Series Convergence – Divergence Tests

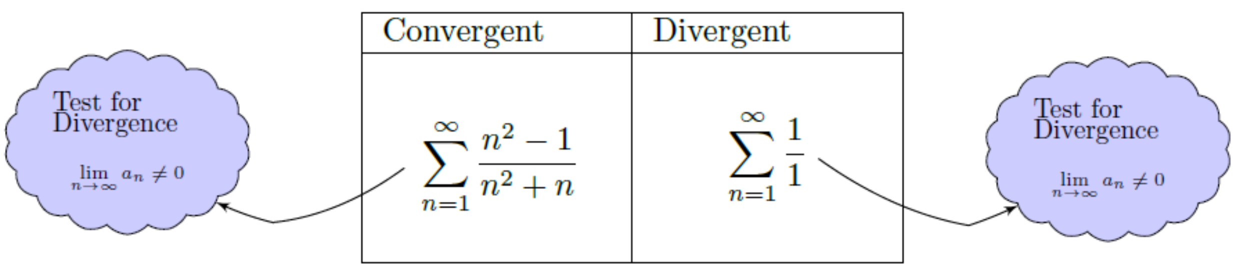
It is usually a good idea to try using the

Test for Divergence as a first step when evaluating a series for convergence or divergence.

If we can show that:

$$\lim_{n \to \infty} a_n \neq 0$$

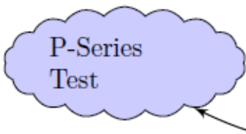
Then we can say that the series diverges without having to do any extra work.



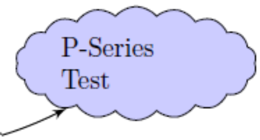
Series Convergence – Divergence Tests

P-Series Test:

- The series be written in the form: $\sum \frac{1}{n^p}$



Convergent	Divergent
$\sum_{n=1}^{\infty} \frac{15}{3n^3}$	$\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$



Series Convergence – Divergence Tests

Geometric Series Test:

- When the series can be written in the form:

$$\sum a_n r^{n-1} \text{ or } \sum a_n r^n$$

Convergent	Divergent
$\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$	$\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$

Geometric Series Test

Direct Comparison Test:

- When the given series, a_n
  looks like a known, or more simple, series, b_n

Direct Comparison Test

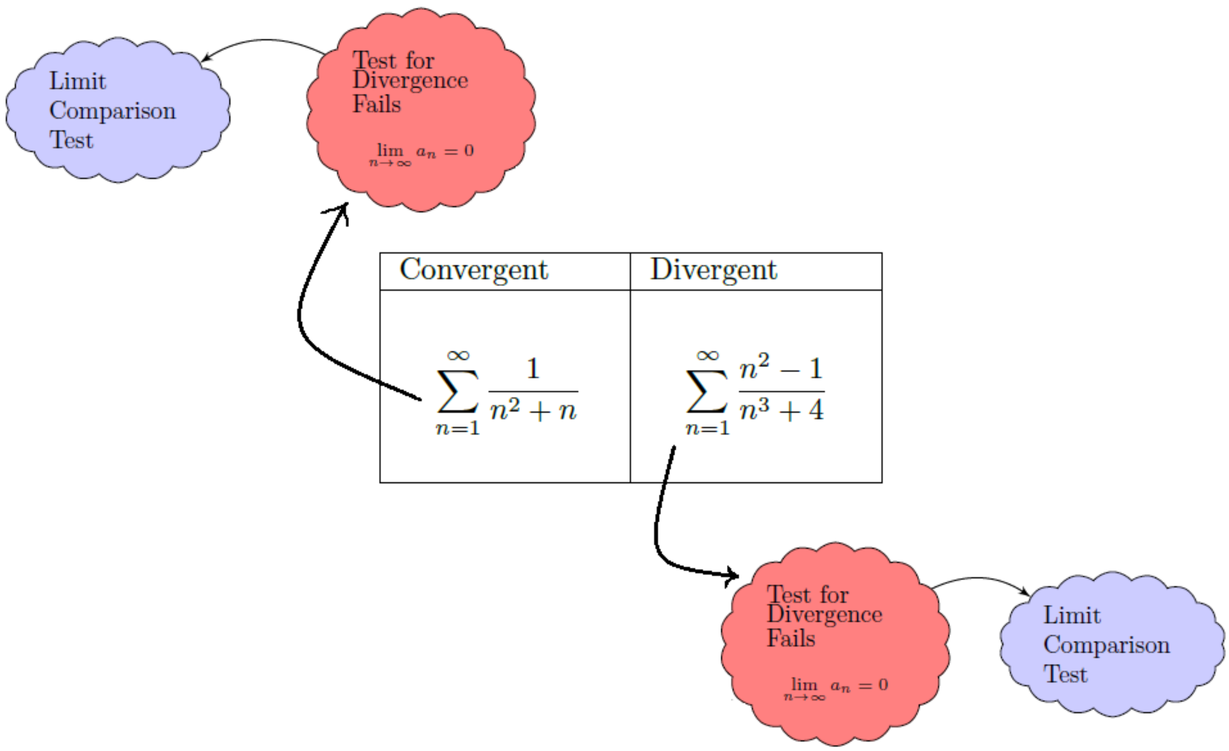
Convergent	Divergent
$\sum_{n=1}^{\infty} \frac{tan^{-1}(n)}{n\sqrt{n}}$	$\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$

Direct Comparison Test

Series Convergence – Divergence Tests

Limit Comparison Test:

- When you can see that the series looks like another convergent or divergent series, b_n
- But it is hard to say whether $b_n > a_n$ or $b_n < a_n$



Series Convergence – Divergence Tests

Root Test:

- When the series can be written in the form:

$$\sum (a_n)^n$$

Convergent	Divergent
$\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$ (Root Test)	$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$ (Root Test)

Series Convergence – Divergence Tests

Alternating Series Test:

- When the series can be written in the form:

$$\sum(-1)^{n+1}a_n \text{ or } \sum(-1)^n a_n$$

Alternating Series Test

Convergent	Divergent
$\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^2-1}{n^2+n}$	$\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{1}$

Alternating Series Test

Series Convergence – Divergence Tests

Ratio Test:

- Whenever we are given something involving a factorial, e.g. $n!$
- Whenever we are given something involving

  a constant raised to the n^{th} power, e.g. $\sum \frac{n+5}{5^n}$

Ratio Test

Convergent	Divergent
$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$	$\sum_{n=1}^{\infty} \frac{1}{1}$

Ratio Test

Series Convergence – Divergence Tests

Integral Test:

- If the sequence is:
  - continuous
  - positive
  - decreasing (we can use the First Derivative Test here)