

Series Convergence – Divergence Tests

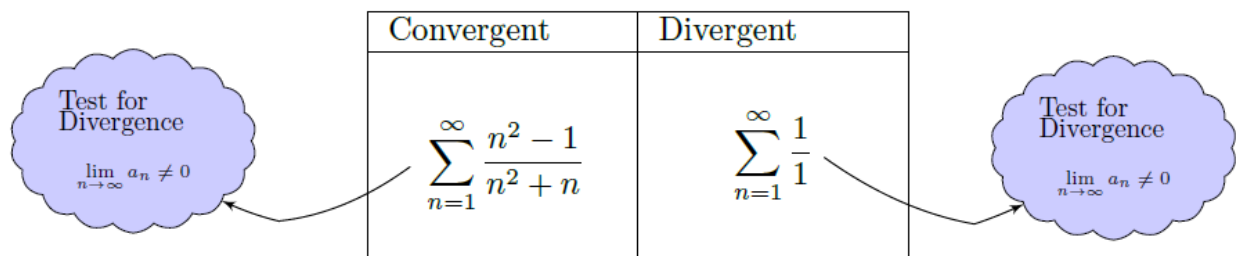
It is usually a good idea to try using the

Test for Divergence as a first step when evaluating a series for convergence or divergence.

If we can show that:

$$\lim_{n \rightarrow \infty} a_n \neq 0$$



Then we can say that the series diverges without having to do any extra work.



Series Convergence – Divergence Tests

P-Series Test:

- The series be written in the form: $\sum \frac{1}{n^p}$

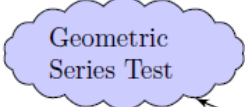
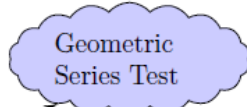
Convergent	Divergent
 $\sum_{n=1}^{\infty} \frac{15}{3n^3}$	 $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

Series Convergence – Divergence Tests

Geometric Series Test:

- When the series can be written in the form:

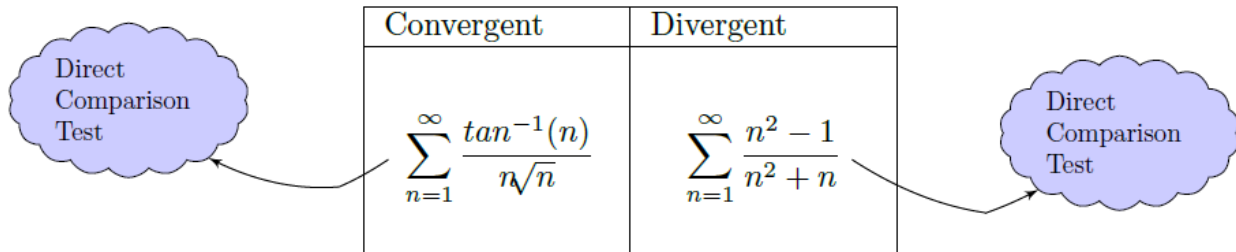
$$\sum a_n r^{n-1} \text{ or } \sum a_n r^n$$

Convergent	Divergent
 $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$	 $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$

Series Convergence – Divergence Tests

Direct Comparison Test:

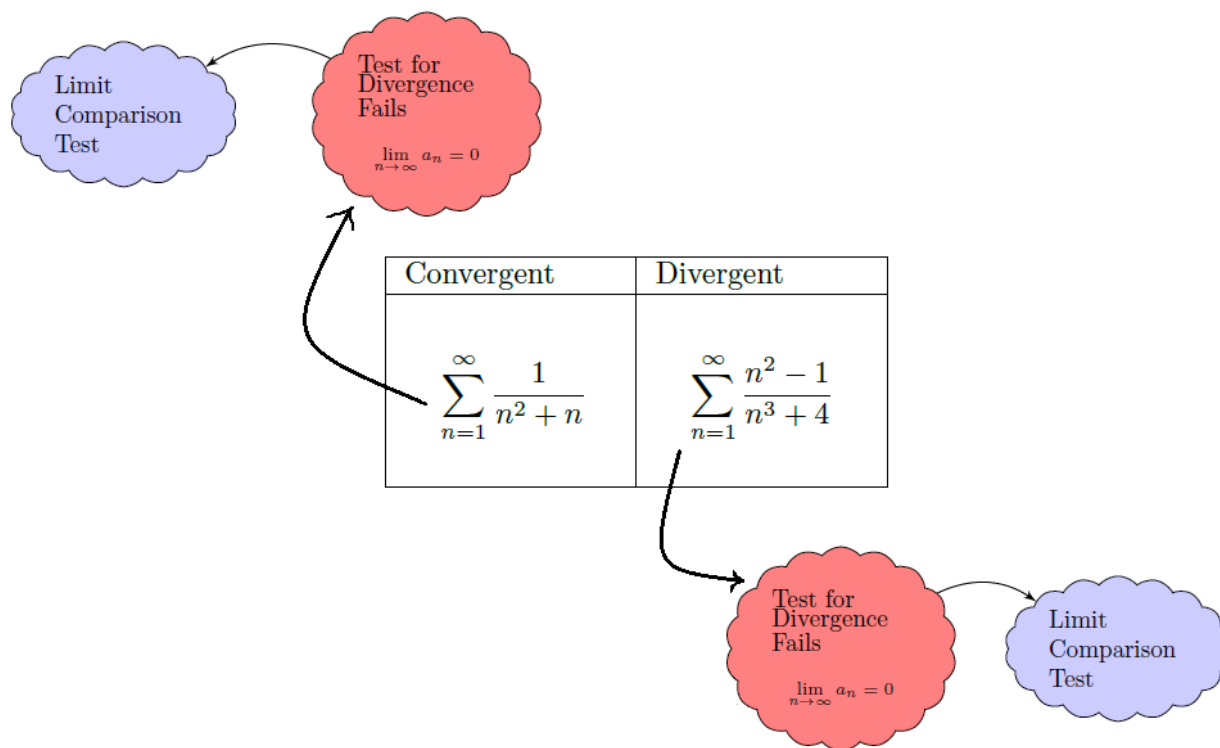
- When the given series, a_n
looks like a known, or more simple, series, b_n



Series Convergence – Divergence Tests

Limit Comparison Test:

- When you can see that the series looks like another convergent or divergent series, b_n
- But it is hard to say whether $b_n > a_n$ or $b_n < a_n$



Series Convergence – Divergence Tests

Root Test:

- When the series can be written in the form:

$$\sum (a_n)^n$$

Convergent	Divergent
$\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$	$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$

Root Test

Root Test

Series Convergence – Divergence Tests

Alternating Series Test:

- When the series can be written in the form:

$$\sum(-1)^{n+1}a_n \text{ or } \sum(-1)^n a_n$$

Convergent	Divergent
$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 - 1}{n^2 + n}$	$\sum_{n=1}^{\infty} (-1)^n \frac{1}{1}$

Alternating Series Test

Alternating Series Test

Series Convergence – Divergence Tests

Ratio Test:

- Whenever we are given something involving a factorial, e.g. $n!$
- Whenever we are given something involving a constant raised to the n^{th} power, e.g. $\sum \frac{n+5}{5^n}$

Convergent	Divergent
$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$	$\sum_{n=1}^{\infty} \frac{1}{1}$

The diagram illustrates the application of the Ratio Test to two series. A table with two columns, 'Convergent' and 'Divergent', contains the series. The convergent column shows the series $\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$, and the divergent column shows the series $\sum_{n=1}^{\infty} \frac{1}{1}$. Two blue cloud-shaped boxes labeled 'Ratio Test' have arrows pointing to the respective series in the table.

Series Convergence – Divergence Tests

Integral Test:

- If the sequence is:
 - continuous
 - positive
 - decreasing (we can use the First Derivative Test here)

Convergent	Divergent
$\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$	$\sum_{n=1}^{\infty} \frac{1}{1}$

The diagram shows a table with two columns: 'Convergent' and 'Divergent'. The 'Convergent' column contains the series $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$. The 'Divergent' column contains the series $\sum_{n=1}^{\infty} \frac{1}{1}$. To the left of the table is a blue cloud labeled 'Integral Test' with an arrow pointing to the convergent series. To the right of the table is another blue cloud labeled 'Integral Test' with an arrow pointing to the divergent series.