

AP Calculus BC

Infinite Series

Direct and Limit Comparison Tests

Use the Direct Comparison Test to determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$

2. $\sum_{n=2}^{\infty} \frac{1}{n-1}$

3. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

4. $\sum_{n=0}^{\infty} \frac{1}{3^n + 1}$

6. $\sum_{n=0}^{\infty} \frac{3^n}{4^n + 5}$

7. $\sum_{n=2}^{\infty} \frac{\ln n}{n+1}$

8. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 1}}$

9. $\sum_{n=0}^{\infty} \frac{1}{n!}$

10. $\sum_{n=1}^{\infty} \frac{1}{e^{-n^2}}$

11. $\sum_{n=1}^{\infty} \frac{4^n}{3^n - 1}$

Answers

Use the Direct Comparison Test to determine the convergence or divergence of the series.

$$1. \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

Converges

$$2. \sum_{n=2}^{\infty} \frac{1}{n-1}$$

Diverges

$$3. \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$$

Diverges

$$4. \sum_{n=0}^{\infty} \frac{1}{3^n + 1}$$

Converges

$$6. \sum_{n=0}^{\infty} \frac{3^n}{4^n + 5}$$

Converges

$$7. \sum_{n=2}^{\infty} \frac{\ln n}{n+1}$$

Diverge

$$8. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 1}}$$

Converge

$$9. \sum_{n=0}^{\infty} \frac{1}{n!}$$

Converge

$$10. \sum_{n=1}^{\infty} \frac{1}{e^{-n^2}}$$

Diverges

$$11. \sum_{n=1}^{\infty} \frac{4^n}{3^n - 1}$$

Diverges

Use the Limit Comparison Test to determine the convergence or divergence of the series.

$$12. \sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

$$13. \sum_{n=1}^{\infty} \frac{2}{3^n-5}$$

$$14. \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

$$15. \sum_{n=3}^{\infty} \frac{3}{\sqrt{n^2-4}}$$

$$16. \sum_{n=3}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$$

$$17. \sum_{n=3}^{\infty} \frac{n+3}{n(n+2)}$$

$$18. \sum_{n=3}^{\infty} \frac{1}{n(n^2+1)}$$

$$19. \sum_{n=0}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

$$20. \sum_{n=3}^{\infty} \frac{n}{(n+1)2^{n-1}}$$

$$21. \sum_{n=3}^{\infty} \sin\left(\frac{1}{n}\right)$$

Answers

Use the Limit Comparison Test to determine the convergence or divergence of the series.

$$12. \sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

Diverges

$$13. \sum_{n=1}^{\infty} \frac{2}{3^n-5}$$

Converges

$$14. \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

Diverges

$$15. \sum_{n=3}^{\infty} \frac{3}{\sqrt{n^2-4}}$$

Diverges

$$16. \sum_{n=3}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$$

Converges

$$17. \sum_{n=3}^{\infty} \frac{n+3}{n(n+2)}$$

Diverges

$$18. \sum_{n=3}^{\infty} \frac{1}{n(n^2+1)}$$

Converges

$$19. \sum_{n=0}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

Converges

$$20. \sum_{n=3}^{\infty} \frac{n}{(n+1)2^{n-1}}$$

Converges

$$21. \sum_{n=3}^{\infty} \sin\left(\frac{1}{n}\right)$$

Diverges

In exercise 22-28, test for convergence or divergence using each test at least once. Identify which test you used.

(a) nth term Test for divergence

(d) Integral Test

(b) p-test

(e) Direct Comparison Test

(c) Geometric Series Test

(f) Limit Comparison Test

$$22. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$$

$$23. \sum_{n=0}^{\infty} 5 \left(-\frac{1}{5} \right)^n$$

$$24. \sum_{n=1}^{\infty} \frac{1}{3^n + 1}$$

$$25. \sum_{n=4}^{\infty} \frac{1}{3n^2 - 2n - 15}$$

$$26. \sum_{n=1}^{\infty} \frac{n}{2n + 3}$$

$$27. \sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)^2}$$

Answers

In exercise 22-28, test for convergence or divergence using each test at least once. Identify which test you used.

(a) nth term Test for divergence

(d) Integral Test

(b) p-test

(e) Direct Comparison Test

(c) Geometric Series Test

(f) Limit Comparison Test

$$22. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$$

Diverges
p-test

$$23. \sum_{n=0}^{\infty} 5 \left(-\frac{1}{5} \right)^n$$

Converges
Geometric Series Test

$$24. \sum_{n=1}^{\infty} \frac{1}{3^n + 1}$$

Converges
Direct Comparison Test

$$25. \sum_{n=4}^{\infty} \frac{1}{3n^2 - 2n - 15}$$

Converges
Limit Comparison Test

$$26. \sum_{n=1}^{\infty} \frac{n}{2n+3}$$

Diverges
 n^{th} term test

$$27. \sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)^2}$$

Converges
Integral Test