

## THE ROOT TEST

### **THEOREM 9.18** Root Test

Let  $\sum a_n$  be a series.

1.  $\sum a_n$  converges absolutely if  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$ .
2.  $\sum a_n$  diverges if  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} > 1$  or  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \infty$ .
3. The Root Test is inconclusive if  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = 1$ .

Sample Problem #2: USING THE ROOT TEST

Determine the convergence or divergence of the series:

$$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$$

$$a_n = \frac{e^{2n}}{n^n} = \left(\frac{e^2}{n}\right)^n \quad \sqrt[n]{|a_n|} = \frac{e^2}{n}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \frac{e^2}{n} = 0 < 1$$

$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$  is absolutely convergent