Calculus

Quiz 5: Differentiation Rules

Problem 1 (5 points). Differentiate the function $f(x) = \sqrt{e^{-x} + 2}$.

Problem 2 ($1 + 1 + 3 = 5$ points). Consider the function $y = x^{10} + \sin(x)$. Find each of the following.

(a) $\dfrac{dy}{dx} =$

(b) $\dfrac{d^2 y}{dx^2} =$

(c) $\dfrac{d^{99} y}{dx^{99}} =$

Answers

Math 1A: Calculus **Instructor: Alexander Paulin**

Quiz 5: Differentiation Rules

Your name: *Discussions 201, 203 // 2018-10-12*

Problem 1 (5 points). Differentiate the function $f(x) = \sqrt{e^{-x} + 2}$.

Solution: Apply the power rule and the chain rule (twice):

$$f'(x) = \frac{1}{2}(e^{-x}+2)^{-1/2}\frac{d}{dx}(e^{-x}+2)$$
$$= \frac{e^{-x}\frac{d}{dx}(-x)}{2\sqrt{e^{-x}+2}}$$
$$= \boxed{\frac{-1}{2e^{x}\sqrt{e^{-x}+2}}}.$$

□

Problem 2 (1 + 1 + 3 = 5 points). Consider the function $y = x^{10} + \sin(x)$. Find each of the following.

(a) $\frac{dy}{dx} = 10x^9 + \cos(x)$

(b) $\frac{d^2y}{dx^2} = 90x^8 - \sin(x)$

(c) $\frac{d^{99}y}{dx^{99}} = -\cos(x)$

Problem 3 (5 points). Find values of a and b that make the below function differentiable at $t = 0$.

$$f(t) = \begin{cases} te^t + 3\tan t & \text{for } t \leq 0 \\ \dfrac{at+b}{t+1} & \text{for } t > 0. \end{cases}$$

Answers

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$$f(t) = \begin{cases} te^t + 3\tan t & \text{for } t \le 0 \\ \dfrac{at+b}{t+1} & \text{for } t > 0. \end{cases}$$

Solution: First, we need $b = 0$ in order for the function to be continuous:

$$\lim_{t\to 0+} f(t) = b$$
$$f(0) = 0e^0 + 3\tan 0 = 0.$$

We also need the derivatives of the two pieces to match up at $t = 0$.

$$\frac{d}{dt}(te^t + 3\tan t) = te^t + e^t + 3\sec^2(t),$$
$$\frac{d}{dt}\left(\frac{at}{t+1}\right) = \frac{a(t+1) - at}{(t+1)^2}$$
$$= \frac{a}{(t+1)^2}.$$

Equating these at $t = 0$ gives the condition

$$0e^0 + e^0 + 3\sec^2(0) = \frac{a}{1^2}$$
$$4 = a$$

so $\boxed{a = 4}$ and $\boxed{b = 0}$.