Worksheet 8.1—Polar Intro & Derivatives
Show all work. No calculator except unless specifically stated.

Short Answer

Convert the following equations to polar form.

1. $y = 4$

$r\sin\theta = 4$

$r = \dfrac{4}{\sin\theta}$

$r = 4\csc\theta$

2. $3x - 5y + 2 = 0$

$3(r\cos\theta) - 5(r\sin\theta) + 2 = 0$

$r(3\cos\theta - 5\sin\theta) = -2$

$r = \dfrac{-2}{3\cos\theta - 5\sin\theta}$

oR

$r = \dfrac{2}{5\sin\theta - 3\cos\theta}$

3. $x^2 + y^2 = 25$

$r^2 = 25$

$r = -5$ or $r = 5$

Convert the following equations to rectangular form.

4. $r = 3\sec\theta$

$r = 3\left(\frac{r}{x}\right)$

$1 = \frac{3}{x}$

$x = 3$

5. $r = 2\sin\theta$

$r = 2\left(\frac{y}{r}\right)$

$r^2 = 2y$

$x^2 + y^2 = 2y$

6. $\theta = \dfrac{5\pi}{6}$

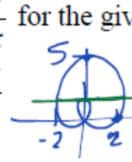
$\tan\theta = \tan\frac{5\pi}{6}$

$\frac{y}{x} = -\frac{\sqrt{3}}{3}$

$y = -\frac{\sqrt{3}}{3}x$

For the following, find $\dfrac{dy}{dx}$ for the given value of θ.

7. $r = 2 + 3\sin\theta$, $\theta = \dfrac{3\pi}{2}$



limaçon

$m = 0$ @ $\theta = \frac{3\pi}{2}$

(Top of inner loop)

$x(\theta) = r\cos\theta$

$x(\theta) = (2 + 3\sin\theta)\cos\theta$

$x'(\theta) = (3\cos\theta)(\cos\theta) + (2 + 3\sin\theta)(-\sin\theta)$

$= 3\cos^2\theta - 2\sin\theta - 3\sin^2\theta$

$y(\theta) = r\sin\theta$

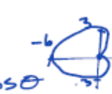
$y(\theta) = (2 + 3\sin\theta)\sin\theta$

$y'(\theta) = 3\cos\theta\sin\theta + (2 + 3\sin\theta)\cos\theta$

$\left.\dfrac{dy}{dx}\right|_{\theta = \frac{3\pi}{2}} = \dfrac{y'\left(\frac{3\pi}{2}\right)}{x'\left(\frac{3\pi}{2}\right)} = \dfrac{0 + (2-3)(0)}{0 + 2 - 3} = 0$

8. $r = 3(1 - \cos\theta)$, $\theta = \dfrac{\pi}{2}$

$r = 3 - 3\cos\theta$



cardioid

$x(\theta) = (3 - 3\cos\theta)\cos\theta$

$x'(\theta) = (3\sin\theta)\cos\theta + (3 - 3\cos\theta)(-\sin\theta)$

$x'\left(\frac{\pi}{2}\right) = 0 + (3)(-1)$
$= -3$

$y(\theta) = (3 - 3\cos\theta)\sin\theta$

$y'(\theta) = (3\sin\theta)\sin\theta + (3 - 3\cos\theta)(\cos\theta)$

$y'\left(\frac{\pi}{2}\right) = 3 + (3)(0)$
$= 3$

So, $\left.\dfrac{dy}{dx}\right|_{\theta = \frac{\pi}{2}} = \dfrac{y'\left(\frac{\pi}{2}\right)}{x'\left(\frac{\pi}{2}\right)} = \dfrac{3}{-3} = -1$

9. $r = 4\sin\theta$, $\theta = \frac{\pi}{3}$

$x = (4\sin\theta)\cos\theta$
$x' = 4\cos^2\theta - 4\sin^2\theta$
$x' = 4\cos 2\theta$
$x'(\frac{\pi}{3}) = 4\cos\frac{2\pi}{3}$
$= 4(-\frac{1}{2})$
$= -2$

$y = (4\sin\theta)\sin\theta$
$y = 4(\sin\theta)^2$
$y' = 8\sin\theta\cos\theta$
$y' = 4\sin 2\theta$
$y'(\frac{\pi}{3}) = 4\sin\frac{2\pi}{3}$
$= 4(\frac{\sqrt{3}}{2})$
$= 2\sqrt{3}$

So, $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

10. $r = 2\sin(3\theta)$, $\theta = \frac{\pi}{4}$

$x = 2\sin(3\theta)\cos\theta$
$x' = 6\cos(3\theta)\cos\theta - 2\sin(3\theta)\sin\theta$
$x'(\frac{\pi}{4}) = 6\cos(\frac{3\pi}{4})\cos\frac{\pi}{4} - 2\sin\frac{3\pi}{4}\cdot\sin\frac{\pi}{4}$
$= 6(-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) - 2(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2})$
$= -3 - 1$
$= -4$

$y = 2\sin(3\theta)\sin\theta$
$y' = 6\cos(3\theta)\sin\theta + 2\sin(3\theta)\cos\theta$
$y'(\frac{\pi}{4}) = 6\cos\frac{3\pi}{4}\sin\frac{\pi}{4} + 2\sin\frac{3\pi}{4}\cos\frac{\pi}{4}$
$= 6(-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) + 2(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2})$
$= -3 + 1$
$= -2$

So, $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{-2}{-4} = \frac{1}{2}$

11. Find the point of horizontal and vertical tangency for $r = 1 + \sin\theta$. Give your answers in polar form (r, θ).

$x = (1+\sin\theta)\cos\theta$ $\quad y = (1+\sin\theta)\sin\theta$
$x' = \cos^2\theta - (1+\sin\theta)\sin\theta$ $\quad y' = \cos\theta\sin\theta + (1+\sin\theta)\cos\theta$
$x' = (\cos^2\theta) - \sin\theta - \sin^2\theta$ $\quad y' = \cos\theta\sin\theta + \cos\theta + \cos\theta\sin\theta$
PID
$x' = (1 - \sin^2\theta) - \sin\theta - \sin^2\theta$ $\quad y' = 2\cos\theta\sin\theta + \cos\theta$
$x' = -2\sin^2\theta - \sin\theta + 1$

Horz

$\frac{dy}{dx} = \frac{0}{\neq 0}$

$y'(\theta) = 0$
$2\cos\theta\sin\theta + \cos\theta = 0$
$\cos\theta(2\sin\theta + 1) = 0$
$\cos\theta = 0 \quad \sin\theta = -\frac{1}{2}$
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \quad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$
*

Vert

$\frac{dy}{dx} = \frac{\neq 0}{0}$

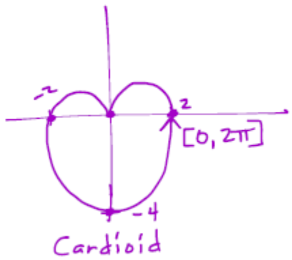
$x'(\theta) = 0$
$-2\sin^2\theta - \sin\theta + 1 = 0$
$2\sin^2\theta + \sin\theta - 1 = 0$
$(2\sin\theta - 1)(\sin\theta + 1) = 0$
$\sin\theta = \frac{1}{2} \quad \sin\theta = -1$
$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta = \frac{3\pi}{2}$
*

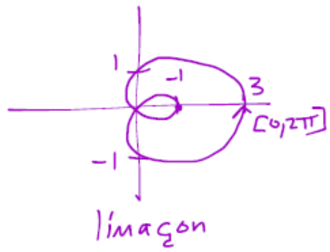
* $\frac{3\pi}{2}$ gives
$\frac{dy}{dx} = \frac{0}{0}$
So throw out
(Not a vert or horz tangent, but a cusp

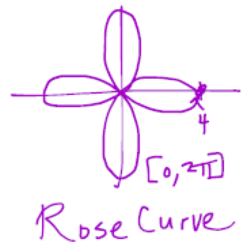
Tables Not shown

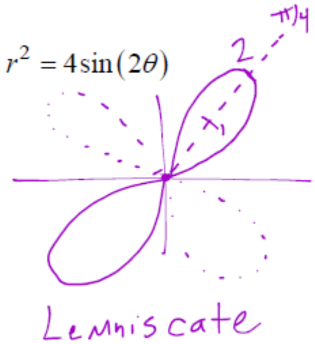
Make a table (of values, not one at which to eat) and sketch the graph.

12. $r = 2 - 2\sin\theta$ 13. $r = 1 + 2\cos\theta$ 14. $r = 4\cos(2\theta)$ 15. $r^2 = 4\sin(2\theta)$









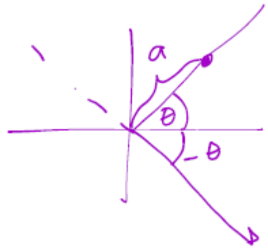
Multiple Choice

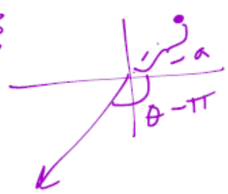
16. If $a \neq 0$ and $\theta \neq 0$, all of the following must represent the same point in polar coordinates *except* which ordered pair?

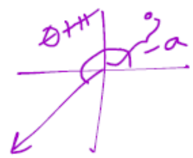
B

(A) (a, θ) (B) $(-a, -\theta)$ (C) $(-a, \theta - \pi)$ (D) $(-a, \theta + \pi)$ (E) $(a, \theta - 2\pi)$

A & B not same







17. Which of the following gives the slope of the polar curve $r = f(\theta)$ graphed in the xy-plane?

(A) $\frac{dr}{d\theta}$ (B) $\frac{dy}{d\theta}$ (C) $\frac{dx}{d\theta}$ (D) $\frac{dy/d\theta}{dx/d\theta}$ (E) $\frac{dy}{dx} \cdot \frac{dr}{d\theta}$

D

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

18. Which of the following represents the graph of the polar curve $r = 2\sec\theta$?

D

$r = 2\left(\frac{r}{x}\right)$

$1 = \frac{2}{x}$

$x = 2$