Section 13.4 – Derivatives of Parametric Equations (FDWK)

Find *dy/dx* in terms of *t* without eliminating the parameter.

1. $x = 4\sin t, \quad y = 2\cos t$

2. $x = \cos t, \quad y = \sqrt{3}\cos t$

3. $x = -\sqrt{t+1}, \quad y = \sqrt{3t}$

4. $x = \frac{1}{t}, \quad y = -2 + \ln t$

5. $x = t^2 - 3t, \quad y = t^3$

6. $x = t^2 + t, \quad y = t^2 - t$

Find the points at which the tangent to the curve is (a) horizontal, (b) vertical.

7. $x = \sec t, \quad y = \tan t$

8. $x = 2 + \cos t, \quad y = -1 + \sin t$

Answers

Find dy/dx in terms of t without eliminating the parameter.

1. $x = 4\sin t, \quad y = 2\cos t$

$\frac{dy}{dx} = \frac{2(-\sin t)}{4\cos t} = -\frac{1}{2}\tan t$

2. $x = \cos t, \quad y = \sqrt{3}\cos t$

$\frac{dy}{dx} = \frac{\sqrt{3}(-\sin t)}{-\sin t} = \sqrt{3}$

3. $x = -\sqrt{t+1}, \quad y = \sqrt{3t}$

$\frac{dy}{dx} = \frac{\frac{1}{\cancel{2}}(3t)^{-1/2}(3)}{-\frac{1}{\cancel{2}}(t+1)^{-1/2}} = -\frac{3\sqrt{t+1}}{\sqrt{3t}}$

4. $x = \frac{1}{t}, \quad y = -2 + \ln t$

$\frac{dy}{dx} = \frac{1/t}{-1/t^2} = -t$

5. $x = t^2 - 3t, \quad y = t^3$

$\frac{dy}{dx} = \frac{3t^3}{2t-3}$

6. $x = t^2 + t, \quad y = t^2 - t$

$\frac{dy}{dt} = \frac{2t-1}{2t+1}$

Find the points at which the tangent to the curve is (a) horizontal, (b) vertical.

7. $x = \sec t, \quad y = \tan t$

$\frac{dy}{dx} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t}$

a) : $m = 0$ IF $\sec t = 0$

$\frac{1}{\cos t} = 0$

$\boxed{\text{NO VALUES}}$

b) m UND IF $\tan t = 0$

$\frac{\sin t}{\cos t} = 0$

$\sin t = 0$

$\boxed{t = 0 + \pi k}$

8. $x = 2 + \cos t, \quad y = -1 + \sin t$

$\frac{dy}{dx} = \frac{\cos t}{-\sin t}$

a) $m = 0$ IF $\cos t = 0 \Rightarrow \boxed{t = \pi/2 + \pi k}$

b) m UND IF $\sin t = 0 \Rightarrow \boxed{t = 0 + \pi k}$

9. $x = 2 - t, \quad y = t^3 - 4t$

10. $x = e^{x^2}, \quad y = xe^x$

Find the equation of the tangent to the given curve at the given point.

11. $x = 2\sin t, \quad y = 2\cos t, \quad t = 3\pi / 4$

12. $x = 3t, \quad y = 8t^3, \quad t = -1/2$

13. $x = 2e^t, \quad y = \frac{1}{3}e^{-t}, \quad t = 0$

14. $x = t\ln t, \quad y = \ln t, \quad t = e$

Answers

9. $x=2-t,\quad y=t^3-4t$

$\frac{dy}{dx}=\frac{3t^2-4}{-1}$

a) $m=0$ IF $3t^2-4=0$

$t^2=4/3 \Rightarrow \boxed{t=\pm 2/\sqrt{3}}$

b) m UND IF $-1=0$

$\boxed{\text{NO VALUES}}$

10. $x=e^{t},\quad y=te^{t}$

$\frac{dy}{dx}=\frac{xe^x+e^x}{2xe^{x^2}}$

a) $m=0$ IF $xe^x+e^x=0$

$e^x(x+1)=0$ @ $\boxed{t=-1}$

b) m UND IF $2te^{t^2}=0$ @ $\boxed{t=0}$

Find the equation of the tangent to the given curve at the given point.

11. $x=2\sin t,\quad y=2\cos t,\quad t=3\pi/4$

$x(3\pi/4)=2(\sqrt{2}/2)=\sqrt{2}$

$y(3\pi/4)=2(-\sqrt{2}/2)=-\sqrt{2}$ $\Big\}$ $(\sqrt{2},-\sqrt{2})$

$\frac{dy}{dx}=\frac{-2\sin t}{2\cos t}=-\tan t\Big|_{t=3\pi/4}=1$

$\boxed{y+\sqrt{2}=1(x-\sqrt{2})}$

12. $x=3t,\quad y=8t^3,\quad t=-1/2$

$x(-1/2)=-3/2$

$y(-1/2)=-1$ $\Big\}$ $(-3/2,-1)$

$\frac{dy}{dx}=\frac{24t^2}{3}\Big|_{t=-1/2}=\frac{24(1/4)}{3}=2$

$\boxed{y+1=2(x+3/2)}$

13. $x=2e^t,\quad y=\frac{1}{3}e^{-t},\quad t=0$

$x(0)=2$

$y(0)=1/3$ $\Big\}$ $(2,1/3)$

$\frac{dy}{dx}=\frac{-\frac{1}{3}e^{-t}}{2e^t}\Big|_{t=0}=\frac{-1/3}{2}=-\frac{1}{6}$

$\boxed{y-1/3=-\frac{1}{6}(x-2)}$

14. $x=t\ln t,\quad y=\ln t,\quad t=e$

$x(e)=e\ln e=e$

$y(e)=1$ $\Big\}$ $(e,1)$

$\frac{dy}{dx}=\frac{1/t}{t\cdot\frac{1}{t}+\ln t}\Big|_{t=e}=\frac{1/e}{1+\ln e}=\frac{1/e}{2}=\frac{1}{2e}$

$\boxed{y-1=\frac{1}{2e}(x-e)}$