ y = \frac{t^2}{6} + \frac{2t}{3} - \frac{1}{3}$

$y = \frac{x^2}{6} + \frac{2x}{3} - \frac{1}{3}$

$y = \frac{x^2 + 4x - 2}{6}$

7) $x = -2t - 3,\ y = 2t^2 + 2t - \frac{5}{2}$

$t = \frac{x+3}{-2}$ $\quad y = 2\left(\frac{x+3}{-2}\right)^2 + 2\left(\frac{x+3}{-2}\right) - \frac{5}{2}$

$y = \frac{x^2+6x+9}{2} - x - 3 - \frac{5}{2}$

$y = \frac{x^2 + 4x - 2}{2}$

8) $x = 2\sin t,\ y = 4\cos t$

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

$\frac{x^2}{4} + \frac{y^2}{16} = 1$

9) $x = \sec t,\ y = 4\tan t$

$\tan^2 x + 1 = \sec^2 x$

$\left(\frac{y}{4}\right)^2 + 1 = x^2$

$x^2 - \frac{y^2}{16} = 1$

10) $x = 4\cos t - 1,\ y = 3\sin t + 1$

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{x+1}{4}\right)^2 + \left(\frac{y-1}{3}\right)^2 = 1$

$1 = \frac{(x+1)^2}{16} + \frac{(y-1)^2}{9}$

Use the parameter to write each rectangular equation as a pair of parametric equations.

11) $x = \frac{y^2}{6},\ t = y$ solve for y and plug into y

$y = t$

$x = \frac{(t)^2}{6} \rightarrow$ $x = \frac{t^2}{6}$

12) $y = -\frac{x^2}{4} + x + 1,\ t = -\frac{x}{3} + \frac{1}{3}$ solve for x

$3t = -x + 1$

$x = 1 - 3t$ plug in to x

$y = -\frac{(1-3t)^2}{4} + (1-3t) + 1$

$y = \frac{-1 + 6t - 9t^2}{4} + 2 - 3t = \frac{7 - 6t - 9t^2}{4}$

Critical thinking questions:

13) Write a set of parametric equations that represent $y = x^2 - 4x$. Then write a second set of parametric equations that represent the same function, but with a slower speed

① Let t = anything in terms of x

② Solve for x, plug into original equation

③ Solve for y.

$t = x$

$y = t^2 - 4t$

$t = 2x + 1 \rightarrow x = \frac{t-1}{2}$ | slower speed because $m = \frac{1}{2}$

$y = \left(\frac{t-1}{2}\right)^2 - 4\left(\frac{t-1}{2}\right)$ $\quad y = \frac{t^2 - 2t + 1}{4} - \frac{4t+1}{2}$

$y = \frac{t^2 - 10t + 9}{4}$

14) Write a set of parametric equations that represent $y = x^2 - 1$. Then write a second set of parametric equations that represent the same function, but with a faster speed and an opposite orientation.

$x = t + 1$

$t = x - 1$

$y = (t+1)^2 - 1$

$y = t^2 + 2t$

$x = -2t + 4$ ← faster speed and opposite direction of $m = 1$.

$y = (-2t+4)^2 - 1$

$y = 4t^2 - 16t + 15,\ x = -2t + 4$