7-5 Parametric Equations

Write each pair of parametric equations in rectangular form. Then graph the equation and state any restrictions on the domain.

9. $x = 2t - 5, y = t^2 + 4$

SOLUTION:

Solve for t in the parametric equation for x.

$$x = 2t - 5$$
$$x + 5 = 2t$$
$$\frac{x+5}{2} = t$$

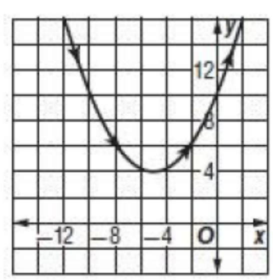
Substitute for t in the parametric equation for y.

$$\begin{aligned} y &= t^2 + 4 \\ &= \left(\frac{x+5}{2}\right)^2 + 4 \\ &= \frac{x^2+10x+25}{4} + 4 \\ &= \frac{x^2}{4} + \frac{10x}{4} + \frac{25}{4} + 4 \\ &= 0.25x^2 + 2.5x + 10.25 \end{aligned}$$

Make a table of values to graph y.

x	y	x	y
−13	20	−3	5
−11	13	−1	8
−9	8	1	13
−7	5	3	20
−5	4		

Plot the (x, y) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t increases. Since substituting larger values for t in $x = 2t - 5$ produces increasing values of x, the orientation moves from left to right.



11. $x = t^2 - 2, y = 5t$

SOLUTION:

Solve for t in the parametric equation for y.

$y = 5t$

$\frac{y}{5} = t$

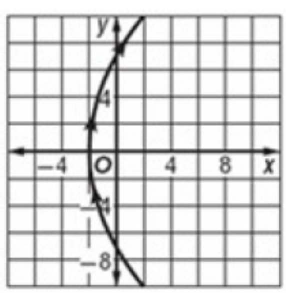
Substitute for t in the parametric equation for x.

$$x = t^2 - 2$$
$$x = \left(\frac{y}{5}\right)^2 - 2$$
$$x + 2 = \left(\frac{y}{5}\right)^2$$
$$\pm\sqrt{x+2} = \frac{y}{5}$$
$$\pm 5\sqrt{x+2} = y$$

Make a table of values to graph y.

x	**y**
−2	0
−1	−5, 5
0	−7.1, 7.1

Plot the (x, y) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t increases. Since substituting larger values for t in $y = 5t$ produces increasing values of y, the orientation moves from bottom to top.



13. $x = -t - 4, y = 3t^2$

SOLUTION:

Solve for t in the parametric equation for x.

$$\begin{aligned} x &= -t - 4 \\ x + 4 &= -t \\ -x - 4 &= t \end{aligned}$$

Substitute for t in the parametric equation for y.

$y = 3t^2$

$y = 3(-x - 4)^2$

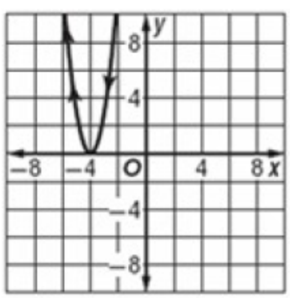
$y = 3(x^2 + 8x + 16)$

$y = 3x^2 + 24x + 48$

Make a table of values to graph y.

x	y
−6	12
−5	3
−4	0
−3	3
−2	12

Plot the (x, y) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t increases. Since substituting larger values for t in $x = -t - 4$ produces decreasing values of x, the orientation moves from right to left.



15. $x = 4t^2$, $y = \frac{6t}{5} + 9$

SOLUTION:

Solve for t in the parametric equation for y.

$$y = \frac{6t}{5} + 9$$
$$y - 9 = \frac{6t}{5}$$
$$\frac{5(y-9)}{6} = t$$

Substitute for t in the parametric equation for x.

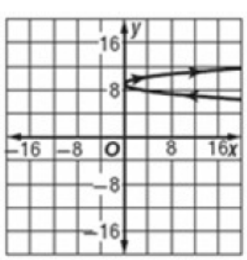
$$x = 4t^2$$
$$x = 4\left(\frac{5(y-9)}{6}\right)^2$$
$$\frac{x}{4} = \left(\frac{5(y-9)}{6}\right)^2$$
$$\pm\sqrt{\frac{x}{4}} = \frac{5(y-9)}{6}$$
$$\pm\frac{6\sqrt{x}}{5\sqrt{4}} = y - 9$$
$$\pm\frac{3\sqrt{x}}{5} = y - 9$$
$$\pm\frac{3\sqrt{x}}{5} + 9 = y$$

Make a table of values to graph y.

x	y
0	9
4	7.8, 10.2
8	7.3, 10.7
12	6.9, 11.1
16	6.6, 11.4

Plot the (x, y) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t increases.

Since substituting larger values for t in $y = \frac{6t}{5} + 9$ produces increasing values of y, the orientation moves from bottom to top.



17. **MOVIE STUNTS** During the filming of a movie, a stunt double leaps off the side of a building. The pulley system connected to the stunt double allows for a vertical fall modeled by $y = -16t^2 + 15t + 100$, and a horizontal movement modeled by $x = 4t$, where x and y are measured in feet and t is measured in seconds. Write and graph an equation in rectangular form to model the stunt double's fall for $0 \le t \le 3$.

SOLUTION:

Solve for t.

$x = 4t$

$\frac{x}{4} = t$

Substitute for t.

$y = -16t^2 + 15t + 100$

$y = -16\left(\frac{x}{4}\right)^2 + 15\left(\frac{x}{4}\right) + 100$

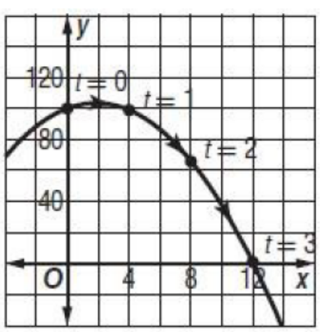
$y = -16\left(\frac{x^2}{16}\right) + 15\left(\frac{x}{4}\right) + 100$

$y = -x^2 + \frac{15x}{4} + 100$

Make a table of values for $0 \le t \le 3$.

t	x	y
0	0	100
1	4	99
2	8	66
3	12	1

Plot the (x, y) coordinates for each t–value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as t moves from 0 to 3.



Write each pair of parametric equations in rectangular form. Then state the restriction on the domain.

34. $x = \sqrt{t} + 4$
$y = 4t + 3$

SOLUTION:

Solve for t.

$$x = \sqrt{t} + 4$$
$$x - 4 = \sqrt{t}$$
$$(x - 4)^2 = t$$

Substitute.

$$y = 4t + 3$$
$$y = 4(x - 4)^2 + 3$$
$$y = 4(x^2 - 8x + 16) + 3$$
$$y = 4x^2 - 32x + 67$$

From the parametric equation $x = \sqrt{t} + 4$, the only possible values for x are values greater than or equal to four. The domain of the rectangular equation needs to be restricted to $x \geq 4$.

37. $x = \log(t - 4)$
$y = t$

SOLUTION:

Solve for t.

$$x = \log(t - 4)$$
$$10^x = t - 4$$
$$10^x + 4 = t$$

Substitute.

$y = t$

$y = 10^x + 4$

From the parametric equation $x = \log(t - 4)$, x can be all real numbers. Therefore, there is no restriction on the domain.