HW 5.3 Concavity & Points of Inflection

For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.

1) $f(x) = 2x^2 - 12x + 20$

2) $f(x) = -x^3 + 2x^2 + 1$

3) $f(x) = x^3 - 3x^2 + 3$

4) $f(x) = x^4 - x^3 - 3x^2 + 4$

5) $f(x) = \dfrac{3}{x + 1}$

6) $f(x) = \dfrac{x^2}{2x + 2}$

7) $f(x) = \dfrac{3x}{x + 1}$

8) $f(x) = \dfrac{x}{x + 1}$

For each problem, find the x-coordinates of all critical points and find the open intervals where the function is increasing and decreasing.

9) $f(x) = -2x^2 - 8x - 9$

10) $f(x) = x^3 - 4x^2 + 5$

11) $f(x) = -x^4 + 2x^2$

12) $f(x) = \dfrac{1}{x - 1}$

Answers

Answers to HW 5.3 Concavity & Points of Inflection (ID: 1)

1) No inflection points exist.
   Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.

2) Inflection point at: $x = \frac{2}{3}$
   Concave up: $\left(-\infty, \frac{2}{3}\right)$ Concave down: $\left(\frac{2}{3}, \infty\right)$

3) Inflection point at: $x = 1$
   Concave up: $(1, \infty)$ Concave down: $(-\infty, 1)$

4) Inflection points at: $x = -\frac{1}{2}, 1$
   Concave up: $\left(-\infty, -\frac{1}{2}\right), (1, \infty)$ Concave down: $\left(-\frac{1}{2}, 1\right)$

5) No inflection points exist.
   Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$

6) No inflection points exist.
   Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$

7) No inflection points exist.
   Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$

8) No inflection points exist.
   Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$

9) Critical point at: $x = -2$
   Increasing: $(-\infty, -2)$ Decreasing: $(-2, \infty)$

10) Critical points at: $x = 0, \frac{8}{3}$
    Increasing: $(-\infty, 0), \left(\frac{8}{3}, \infty\right)$ Decreasing: $\left(0, \frac{8}{3}\right)$

11) Critical points at: $x = -1, 0, 1$
    Increasing: $(-\infty, -1), (0, 1)$ Decreasing: $(-1, 0), (1, \infty)$

12) No critical points exist.
    Increasing: No intervals exist. Decreasing: $(-\infty, 1), (1, \infty)$

For each problem, find the open intervals where the function is increasing and decreasing.

13) $f(x) = 2x^2 + 16x + 27$

14) $f(x) = x^3 - x^2 + 4$

15) $f(x) = x^4 + 2x^3 - 2x^2 + 2$

16) $f(x) = \dfrac{3}{x + 2}$

For each problem, use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y.

17) $3y^2 + 2 = 5x^3$

18) $3x^3 + 5y^2 = 2$

19) $3y^3 + y^2 = 4x$

20) $3x^2 = 4y^2 + y$

21) $x^2 - 3y = 5x^3y^2$

22) $3x^2 - y^2 = xy^3$

23) $-3x^3y^3 + 1 = x$

24) $2 = 2x^2 + 3x^3y^3$

For each problem, find the indicated derivative with respect to x.

25) $f(x) = -5x^5 - 3x^3 + x^2$ Find f''

26) $f(x) = x^4 - 5x^2 + 5x$ Find f'''

27) $f(x) = 5x^3 + 5x^2 - 4x$ Find $f^{(4)}$

28) $f(x) = 5x^5 - 4x^4 - 4x^2$ Find f'''

Differentiate each function with respect to x.

29) $y = \dfrac{-2x - 3}{(-2x^2 + 5)^{-3}}$

30) $y = (3x^2 - 1)^{-3}(-x^3 - 2)$

Answers

13) Increasing: $(-4, \infty)$ Decreasing: $(-\infty, -4)$

14) Increasing: $(-\infty, 0), \left(\frac{2}{3}, \infty\right)$ Decreasing: $\left(0, \frac{2}{3}\right)$

15) Increasing: $(-2, 0), \left(\frac{1}{2}, \infty\right)$ Decreasing: $(-\infty, -2), \left(0, \frac{1}{2}\right)$

16) Increasing: No intervals exist. Decreasing: $(-\infty, -2), (-2, \infty)$

17) $\frac{dy}{dx} = \frac{5x^2}{2y}$ 18) $\frac{dy}{dx} = -\frac{9x^2}{10y}$ 19) $\frac{dy}{dx} = \frac{4}{9y^2 + 2y}$ 20) $\frac{dy}{dx} = \frac{6x}{8y + 1}$

21) $\frac{dy}{dx} = \frac{15x^2y^2 - 2x}{-3 - 10x^3y}$ 22) $\frac{dy}{dx} = \frac{y^3 - 6x}{-2y - 3y^2x}$ 23) $\frac{dy}{dx} = \frac{-1 - 9x^2y^3}{9x^3y^2}$ 24) $\frac{dy}{dx} = \frac{-4 - 9xy^3}{9x^2y^2}$

25) $f''(x) = -100x^3 - 18x + 2$ 26) $f'''(x) = 24x$ 27) $f^{(4)}(x) = 0$

28) $f'''(x) = 300x^2 - 96x$

29) $\frac{dy}{dx} = \frac{(-2x^2 + 5)^{-3} \cdot -2 - (-2x - 3) \cdot -3(-2x^2 + 5)^{-4} \cdot -4x}{((-2x^2 + 5)^{-3})^2}$

$= 2(-2x^2 + 5)^2(14x^2 - 5 + 18x)$

30) $\frac{dy}{dx} = (3x^2 - 1)^{-3} \cdot -3x^2 + (-x^3 - 2) \cdot -3(3x^2 - 1)^{-4} \cdot 6x$

$= \frac{3x(3x^3 + x + 12)}{(3x^2 - 1)^4}$