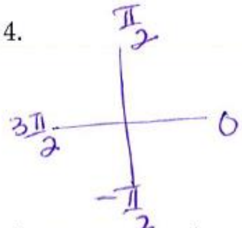
Parametric Equations Day #5

WARM UP: Provide equations and a window to display the RIGHT HALF of a circle with center

at (0, 0) and radius 4.

$x = 4\cos\theta$

$y = 4\sin\theta$



x: [-5, 5]

y: [-5, 5]

t: [-π/2, π/2] tstep: π/12

In order to simulate rotating objects we use angular velocity. The angular velocity of a rotating object is described in terms of the degrees or radians through which an object turns in a unit of time. For example 3600° degrees per second or 20π per second each describe the angular velocity of an object making 10 complete revolutions in one second. For a particular time T, the measure of the angle through which the object has turned is $\theta = 3600T$ or $\theta = 20\pi T$.

1. Suppose a 2.25 inch radius CD is making 8 counterclockwise revolutions per second. To track the position on a point P on the CD as a function of time, assume the disk revolves about the origin O and that at T=0, point P is at (2.25, 0).

   a. Stat the angular velocity in degrees per second and in radians per second.

(2.25, 0) $360 \cdot 8 = 2880t$ $2\pi \cdot 8 = 16\pi t$

   b. Write parametric equations that give the location of point P at any time T. Write both the degree and radian forms of the equations.

$x = 2.25\cos(2880t)$ $x = 2.25\cos(16\pi t)$

$y = 2.25\sin(2880t)$ $y = 2.25\sin(16\pi t)$

   c. Use an appropriate window and the radian form of the parametric equations to display exactly 1 second of motion. How many different times does the point rotate around the circle in one second?

x: [-3, 3] t: [0, 1]

y: [-3, 3] tstep = .01

   d. Change the equations and/or window to simulate the rotation of point P around the circle exactly ones. Twice? Three times?

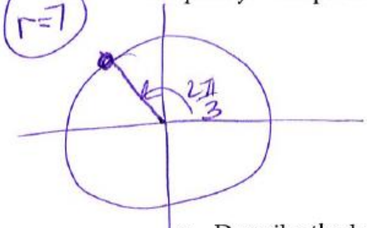
Change t max 1 rev → 1/8 ← # total revolutions

2 rev → 2/8

3 rev → 3/8

2. A pulley rotates counterclockwise at 5π radians per second. Represent the center of the pully by the origin O of a coordinate system and let P be a point on the circumference of the pulley. The parametric equations for the terminal point of a rotation vector OP are

$r=7$



$$\begin{cases} x = 7\cos\left(5\pi T + \frac{2\pi}{3}\right) \\ y = 7\sin\left(5\pi T + \frac{2\pi}{3}\right) \end{cases}$$

a. Describe the location of point P at T=0 by giving the components of OP. What are the direction and length of OP?

$x = 7\cos(5\pi \cdot 0 + \frac{2\pi}{3}) \rightarrow -3.5$

$y = 7\sin(5\pi \cdot 0 + \frac{2\pi}{3}) \rightarrow \frac{7\sqrt{3}}{2}$

$\rightarrow (-3.5, \frac{7\sqrt{3}}{2})$ point on circle

$\rightarrow$ · length Direct $[7, \frac{2\pi}{3}]$

b. Use an appropriate window to simulate the motion of point P.

t: [0,1] tstep = .01
x: [-8,8]
y: [-8,8]

c. Change the equation and/or window so that P makes exactly one revolution.

Change Tmax $\frac{1}{2.5}$ or .4 $\qquad 2.5 \cdot 2\pi = 5\pi$

d. Change the equations and/or window so that P starts at the 12:00 position and makes exactly 2 revolutions.

$x = 7\cos(5\pi t + \frac{\pi}{2})$ $\qquad$ 2rev $\rightarrow$ Tmax = 0.8
$y = 7\sin(5\pi t + \frac{\pi}{2})$

Parametric Equations Day #5

Assignment:

1. Prove the equations and window needed to display a circle with center at the origin, radius of 7, making 3 revolutions per second, starting at the lowest point. Show only one revolution.

$3 \cdot 2\pi t$ $\frac{3\pi}{2}$

$x = 7\cos\left(6\pi t + \frac{3\pi}{2}\right)$

$y = 7\sin\left(6\pi t + \frac{3\pi}{2}\right)$

$t: [0, \frac{1}{3}]$ tstep $= .01$ π 0

$x: [-8, 8]$

$y: [-8, 8]$

\# revolutions $\frac{3\pi}{2}$

2. Mrs. Long has a window fan that makes 2.5 revolutions per second. Each fan blade is 5 inches long, and there is a sticker on the edge of one of the blades. The center of the fan is 8 inches above the base of the fan. Before the fan is turned on, the sticker is in the 9:00 position.

$2.5 \cdot 2\pi$

a. Write a parametric equation that models the path of this sticker when the fan is turning.

$x = 5\cos(5\pi t + \pi)$

$y = 5\sin(5\pi t + \pi) + 8$

b. Give a window that will capture 2 revolutions on the fan blade.

$t: [0, .8]$

tstep $= .01$

$\frac{1}{2.5} = .4 \times 2 = .8$

$\uparrow$ \# revs

c. What are the coordinates and position of the sticker at 3 seconds.

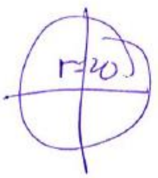
$x = 5\cos(5\pi(3) + \pi) = 5$

$y = 5\sin(5\pi(3) + \pi) + 8 = 8$

$(5, 8)$

3. A Ferris wheel with a 20 foot radius is turning at one revolution per $\boxed{\text{minute.}}$

   a. Find equations and a window that will show one revolution of the wheel. Work in radians.



$x = 20\cos(2\pi t)$ $\quad$ $t: [0, 1]$ $\quad$ tstep = .1

$y = 20\sin(2\pi t)$ $\quad$ $x: [-20, 20]$

$y: [-20, 20]$

   b. At T=0, where is the rider?

3:00 position $\quad$ (20,0)

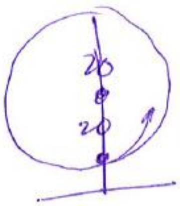
   c. Alter your equations so that the rider will start at the 6:00 position.

$x = 20\cos\left(2\pi t + \frac{3\pi}{2}\right)$

$y = 20\sin\left(2\pi t + \frac{3\pi}{2}\right)$

$\frac{3\pi}{2}$

   d. To make this model more realistic, the center will not be at (0, 0). If the x-axis represents the ground, what is a more realistic position of the center? Adjust your equations to reflect this change.



center @ (0,24)

$x = 20\cos\left(2\pi t + \frac{3\pi}{2}\right)$

$y = 20\sin\left(2\pi t + \frac{3\pi}{2}\right) + 24$

   e. Including all adjustments, find the location of the rider after 30 seconds of motion. Describe the location in terms of both clock positions and as compared to the origin.

1/2 min

Top of ferris wheel $\quad$ 12:00 position

(0,44)

   f. Including all adjustments, find the location of the rider after 10 seconds of motion. Describe the location in terms of both clock position and as compared to the origin.

$10\text{ secs} = \frac{10}{60} = \frac{1}{6}\text{ minute}$

$x = 20\cos\left(2\pi(1/6) + \frac{3\pi}{2}\right) \to 17.32$

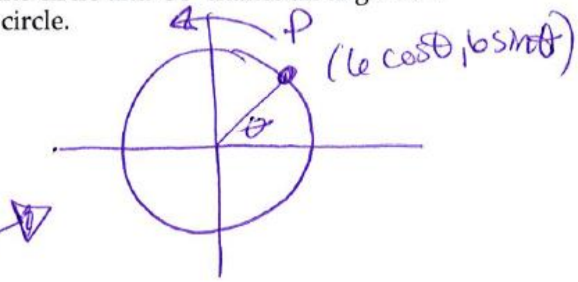
$y = 20\sin\left(2\pi(1/6) + \frac{3\pi}{2}\right) + 24 \to 14$

(17.32, 14)

Parametric Equations Day #5

One of the most common nonlinear motions is circular motion, that of objects turning around a point. The wheels on cars and bicycles, CDs, Ferris wheels, tools, and machines use circular motion in some way. On a computer disk, data is stored on tracks in sectors. The entire recording surface is accessible to the reading head almost instantaneously, simply by rotating the disk.

1) Suppose a circle has radius 6 cm. Point P is on the circle and OP makes an angle of θ with the positive x-axis as P moves around the circle.



a) How long is OP? 6 cm

b) What is the direction of OP? C C

c) What are the components of OP?

d) Write parametric equations describing the coordinates of point P.

$x = 6\cos t$

$y = 6\sin t$

e) Use your parametric equations to produce a graph of the circle. List a window to display the graph of exactly one full revolution.

$t: [0, 2\pi]$ tstep: $\pi/12$

$x: [-7, 7]$

$y: [-7, 7]$

2) Investigate how you could set your calculator to display only the part of the circle described in each case.

a) quarter circle between the positive x- and y-axes

$t: [0, \frac{\pi}{2}]$

b) half circle to the left of the y-axis

$t: [\frac{\pi}{2}, \frac{3\pi}{2}]$

c) half circle below the x-axis

$t: [\pi, 2\pi]$

d) half circle to the right of the y-axis

$t: [-\frac{\pi}{2}, \frac{\pi}{2}]$

e) quarter circle above the lines $y = x$ and $y = -x$

Assignment: $t: [\pi/4, \frac{3\pi}{4}]$

Parametric Equations Day #5

Duplicate

1) Consider the circle with center at the origin and radius 6 cm.

a) Write the parametric equations for this circle. Choose a window that will show exactly one revolution of the circle.

$x = 6\cos t$
$y = 6\sin t$

b) What changes to the equations or window will display a quarter circle above the x-axis and to the right of the y-axis?

c) What changes to the equations or window will display a half circle below the x-axis?

d) What changes to the equations or window will display a half circle to the left of the y-axis?

e) What changes to the equations or window will display a quarter circle below the x-axis and to the left of the y-axis?

f) Were these graphs drawn clockwise or counterclockwise? What changes to the equations or window will draw these graphs in the opposite direction?