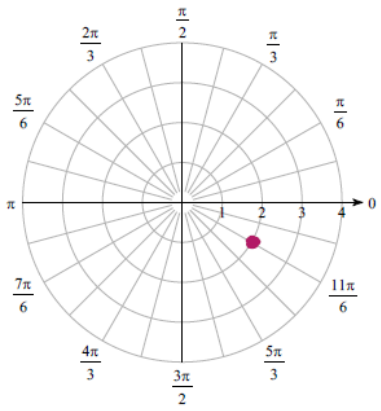
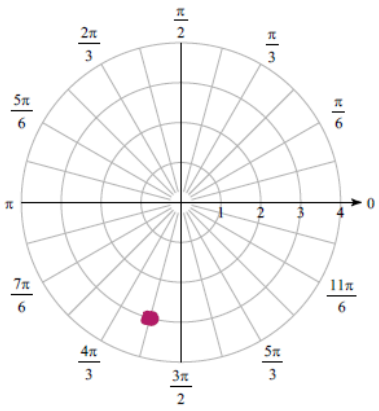
Intro to Polar Form Notes

Plot the point with the given polar coordinates.

1) $\left(-2, \frac{5\pi}{6}\right)$

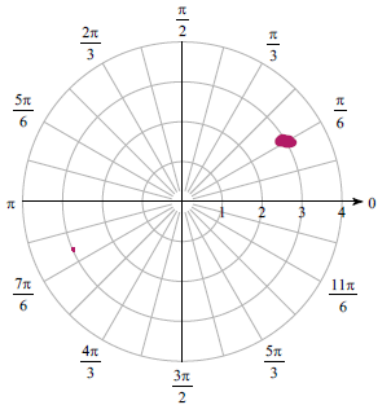


2) $\left(3, -\frac{7\pi}{12}\right)$

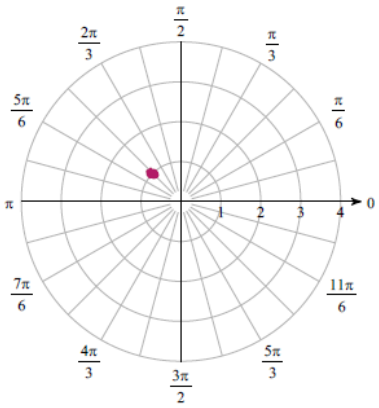


Find 3 other polar coordinates that describe the same point as the provided polar coordinates.

3) $\left(3, -\frac{11\pi}{6}\right)$



4) $\left(-1, -\frac{\pi}{4}\right)$



$(3, \pi/6)$ $(-3, -\frac{5\pi}{6})$ $(-1, \frac{7\pi}{4})$ $(1, \frac{3\pi}{4})$

$(-3, 7\pi/6)$ $(1, -\frac{5\pi}{4})$

Convert each pair of polar coordinates to rectangular coordinates.

5) $\left(4, -\frac{11\pi}{6}\right)$ (r, θ) — 1st quad.

$x = r\cos\theta = 4\cos\left(\frac{-11\pi}{6}\right) = \frac{4\sqrt{3}}{2}$ or $2\sqrt{3}$

$y = r\sin\theta = 4\sin\left(\frac{-11\pi}{6}\right) = 4\left(\frac{1}{2}\right) = 2$

$(2\sqrt{3}, 2)$

6) $\left(-2, \frac{4\pi}{3}\right)$

$x = -2\cos\left(\frac{4\pi}{3}\right) = -2\left(\frac{-1}{2}\right) = 1$

$y = -2\sin\left(\frac{4\pi}{3}\right) = -2\left(\frac{-\sqrt{3}}{2}\right) = \sqrt{3}$

$(1, \sqrt{3})$

Convert each pair of rectangular coordinates to polar coordinates where $r > 0$ and $0 \le \theta < 2\pi$.

7) $(-2, 2\sqrt{3})$ $x^2 + y^2 = r^2$

$(-2)^2 + (2\sqrt{3})^2 = r^2$

$4 + 4\cdot 3 = r^2$

$4 + 12 = r^2$

$16 = r^2$

$r = 4$

$\tan\theta = \frac{2\sqrt{3}}{-2}$

$\tan\theta = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$

$(4, \frac{2\pi}{3})$

8) $(-\sqrt{2}, -\sqrt{2})$

$(-\sqrt{2})^2 + (-\sqrt{2})^2 = 2 + 2 = 4$

$\sqrt{4} = 2 = r$

$\tan\theta = \frac{-\sqrt{2}}{-\sqrt{2}}$

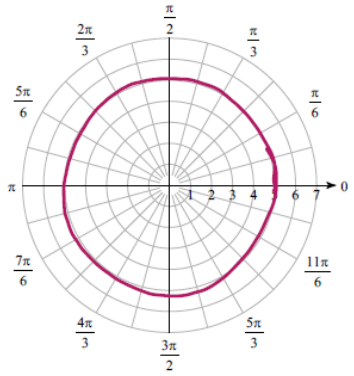
$\tan\theta = 1$

$\theta = \frac{5\pi}{4}$

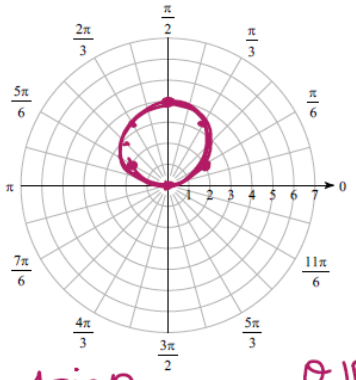
$(2, \frac{5\pi}{4})$

Consider each polar equation. Classify the curve; and sketch the graph.

9) $r = 5$ circle



10) $r = 4\sin\theta$ circle

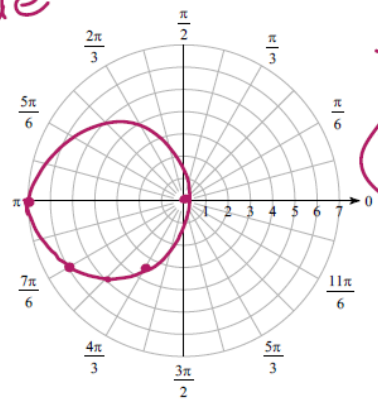


θ	$r = 4\sin\theta$
0	$r = 0$
$\pi/2$	4
π	0
$\frac{3\pi}{2}$	−4
2π	0

θ	$r = 4\sin\theta$
$\pi/4$	$4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$
$\frac{\pi}{6}$	$4\left(\frac{1}{2}\right) = 2$
$\pi/3$	$4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$
$2\pi/3$	$= 2\sqrt{3}$
$5\pi/6$	2

11) $r = -7\cos\theta$

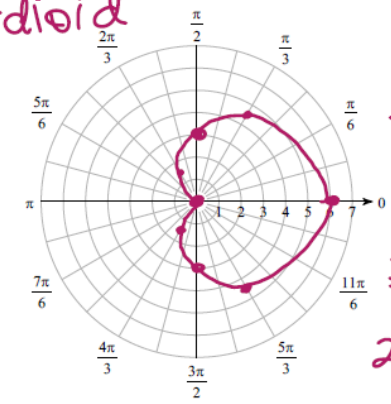
circle



θ	$r = -7\cos\theta$
0	-7
$\pi/6$	$-7\cos\frac{\pi}{6} = -\frac{7\sqrt{3}}{2}$
$\pi/3$	$-7\cos\frac{\pi}{3} = -7\left(\frac{1}{2}\right) = -3.5$
$\pi/4$	$-7\cos\frac{\pi}{4} = -\frac{7\sqrt{2}}{2}$
$\pi/2$	0

12) $r = 3 + 3\cos\theta$

cardioid

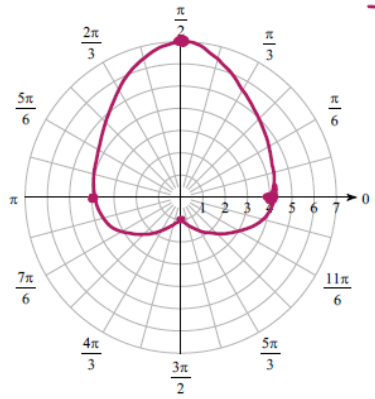


θ	$3 + 3\cos\theta$
0	6
$\frac{\pi}{2}$	3
π	0
$\frac{3\pi}{2}$	3
2π	6

θ	$3 + 3\cos\theta$
$\pi/3$	$3 + 3\left(\frac{1}{2}\right) = 4.5$
$2\pi/3$	$3 + 3\left(-\frac{1}{2}\right) = 1.5$
$4\pi/3$	$3 + 3\left(-\frac{1}{2}\right) = 1.5$
$5\pi/3$	$3 + 3\left(\frac{1}{2}\right) = 4.5$

limacon

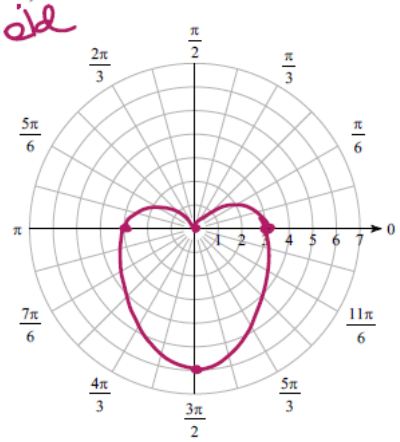
13) $r = 4 + 3\sin\theta$



θ	$r = 4 + 3\sin\theta$
0	4
$\pi/2$	7
π	4
$3\pi/2$	1

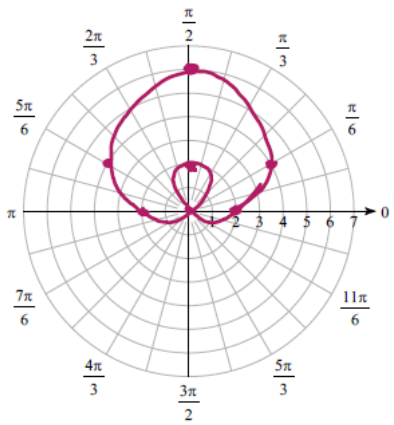
14) $r = 3 - 3\sin\theta$

cardioid



θ	$3 - 3\sin\theta$
0	3
$\pi/2$	0
π	3
$3\pi/2$	6

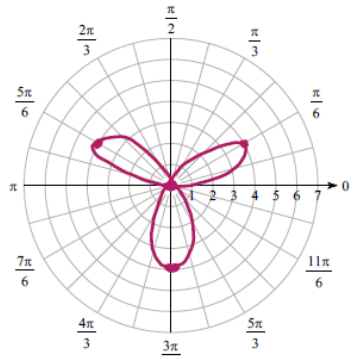
15) $r = 2 + 4\sin\theta$



θ	$2+4\sin\theta$
0	2
$\pi/2$	6
π	2
$3\pi/2$	−2
2π	2

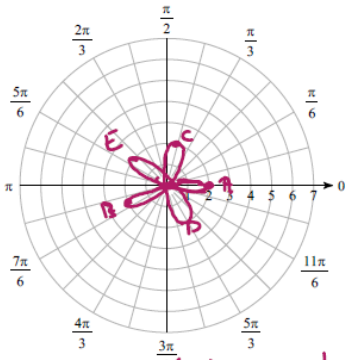
θ	$2+4\sin\theta$
$\pi/6$	4
$5\pi/6$	4
$7\pi/6$	0
$11\pi/6$	0

16) $r = 4\sin(3\theta)$



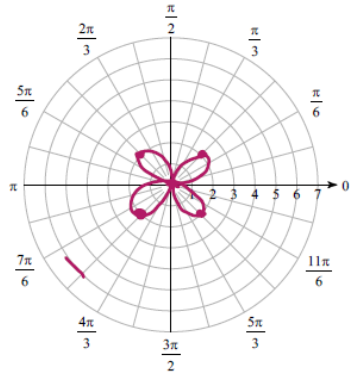
θ	$r = 4\sin(3\theta)$	θ	$4\sin(3\theta)$
0	0	$4\pi/6$	0
$\pi/6$	4	$5\pi/6$	4
$2\pi/6$	0	$6\pi/6$	0
$3\pi/6$	-4		

17) $r = 2\cos(5\theta)$



θ	$2\cos(5\theta)$	θ		θ	
0	2	$4\pi/10$	2	$8\pi/10$	2
$\pi/10$	0	$5\pi/10$	0	$9\pi/10$	0
$2\pi/10$	-2	$6\pi/10$	-2	$10\pi/10$	-2
$3\pi/10$	0	$7\pi/10$	0		

18) $r = 2\sin(2\theta)$



θ	$2\sin(2\theta)$
0	0
$\pi/4$	2
$2\pi/4$	0
$3\pi/4$	-2
$4\pi/4$	0
$5\pi/4$	2
$6\pi/4$	0
$7\pi/4$	-2
$8\pi/4$	0

$\frac{2\pi}{2} = (\pi)/4$

Convert each equation from rectangular to polar form.

19) $y = -x\sqrt{3}$

$r\sin\theta = -r\cos\theta\sqrt{3}$

$\sin\theta = -\cos\theta\sqrt{3}$

$\tan\theta = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$

20) $y = 5x$

$r\sin\theta = 5r\cos\theta$

$\sin\theta = 5\cos\theta$

$\tan\theta = 5$

21) $(x+3)^2 + (y-1)^2 = 10$

$x^2 + 6x + 9 + y^2 - 2y + 1 = 10$

$\frac{r^2}{r} + \frac{6r\cos\theta}{r} - \frac{2r\sin\theta}{r} = \frac{0}{r}$

$r + 6\cos\theta - 2\sin\theta = 0$

$r = 2\sin\theta - 6\cos\theta$

22) $y = \frac{x^2}{3}$

$r\sin\theta = \frac{(r\cos\theta)^2}{3}$

$\frac{3r\sin\theta}{r} = \frac{r^2\cos^2\theta}{r}$

$\frac{3\sin\theta}{\cos^2\theta} = \frac{r\cos^2\theta}{\cos^2\theta}$

$3\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta} = r$

$3\tan\theta\cdot\sec\theta = r$

Convert each equation from polar to rectangular form.

23) $\theta = \frac{3\pi}{4}$

$\tan\theta = \tan\frac{3\pi}{4}$

$\frac{y}{x} = -1$

$y = -x$

24) $r = 4\cos\theta + 2\sin\theta$ $\cdot r$

$r^2 = 4r\cos\theta + 2r\sin\theta$

$x^2 + y^2 = 4x + 2y$

$x^2 - 4x + 4 + y^2 - 2y + 1 = 0 + 4 + 1$

$(x-2)^2 + (y-1)^2 = 5$

25) $r^2 = 4\csc(2\theta)$

$r^2 = \frac{4}{\sin(2\theta)}$

$r^2\sin(2\theta) = 4$

$r^2 2\sin\theta\cdot\cos\theta = 4$

$2r\sin\theta \cdot r\cdot\cos\theta = 4$

$2y \cdot x = 4$ or $xy = 2$

26) $r^2 = 3\sec(2\theta)$

$r^2 = \frac{3}{\cos(2\theta)}$

$r^2\cos(2\theta) = 3$

$r^2(\cos^2\theta - \sin^2\theta) = 3$

$r^2\cos^2\theta - r^2\sin^2\theta = 3$

$x^2 - y^2 = 3$