SHORT CUT TO APPLY INTEGRAL TEST

p-series

A series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$ is called a *p*-series, where *p* is a positive constant.

For $p = 1$, the series $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ is called the **harmonic series**.

WILL BE HELPFUL WHEN APPLYING COMPARISON TEST

Based on your experience with *p*-series and their reliance on the number one, fill in chart below.

p-Series Test

The *p*-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$, based on above

a) If $p = 1$, ~~THE SERIES DIVERGES~~ (HARMONIC SERIES)
~~THE TEST PROVIDES INSUFFICIENT INFO (COULD BE DIV OR CONV)~~

b) If $p < 1$, THE SERIES DIVERGES $(0 < p \le 1)$

c) If $p > 1$, THE SERIES CONVERGES

Note: If the *p*-series converges, we cannot find its sum. This is more often the case than not.

Example 13:
Determine of the following converges or diverges:

(a) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

$p = \frac{3}{2} > 1$

$\therefore$ CONVERGES

(b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{n}{n^{1/2}}$

$\sum_{n=1}^{\infty} n^{1/2} \rightarrow \frac{1}{n^{-1/2}}$

$p = \frac{-1}{2} < 1$

$\therefore$ DIVERGES

(c) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$

$\sum_{n=1}^{\infty} \frac{n^{1/2}}{n}$

$\sum_{n=1}^{\infty} n^{-1/2} \rightarrow \frac{1}{n^{1/2}}$

$p = \frac{1}{2} < 1$

$\therefore$ DIVERGES

(d) $\sum_{n=1}^{\infty} \frac{999999999}{n^{1.0000000001}}$