CALCULUS BC
WORKSHEET ON PARAMETRICS AND CALCULUS

Work these on **notebook paper**. Do not use your calculator.

On problems 1 – 5, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1. $x = t^2,\ y = t^2 + 6t + 5$
2. $x = t^2 + 1,\ y = 2t^3 - t^2$
3. $x = \sqrt{t},\ y = 3t^2 + 2t$
4. $x = \ln t,\ y = t^2 + t$
5. $x = 3\sin t + 2,\ y = 4\cos t - 1$

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6. A curve C is defined by the parametric equations $x = t^2 + t - 1,\ y = t^3 - t^2$.
   (a) Find $\frac{dy}{dx}$ in terms of t.
   (b) Find an equation of the tangent line to C at the point where $t = 2$.

7. A curve C is defined by the parametric equations $x = 2\cos t,\ y = 3\sin t$.
   (a) Find $\frac{dy}{dx}$ in terms of t.
   (b) Find an equation of the tangent line to C at the point where $t = \frac{\pi}{4}$.

On problems 8 – 10, find:

(a) $\frac{dy}{dx}$ in terms of t.
(b) all points of horizontal and vertical tangency

8. $x = t + 5,\ y = t^2 - 4t$
9. $x = t^2 - t + 1,\ y = t^3 - 3t$
10. $x = 3 + 2\cos t,\ y = -1 + 4\sin t$

On problems 11 - 12, a curve C is defined by the parametric equations given. For each problem, write an integral expression that represents the length of the arc of the curve over the given interval.

11. $x = t^2,\ y = t^3,\ 0 \le t \le 2$
12. $x = e^{2t} + 1,\ y = 3t - 1,\ -2 \le t \le 2$

Answers

Answers to Worksheet on Parametrics and Calculus

1. $\frac{dy}{dx}=\frac{2t+6}{2t}=1+\frac{3}{t}$; $\frac{d^2y}{dx^2}=\frac{-\frac{3}{t^2}}{2t}=-\frac{3}{2t^3}$

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2. $\frac{dy}{dt}=3t-1$; $\frac{d^2y}{dx^2}=\frac{3}{2t}$

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3. $\frac{dy}{dx}=\frac{6t+2}{\frac{1}{2}t^{-\frac{1}{2}}}=12t^{\frac{3}{2}}+4t^{\frac{1}{2}}$; $\frac{d^2y}{dx^2}=\frac{18t^{\frac{1}{2}}+2t^{-\frac{1}{2}}}{\frac{1}{2}t^{-\frac{1}{2}}}=36t+4$

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4. $\frac{dy}{dx}=\frac{2t+1}{\frac{1}{t}}=2t^2+t$; $\frac{d^2y}{dx^2}=\frac{4t+1}{\frac{1}{t}}=4t^2+t$

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5. $\frac{dy}{dx}=\frac{-4\sin t}{3\cos t}=-\frac{4}{3}\tan t$; $\frac{d^2y}{dx^2}=\frac{-\frac{4}{3}\sec^2 t}{3\cos t}=-\frac{4}{9}\sec^3 t$

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6. (a) $\frac{dy}{dx}=\frac{3t^2-2t}{2t+1}$

(b) When $t=2$, $\frac{dy}{dx}=\frac{3\cdot 2^2-2\cdot 2}{2\cdot 2+1}=\frac{8}{5}$, $x=5$, $y=4$ so the tangent line equation is

$y-4=\frac{8}{5}(x-5)$

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7. (a) $\frac{dy}{dx}=\frac{3\cos t}{-2\sin t}=-\frac{3}{2}\cot t$

(b) When $t=\frac{\pi}{4}$, $\frac{dy}{dx}=-\frac{3}{2}\cot\frac{\pi}{4}=-\frac{3}{2}$, $x=\sqrt{2}$, $y=\frac{3\sqrt{2}}{2}$ so the tangent line equation is

$y-\frac{3\sqrt{2}}{2}=-\frac{3}{2}\left(x-\sqrt{2}\right)$

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8. (a) $\frac{dy}{dx} = \frac{2t-4}{1}$

(b) A horizontal tangent occurs when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$ so a horizontal tangent occurs when $2t - 4 = 0$ which is at $t = 2$. When $t = 2, x = 7$ and $y = -4$ so a horizontal tangent occurs at the point $(7, -4)$. A vertical tangent occurs when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$. Since $1 \neq 0$, there is no point of vertical tangency on this curve.

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9. (a) $\frac{dy}{dx} = \frac{3t^2-3}{2t-1}$

(b) A horizontal tangent occurs when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$ so a horizontal tangent occurs when $3t^2 - 3 = 0$ which is at $t = \pm 1$. When $t = 1$, $x = 1$ and $y = -2$, and when $t = -1$, $x = 3$ and $y = 2$ so a horizontal tangent occurs at the points $(1, -2)$ and $(3, 2)$

A vertical tangent occurs when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$ so a vertical tangent occurs when $2t - 1 = 0$ so $t = \frac{1}{2}$. When $t = \frac{1}{2}$, $x = \frac{3}{4}$ and $y = -\frac{11}{8}$ so a vertical tangent occurs at the point $\left(\frac{3}{4}, -\frac{11}{8}\right)$.

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10. (a) $\frac{dy}{dx} = \frac{4\cos t}{-2\sin t}$

(b) A horizontal tangent occurs when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$ so a horizontal tangent occurs when $4\cos t = 0$ which is at $t = \frac{\pi}{2}$ and $\frac{3\pi}{2}$. When $t = \frac{\pi}{2}$, $x = 3$ and $y = 3$, and when $t = \frac{3\pi}{2}$, $x = 3$ and $y = -5$ so a horizontal tangent occurs at the points $(3, 3)$ and $(3, -5)$.

A vertical tangent occurs when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$ so a vertical tangent occurs when $-2\sin t = 0$ so $t = 0$ and π. When $t = 0$, $x = 5$ and $y = -1$ and when $t = \pi$, $x = 1$ and $y = -1$ so a vertical tangent occurs at the points $(5, -1)$ and $(1, -1)$.

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11. $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^2 \sqrt{(2t)^2 + (3t^2)^2}\, dt$ or $\int_0^2 \sqrt{4t^2 + 9t^4}\, dt$

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12. $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_{-2}^2 \sqrt{(2e^{2t})^2 + (3)^2}\, dt$ or $\int_{-2}^2 \sqrt{4e^{4t} + 9}\, dt$