2017fa 140 Test4 Ch05 10Q Integration

Short Answer

1. Find the indefinite integral $\int (-8t+7)dt$.

2. Find the indefinite integral $\int 11\tan^2 x + 16 \; dx$.

3. Evaluate the integral.

$$\int_{7}^{6} -24x \; dx$$

given, $\int_{6}^{7} x^3 dx = \frac{1105}{4}$, $\int_{6}^{7} x^2 dx = \frac{127}{3}$, $\int_{6}^{7} x \; dx = \frac{13}{2}$, $\int_{6}^{7} dx = 1$.

Short Answer

1. Find the indefinite integral $\int (-8t+7)dt$. $= -8\int t dt + 7\int dt$

$= -8 \cdot \frac{1}{2} \cdot t^2 + 7t + C$

$= -4t^2 + 7t + C$

- Split integrand
- Antiderivative
- Simplify
- + C

2. Find the indefinite integral $\int 11\tan^2 x + 16 \; dx$. $= 11\int \tan^2 x dx + 16\int dx$

$= 11\int (\sec^2 x - 1)dx + 16\int dx$

$= 11\int \sec^2 x dx - 11\int dx + 16\int dx$

$= 11\tan x + 5x + C$

- Split integrand
- Antiderivative
- Simplify
- + C

3. Evaluate the integral.

$$\int_7^6 -24x \; dx \quad = +\int_6^7 24x dx = 24\int_6^7 x dx = 24 \cdot \frac{13}{2} = 156$$

$$\text{given, } \int_6^7 x^3 dx = \frac{1105}{4}, \int_6^7 x^2 dx = \frac{127}{3}, \int_6^7 x \, dx = \frac{13}{2}, \int_6^7 dx = 1.$$

- Reverse boundaries
- Constant multiple, 24
- 13/2
- Simplify

4. The force F (in newtons) of a hydraulic cylinder in a press is proportional to the square of $\sec x$ where x is the distance (in meters) that the cylinder is extended in its cycle. The domain of F is $\left[0, \pi/3\right]$ and $F(0) = 400$. Find the average force exerted by the press over the interval $\left[0, \pi/3\right]$. Round your answer to the nearest whole number.

5. Find the indefinite integral of the following function and check the result by differentiation.

$$\int (5-s)\sqrt{s}\,ds$$

6. Find $F'(x)$ given $F(x) = \int_1^{6x} \frac{7}{t}\,dt$.

7. Find the integral $\int \frac{\sin x}{13+\cos^2 x}\,dx$.

4. The force F (in newtons) of a hydraulic cylinder in a press is proportional to the square of $\sec x$ where x is the distance (in meters) that the cylinder is extended in its cycle. The domain of F is $[0, \pi/3]$ and $F(0) = 400$. Find the average force exerted by the press over the interval $[0, \pi/3]$. Round your answer to the nearest whole number.

- `k = 400`
- `Formula`
- `Antiderivative`
- `Simplify, 662`

$$F(x) = k\sec^2 x$$
$$F(0) = k\sec^2 0 = k = 400$$

$$\overline{F(0,\pi/3)} = \frac{1}{\pi/3-0}\int_0^{\pi/3} 400\sec^2 x\,dx$$
$$= \frac{1200}{\pi}\left[\tan x\right]_0^{\pi/3}$$
$$= \frac{1200}{\pi}\left[\sqrt{3}-0\right] \approx 661.595 \approx 662$$

5. Find the indefinite integral of the following function and check the result by differentiation.

- `Split integrand`
- `Antiderivative`
- `Simplify`
- `+ C`

$$\int (5-s)\sqrt{s}\,ds = 5\int s^{1/2}ds - \int s^{3/2}ds$$
$$= 5\cdot\frac{2}{3}\cdot s^{3/2} - \frac{2}{5}\cdot s^{5/2} + C$$
$$= \frac{10}{3}s^{3/2} - \frac{2}{5}s^{5/2} + C$$

6. Find $F'(x)$ given $F(x) = \int_1^{6x} \frac{7}{t}\,dt$.

- `Constant multiple, 7`
- `Antiderivative, ln`
- `FTC`
- `Simplify`

$$F'(x) = \frac{d}{dx}\left[\int_1^{6x}\frac{7}{t}dt\right] = 7\frac{d}{dx}\left[\int_1^{6x}\frac{dt}{t}\right]$$
$$= 7\frac{d}{dx}\left[\ln t\right]_1^{6x} = 7\frac{d}{dx}\left[\ln 6x - 0\right]$$
$$= 7\cdot\frac{1}{6x}\cdot 6 = \frac{7}{x}$$

7. Find the integral $\int \frac{\sin x}{13+\cos^2 x}\,dx$.

- `u = cos x`
- `Substitute`
- `Arctangent`
- `+ C`

$$u = \cos x,\ du = -\sin x\,dx$$
$$= \int \frac{-du}{13+u^2} = -\frac{1}{\sqrt{13}}\arctan\frac{\cos x}{\sqrt{13}} + C$$

8. Evaluate the integral $\int_{\sqrt{3}}^{3} \frac{12}{9+x^2}\,dx$.

9. Find the indefinite integral.

$$\int \sinh(6-3x)\,dx$$

10. Evaluate the integral $\int_{0}^{\ln 5} 8\tanh x\,dx$.

8. Evaluate the integral $\int_{\sqrt{3}}^{3} \frac{12}{9+x^2}\,dx$. $= 12\int_{\sqrt{3}}^{3} \frac{dx}{9+x^2} = 12\left[\frac{1}{3}\arctan\frac{x}{3}\right]_{\sqrt{3}}^{3}$

$= \left[(4\arctan 1) - \left(4\arctan\frac{\sqrt{3}}{3}\right)\right]$

$= \left[4\cdot\frac{\pi}{4} - 4\cdot\frac{\pi}{6}\right] = 4\cdot\frac{\pi}{4} - 2\cdot\frac{\pi}{3}$

$= \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

- Arctangent
- FTC
- Simplify
- π/3

9. Find the indefinite integral.

$\int \sinh(6-3x)\,dx = -\frac{1}{3}\cosh(6-3x) + C$

(Try to differentiate as a check.)

- Antiderivative
- Chain rule
- Simplify
- + C

10. Evaluate the integral $\int_{0}^{\ln 5} 8\tanh x\,dx$. $= 8\int_{0}^{\ln 5} \frac{\sinh x}{\cosh x}\,dx$

$u = \cosh x;\ du = \sinh x$

$u_1 = \cosh 0 = 1$

$u_2 = \cosh \ln 5 = \frac{13}{5}$

$= 8\int_{1}^{13/5} \frac{du}{u} = 8\left[\ln\frac{13}{5} - \ln 1\right]$

$= 8\ln\frac{13}{5}$

- u = cosh x
- Substitute
- FTC
- Simplify

SHORT ANSWER

1. ANS:
   $-4t^2 + 7t + C$

   PTS: 1 DIF: Easy REF: 5.1.15
   OBJ: Evaluate the indefinite integral of a function MSC: Skill
   NOT: Section 5.1
2. ANS:
   $11\tan x + 5x + C$

   PTS: 1 DIF: Medium REF: 5.1.39
   OBJ: Evaluate the indefinite integral of a function MSC: Skill
   NOT: Section 5.1
3. ANS:
   156

   PTS: 1 DIF: Easy REF: 5.3.33
   OBJ: Evaluate the definite integral of a function MSC: Skill
   NOT: Section 5.3
4. ANS:
   662 newtons

   PTS: 1 DIF: Medium REF: 5.4.71b
   OBJ: Calculate the average value of a function in applications MSC: Application
   NOT: Section 5.4
5. ANS:
   $\frac{10}{3}s^{\frac{3}{2}} - \frac{2}{5}s^{\frac{5}{2}} + C$

   PTS: 1 DIF: Easy REF: 5.5.37
   OBJ: Evaluate the indefinite integral of a function MSC: Skill
   NOT: Section 5.5
6. ANS:
   $F'(x) = \frac{7}{x}$

   PTS: 1 DIF: Easy REF: 5.7.71
   OBJ: Calculate the derivative of an integral function MSC: Skill
   NOT: Section 5.7

7. ANS:

$$-\frac{\sqrt{13}}{13}\arctan\left(\frac{\sqrt{13}}{13}\cos x\right)+C$$

PTS: 1 DIF: Medium REF: 5.8.16
OBJ: Evaluate the indefinite integral involving an inverse trigonometric function
MSC: Skill NOT: Section 5.8

8. ANS:

$$\frac{1}{3}\pi$$

PTS: 1 DIF: Easy REF: 5.8.28
OBJ: Evaluate a definite integral involving inverse trigonometric functions
MSC: Skill NOT: Section 5.8

9. ANS:

$$\frac{-1}{3}\cosh(6-3x)+C$$

PTS: 1 DIF: Medium REF: 5.9.47
OBJ: Evaluate the indefinite integral involving hyperbolic functions
MSC: Skill NOT: Section 5.9

10. ANS:

$$8\ln\left(\frac{13}{5}\right)$$

PTS: 1 DIF: Medium REF: 5.9.59
OBJ: Evaluate a definite integral involving hyperbolic functions MSC: Skill
NOT: Section 5.9