Integration Techniques

MAT 141 Homework
Problem Set 1

Name: _____

Date: _____

1. (*25 points*) Evaluate the integral.

a. (*5 pts*) $\int ne^{-n}\,dn$

b. (*5 pts*) $\int (x^2+1)\sin x\,dx$

c. (*5 pts*) $\int (\ln x)^3\,dx$

d. (*5 pts*) $\int \tan^{-1} x\,dx$

e. (*5 pts*) $\int x^5 e^{x^2}\,dx$ [*Hint*: Use a substitution first.]

Integration Techniques

Answers

1. *(25 points)* Evaluate the integral.

a. *(5 pts)* $-ne^{-n} - e^{-n} + C$

b. *(5 pts)* $-x^2 \cos x + 2x \sin x + \cos x + C$

c. *(5 pts)* $x[(\ln x)^3 - 3(\ln x)^2 + 6 \ln x - 6] + C$

d. *(5 pts)* $x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2) + C$

e. *(5 pts)* $\frac{1}{2}(x^4 e^{x^2} - 2x^2 e^{x^2} + 2e^{x^2}) + C$

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2. (*12 points*) Evaluate the integral.

a. (*6 pts*) $\displaystyle\int_1^2 x \ln x \, dx$

b. (*6 pts*) $\displaystyle\int_0^1 e^n \sin(n\pi) \, dn$

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Answers

2. (*12 points*) Evaluate the integral.

a. (*6 pts*) $\dfrac{8\ln 2 - 3}{4} \approx 0.636$

b. (*6 pts*) $\dfrac{(e+1)\pi}{\pi^2 + 1} \approx 1.075$

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3. (*15 points*) Evaluate the integral.

a. (*5 pts*) $\displaystyle\int_0^{\pi/4} \cos^2 x \sin^3 x \, dx$

b. (*5 pts*) $\displaystyle\int \sin^4 t \cos^3 t \, dt$

c. (*5 pts*) $\displaystyle\int \tan^5 \theta \sec^3 \theta \, d\theta$

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3. *(15 points)* Evaluate the integral.

a. *(5 pts)* $\dfrac{16-7\sqrt{2}}{120} \approx 0.051$

b. *(5 pts)* $\dfrac{\sin^5 t}{5} - \dfrac{\sin^7 t}{7} + C$

c. *(5 pts)* $\dfrac{\sec^7 \theta}{7} - \dfrac{2\sec^5 \theta}{5} + \dfrac{\sec^3 \theta}{3} + C$

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4. (*10 points*) Evaluate the integral.

a. (*5 pts*) $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

b. (*5 pts*) $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{(x^2+1)^2}$

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4. (*10 points*) Evaluate the integral.

a. (*5 pts*) $\frac{9}{2}\arcsin\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C$

b. (*5 pts*) $\frac{4\pi + 3\sqrt{3}}{24} \approx 0.74$

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5. (*10 points*) Evaluate the integral.

a. (*5 pts*) $\displaystyle\int \frac{5x-2}{(x-2)^2}\,dx$

b. (*5 pts*) $\displaystyle\int \frac{2x-1}{16x^4-1}\,dx$

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5. (*10 points*) Evaluate the integral.

a. (*5 pts*) $5\ln|x-2| - \dfrac{8}{x-2} + C$

b. (*5 pts*) $\dfrac{1}{8}(\ln(2x+1)^2 - \ln(4x^2+1) + 2\arctan(2x)) + C$

6. (*5 points*) Use partial fractions to prove the integration formula.

$$\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a} \ln \left| \frac{a + x}{a - x} \right| + C$$

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6. (*5 points*) Use partial fractions to prove the integration formula.

Begin with a partial fraction decomposition.

$$\frac{1}{a^2 - x^2} = \frac{1}{(a+x)(a-x)} = \frac{A}{a+x} + \frac{B}{a-x}$$

Multiplying through by $(a-x)(a+x)$ gives the equation $1 = (-A+B)x + (Aa+Ba)$.

This means that $-A + B = 0$, or $A = B$. Also, $Aa + Ba = 1$. Since $A = B$,

we get $Aa + Ba = 1 \;\Rightarrow\; Aa + Aa = 1$

7. (*18 points*) Evaluate the limit using L'Hôpital's Rule.

a. (*3 pts*) $\lim_{t\to 0} \dfrac{\sin t^2}{t}$

b. (*3 pts*) $\lim_{\theta\to \pi/2} \dfrac{2\theta - \pi}{\cos(2\pi - \theta)}$

c. (*3 pts*) $\lim_{t\to 0} \dfrac{t(1-\cos t)}{t - \sin t}$

d. (*3 pts*) $\lim_{x\to 0} \dfrac{2^x - 1}{3^x - 1}$

e. (*3 pts*) $\lim_{t\to \infty} \dfrac{\log_2 t}{\log_3(t+3)}$

f. (*3 pts*) $\lim_{x\to 0^+} \dfrac{\ln(e^x - 1)}{\ln x}$

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Answers

7. (*18 points*) Evaluate the limit using L'Hôpital's Rule.

a. (*3 pts*) 0

b. (*3 pts*) -2

c. (*3 pts*) 3

d. (*3 pts*) $\dfrac{\ln 2}{\ln 3}$

e. (*3 pts*) $\dfrac{\ln 3}{\ln 2}$

f. (*3 pts*) 1

8. (*20 points*) Determine if each integral is convergent or divergent. Evaluate those that are convergent.

a. (*5 pts*) $\int_4^\infty \frac{1}{(x-3)^{1/2}}dx$

b. (*5 pts*) $\int_1^\infty \frac{1}{(3x-1)^4}dx$

c. (*5 pts*) $\int_{-\infty}^0 e^\alpha d\alpha$

d. (*5 pts*) $\int_3^\infty \frac{2}{x^2-4}dx$

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Answers

8. (*20 points*) Determine if each integral is convergent or divergent.

Evaluate those that are convergent.

a. (*5 pts*) Divergent

b. (*5 pts*) Convergent, $\frac{1}{72}$

c. (*5 pts*) Convergent, 1

d. (*5 pts*) Convergent, $\frac{\ln 5}{2}$

9. (*5 points*) Show by the Comparison Theorem that $\int_3^{\infty} \frac{\cos^2 x}{x^2} dx$ is convergent.

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9. (*5 points*) Since $-1 \leq \cos x \leq 1$ we get $\cos^2 x \leq 1$.

Therefore, $\dfrac{\cos^2 x}{x^2} \leq \dfrac{1}{x^2}$.

We know that, since p=2, the integral $\displaystyle\int_3^\infty \frac{dx}{x^2}$ is convergent. By comparison, this makes $\displaystyle\int_3^\infty \frac{\cos^2 x}{x^2}\, dx$ convergent as well.

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10. (*18 points*) Use a table of integrals to evaluate the following integrals.

a. (*6 pts*) $\int \sqrt{x^2 + 6x}\, dx$

b. (*6 pts*) $\int x^2 e^{-3x}\, dx$

c. (*6 pts*) $\int \frac{dx}{\sqrt{2ax - x^2}}\, dx;\ a > 0$

Integration Techniques

Answers

10. (*18 points*) Use a table of integrals to evaluate the following integrals.

a. (*6 pts*) $\frac{1}{2}[(x+3)\sqrt{x^2+6x} - 9\ln|x+3+\sqrt{x^2+6x}|] + C$

b. (*6 pts*) $-\frac{9x^2+6x+2}{27e^{3x}} + C$

c. (*6 pts*) $\arccos\left(\frac{a-x}{a}\right) + C$