A **Series** is the sum of the terms in a sequence. Finite sequences and series have defined first and last terms, whereas infinite sequences and series continue indefinitely. A series is informally the result of adding any number of terms from a sequence together: $a_1 + a_2 + a_3 + \cdots$. A series can be written more succinctly by using the summation symbol sigma, $\sum.$, the Greek letter "S" for **Esum** (the "E" is both silent and not really there.)

For infinite series, we can look at the sequence **of partial sums**, that is, looking to see what the sums are doing as we add additional terms. In general, the nth partial sum of a series is denoted S_n. This can be explored on a calculator by adding sequential terms to the aggregate sum.

Example 5:

For both $a_n = \frac{1}{n}$ and $b_n = \frac{1}{n^2}$, generate the sequence of partial sums $S_1, S_2, S_3, \ldots, S_n$, for each, then determine if the sequences converges or diverges. Do the results surprise you? Where else have we seen something like this before?

$\cancel{S_n}$ $S_n = \sum_{i=1}^{n}$

$S_n = \sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$

$= 0$ CONVERGES

$S_n = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots$

$= 0$ CONVERGES

Convergence and Divergence of a Series

What does it mean for a series to converge? To diverge? Let's look at a couple series from a special family called **geometric series.**

Example 6:

Given the series $\sum_{n=1}^{\infty} \frac{3}{2^n} = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{128} + \frac{3}{256} + \frac{3}{512} + \frac{3}{1024} + \cdots$, $\frac{3}{2048}, \frac{3}{4096}, \frac{3}{8192}$

find the first **ten** terms of the sequence of partial sums, and list them below, $S_1, S_2, S_3, \ldots, S_{10}$. Based on this sequence of partial sums, do you think the series converges? Diverges? To what? (HINT: first rewrite the rule of sequence so that it looks like an **exponential function** of n.)

$S_1 = \frac{3}{2}$

$S_2 = \frac{3}{2} + \frac{3}{4} = \frac{9}{4}$

$S_3 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} = \frac{21}{8}$

$S_4 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} = \frac{45}{16}$

$S_5 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{\cancel{256}\ 128} = \frac{\cancel{723}}{\cancel{256}}\ \frac{363}{128}$

$S_6 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{128} + \frac{3}{256} = \frac{\cancel{1449}}{\cancel{512}}\ \frac{729}{256}$

$S_7 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{128} + \frac{3}{256} + \frac{3}{512} = \frac{\cancel{2921}}{\cancel{1024}}\ \frac{1461}{512}$

$S_8 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{128} + \frac{3}{256} + \frac{3}{512} + \frac{3}{1024} = \frac{\cancel{5805}}{\cancel{2048}}\ \frac{2925}{1024}$

$S_9 = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{128} + \frac{3}{256} + \frac{3}{512} + \frac{3}{1024} + \frac{3}{2048} = \frac{\cancel{11653}}{\cancel{4096}}\ \frac{5853}{2048}$

$S_{10} = \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{128} + \frac{3}{256} + \frac{3}{512} + \frac{3}{1024} + \frac{3}{2048} + \frac{3}{4096} = \frac{\cancel{23229}}{\cancel{8192}}\ \frac{11709}{4096}$

~~CONVERGES~~ B/C $|r| < 1$ $(r = \frac{1}{2})$ AND IT APPROACHES 0.

CONVERGES

Example 7:

Given the series $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n = \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \frac{243}{32} + \cdots$, find the first <u>**five**</u> terms of the sequence of partial sums, and list them below. Based on this sequence of partial sums, do you think the series converges? Diverges? To what?

$S_1 = \frac{3}{2}$

$S_2 = \frac{3}{2} + \frac{9}{4} = \frac{15}{4}$

$S_3 = \frac{3}{2} + \frac{9}{4} + \frac{27}{8} = \frac{57}{8}$

$S_4 = \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} = \frac{195}{16}$

$S_5 = \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \frac{243}{32} = \frac{633}{32}$

DIVERGES B/C $|r| > 1$ AND IT APPROACHES ∞ $(\neq 0)$.
$(r = \frac{3}{2})$

We are now going to look at several families of infinite series and several tests that will help us determine whether they converge or diverge. For some that converge, we might be able to give the actual sum, or an interval in which we know the sum will be. For others, simply knowing that they converge will have to suffice.