CALCULUS BC
POLAR Quiz Review

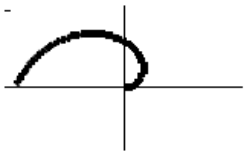
Work the following on notebook paper. Do not use your calculator except on problem 10.

1. Find the slope of the curve $r = 2 - 3\sin\theta$ at the point $(2, \pi)$.

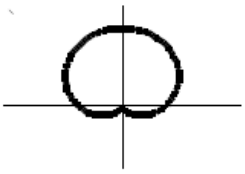
2. Find the equation of the tangent line to the curve $r = 3\sin(2\theta)$ at the point where $\theta = \frac{\pi}{3}$.

On problems 3 – 5, set up an integral to find the area of the shaded region. Do not evaluate.

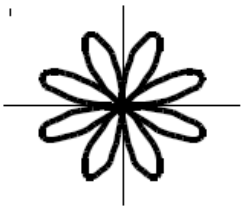
3. $r = \theta$



4. $r = 1 + \sin\theta$



5. $r = 2\sin 4\theta$



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6. Sketch the polar region described by the following integral expression for area:

$$\frac{1}{2}\int_0^{\pi/3} \sin^2(3\theta)\,d\theta$$

Use your calculator on problem 7.

7. Given the polar curve $r = \theta + 2\sin\theta$ for $0 \le \theta \le 2\pi$
   (a) Sketch the graph of the curve.
   (b) Find the angle θ that corresponds to the point(s) on the curve where $x = -1$.
   (c) Find the angle θ that corresponds to the point(s) on the curve where $y = 2$.

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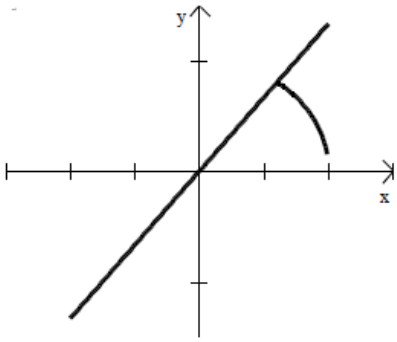
8. The figure shows the graphs of the line $y = \frac{2}{3}x$ and the curve C given by $y = \sqrt{1 - \frac{x^2}{4}}$. Let S be the region in the first quadrant bounded by the two graphs and the x-axis. The line and the curve intersect at point P.



(a) Find the coordinates of P.

(b) Set up and evaluate an integral expression with respect to x that gives the area of S.

(c) Find a polar equation to represent curve C.

(d) Use the polar equation found in (c) to set up and evaluate an integral expression with respect to the polar angle θ that gives the area of S.

Answers

Answers to Polar Quiz Review

1. $\frac{2}{3}$

2. $y-\frac{9}{4}=\frac{\sqrt{3}}{5}\left(x-\frac{3\sqrt{3}}{4}\right)$

3. $\frac{1}{2}\int_0^{\pi}\theta^2 d\theta$

4. $\frac{1}{2}\int_{\pi/2}^{\pi}(1+\sin\theta)^2\, d\theta$

5. $\frac{1}{2}\int_0^{\pi/4}(2\sin 4\theta)^2\, d\theta$

6. 1 petal of $r=\sin 3\theta$

7. (a) graph

   (b) 1.839, 4.295

   (c) 0.921, 2.563

8. (a) (1.2, 0.8)

   (b) 0.927

   (c) $r^2=\frac{4}{4\sin^2\theta+\cos^2\theta}$

   (d) 0.927

9. A curve is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \cos(3\theta)$ for $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$, where r is measured in meters and θ is measured in radians.

(a) Find the area bounded by the curve and the y-axis.

(b) Find the angle θ that corresponds to the point on the curve with y-coordinate -1.

(c) For what values of θ, $\pi \le \theta \le \frac{3\pi}{2}$, is $\frac{dr}{d\theta}$ positive? What does this say about r? What does it say about the curve?

(d) Find the value of θ on the interval $\pi \le \theta \le \frac{3\pi}{2}$ that corresponds to the point on the curve with the greatest distance from the origin. What is the greatest distance? Justify your answer.

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10. A region R in the xy-plane is bounded below by the x-axis and above by the polar curve defined by $r = \frac{4}{1+\sin\theta}$ for $0 \le \theta \le \pi$.

(a) Find the area of R by evaluating an integral in polar coordinates.

(b) The curve resembles an arch of the parabola $8y = 16 - x^2$. Convert the polar equation to rectangular coordinates, and prove that the curves are the same.

(c) Set up and evaluate an integral in rectangular coordinates that gives the area of R.

Answers

9. (a) 19.675

(b) 3.485

(c) $\frac{dr}{d\theta} > 0$ for $(\pi, 4.302)$. (Moves away from origin)

(d) The greatest distance is 5.245 when $\theta = 4.302$.

10. (a) 10.667 (b) $r + r\sin\theta = 4$, $\sqrt{x^2 + y^2} + y = 4$

$x^2 + y^2 = 16 - 8y + y^2$, $8y = 16 - x^2$

(c) 10.667