THE ROOT TEST

THEOREM 9.18 Root Test

Let Σa_n be a series.

1. Σa_n converges absolutely if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$.
2. Σa_n diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$.
3. The Root Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$.

Sample Problem #2: **USING THE ROOT TEST**

Determine the convergence or divergence of the series:

$$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$$

$$a_n = \frac{e^{2n}}{n^n} = \left(\frac{e^2}{n}\right)^n \qquad \sqrt[n]{|a_n|} = e^2/n$$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} e^2/n = 0 < 1$$

$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$ is absolutely convergent