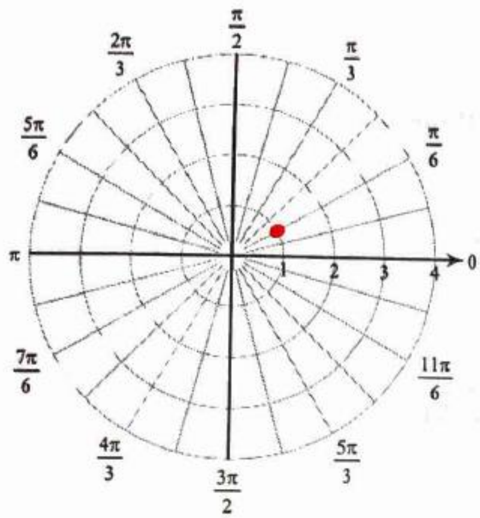
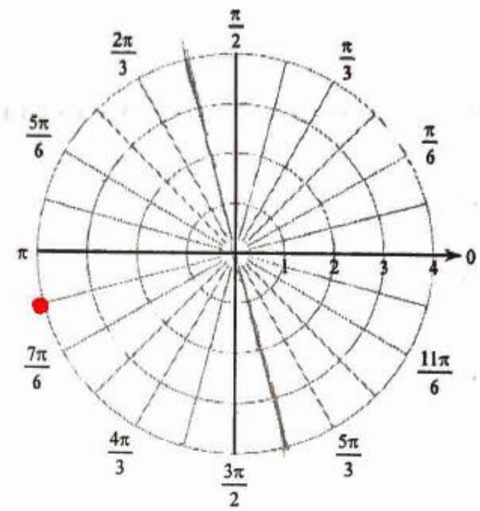
Plot the point with the given polar coordinates.

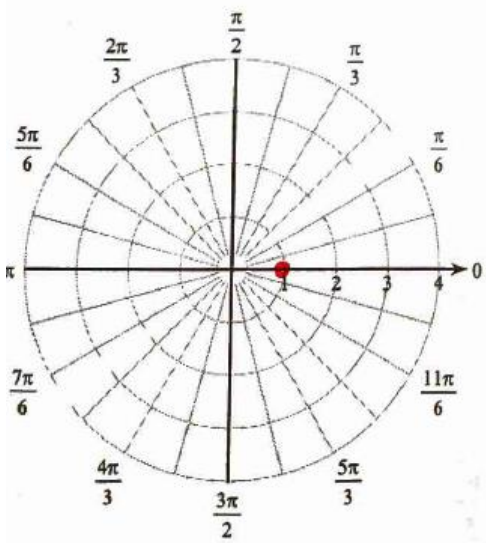
1) $\left(1, \frac{\pi}{6}\right)$



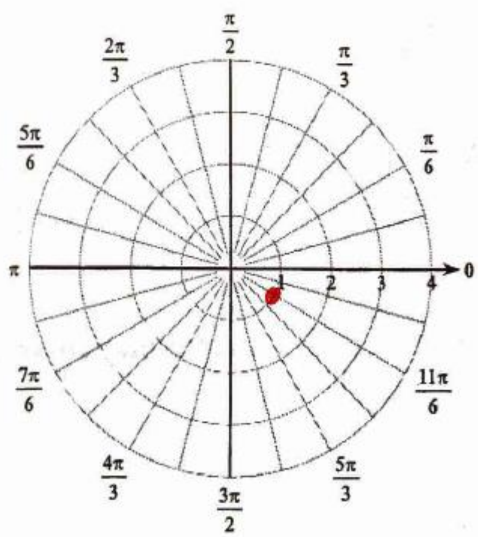
2) $\left(4, \frac{13\pi}{12}\right)$



3) $(1, 0)$



4) $\left(1, \frac{11\pi}{6}\right)$



Find 3 additional pairs of polar coordinates that describe the same point as the provided polar coordinates.

5) $\left(3, \frac{19\pi}{12}\right)$ $\left(-3, \frac{7\pi}{12}\right)$

$\left(3, -\frac{5\pi}{12}\right)\left(-3, -\frac{17\pi}{12}\right)$

6) $\left(4, \frac{\pi}{2}\right)$ $\left(-4, -\pi/2\right)$

$\left(-4, 3\pi/2\right)$

$\left(4, -3\pi/2\right)$

Convert each pair of polar coordinates to rectangular coordinates.

7) $\left(1, \frac{\pi}{2}\right)$ $x = 1\cos \pi/2,\ y = 1\sin \pi/2$

$(0, 1)$

8) $\left(1, \frac{7\pi}{6}\right)$ $x = 1\cos\frac{7\pi}{6},\ y = 1\sin\frac{7\pi}{6}$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

9) $(3, 0)$ $x = 3\cos 0,\ y = 3\sin 0$

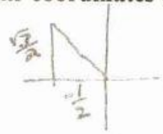
$(3, 0)$

10) $\left(2, \frac{5\pi}{3}\right)$ $x = 2\cos\frac{5\pi}{3},\ y = 2\sin\frac{5\pi}{3}$

$(1, -\sqrt{3})$

Convert each pair of rectangular coordinates to polar coordinates where $r > 0$ and $0 \le \theta < 2\pi$.

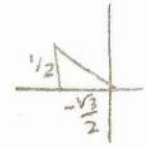
11) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $r = 1$ $\theta = \frac{2\pi}{3}$



$\left(1, \frac{2\pi}{3}\right)$

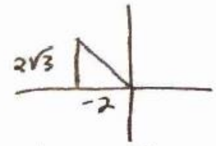
12) $(-3, 0)$ $(3, \pi)$

13) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $r = 1$ $\theta = \frac{5\pi}{6}$



$\left(1, \frac{5\pi}{6}\right)$

14) $(-2, 2\sqrt{3})$



$\left(4, \frac{2\pi}{3}\right)$

$r = 4$ $\theta = \frac{2\pi}{3}$

Convert each equation from rectangular to polar form.

15) $(x+1)^2 + y^2 = 1$

$x^2 + 2x + 1 + y^2 = 1$

$r^2 + 2x = 0$

divide by r $\quad r^2 + 2(r\cos\theta) = 0$

$r = -2\cos\theta$

16) $x = \frac{y^2}{5}$

$5x = y^2$

$5(r\cos\theta) = (r\sin\theta)^2$

$5r\cos\theta = r^2\sin^2\theta$

$r = \frac{5\cos\theta}{\sin^2\theta}$

$r = 5\cot\theta\csc\theta$

Convert each equation from polar to rectangular form.

ellipse 17) $(r = -4\cos\theta + 2\sin\theta)\,r$

$r^2 = -4r\cos\theta + 2r\sin\theta$

$x^2 + y^2 = -4x + 2y$

$x^2 + 4x + \underline{4} + y^2 - 2y + \underline{1} = 0 + \underline{4} + \underline{1}$

$(x+2)^2 + (y-1)^2 = 5$

18) $\tan\theta = 1$

$\frac{y}{x} = 1$

$y = x$

19) $r = 3\tan\theta\sec\theta$

$r = \frac{3y}{x\cos\theta}$

$r\cos\theta \cdot x = 3y$

$x^2 = 3y$

$y = \frac{x^2}{3}$ parabola

20) $(r = 4\sin\theta)\, r$

$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$

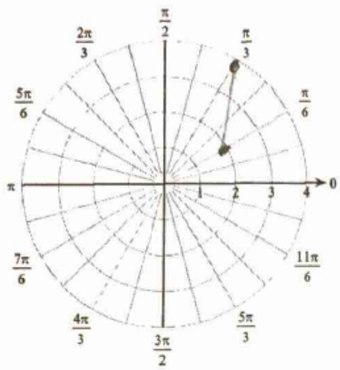
$x^2 + y^2 - 4y + \underline{4} = 0 + \underline{4}$

$x^2 + (y-2)^2 = 4$

circle

Two points are specified using polar coordinates. Find the distance between the points. Hint: convert to rectangular coordinates first then use a formula - you know - one that finds distance. ;)

21) $\left(2, \frac{\pi}{6}\right), \left(4, \frac{\pi}{3}\right)$



Rectangular:

$\left(2\cos\frac{\pi}{6}, 2\sin\frac{\pi}{6}\right) \quad \left(4\cos\frac{\pi}{3}, 4\sin\frac{\pi}{3}\right)$

$(\sqrt{3}, 1) + (2, 2\sqrt{3})$

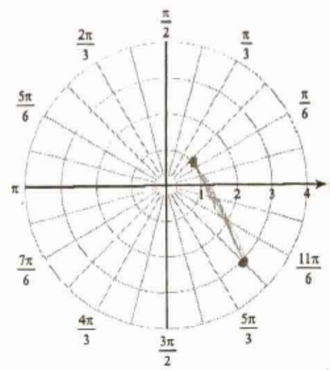
$d = \sqrt{(\sqrt{3}-2)^2 + (1-2\sqrt{3})^2}$

$d = \sqrt{(3-4\sqrt{3}+4) + (1-4\sqrt{3}+12)}$

$d = \sqrt{20-8\sqrt{3}}$

$d \approx 2.48$

22) $\left(3, \frac{7\pi}{4}\right), \left(1, \frac{\pi}{4}\right)$



Rectangular:

$\left(3\cos\frac{7\pi}{4}, 3\sin\frac{7\pi}{4}\right) \quad \left(1\cos\frac{\pi}{4}, 1\sin\frac{\pi}{4}\right)$

$\left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right) \quad \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$d = \sqrt{\left(\frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)^2}$

$d = \sqrt{(\sqrt{2})^2 + (-2\sqrt{2})^2}$

$d = \sqrt{2+8}$

$d = \sqrt{10}$

$d \approx 3.16$