Sequences and Series

1. (*18 points*) Give the first six terms for the following sequences.

a. (*6 pts*) $a_n = \dfrac{(-1)^{n-1}n^2}{n!}$

b. (*6 pts*) $a_n = \dfrac{n}{2^n}$

c. (*6 pts*) $a_n = n \sin \dfrac{n\pi}{6}$

Sequences and Series

Answers

1. (*18 points*) Give the first six terms for the following sequences.

a. (*6 pts*) $\left\{1, -2, \frac{3}{2}, -\frac{2}{3}, \frac{5}{24}, -\frac{1}{20}\right\}$

b. (*6 pts*) $\left\{\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \frac{5}{32}, \frac{3}{32}\right\}$

c. (*6 pts*) $\left\{\frac{1}{2}, \sqrt{3}, 3, 2\sqrt{3}, \frac{5}{2}, 0\right\}$

2. (*9 points*) Give the general term a_n for the following sequences, assuming the pattern holds.

a. (*3 pts*) $\left\{-\frac{1}{3}, \frac{1}{2}, -\frac{3}{5}, \frac{2}{3}, -\frac{5}{7}, \cdots\right\}$

b. (*3 pts*) $\left\{\frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \frac{25}{11}, \cdots\right\}$

c. (*3 pts*) $\{0, 1, 0, -1, 0, 1, 0, -1, 0, \cdots\}$

Sequences and Series

Answers

2. *(9 points)*

Give the general term a_n for the following sequences, assuming the pattern holds.

a. *(3 pts)* $a_n = \dfrac{(-1)^n n}{n+2}$

b. *(3 pts)* $a_n = \dfrac{n^2}{2n+1}$

c. *(3 pts)* $a_n = \sin\left(\dfrac{(n-1)\pi}{2}\right)$

3. *(20 points)*

State whether the sequence is convergent or divergent. If it is convergent, find the limit.

a. *(5 pts)* $a_n = \dfrac{n^3 - 2}{2n^3 + n + 9}$

b. *(5 pts)* $a_n = \dfrac{2^n}{n^2 + n}$

c. *(5 pts)* $\{4n^3 e^{-n}\}$

d. *(5 pts)* $\{\ln(3n^4 + 1) - \ln(7n^4 - n^2 - 11)\};\ n \geq 2$

Sequences and Series

Answers

3. (*20 points*)

State whether the sequence is convergent or divergent. If it is convergent, find the limit.

a. (*5 pts*) Convergent, $\frac{1}{2}$

b. (*5 pts*) Divergent

c. (*5 pts*) Convergent, 0

d. (*5 pts*) Convergent, $\ln \frac{3}{7}$

4. (*4 points*) Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$

whose partial sums are $s_n = \dfrac{2n^3 + 7}{5n^3 - 1}$.

Sequences and Series

Answers

4. *(4 points)* $\frac{2}{5}$

Sequences and Series

5. (*8 points*) Is the geometric series convergent?

If it is, give the sum.

a. (*4 pts*) $2 - 5 + \frac{25}{2} - \frac{125}{4} + \cdots$

b. (*4 pts*) $\frac{1}{3} + \frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \cdots$

Sequences and Series

Answers

5. (*8 points*) Is the geometric series convergent? If it is, give the sum.

a. (*4 pts*) Divergent

b. (*4 pts*) Convergent, $\frac{4}{3}$

6. *(20 points)*

Confirm that the integral test can be applied to each series. Then use the integral test to determine if the series is convergent or divergent.

a. *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} n^{-0.7}$

b. *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} \frac{2}{5n-1}$

c. *(5 pts)* $\displaystyle 1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\cdots$

d. *(5 pts)* $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$

Sequences and Series

Answers

6. (*20 points*)

Confirm that the integral test can be applied to each series. Then use the integral test to determine if the series is convergent or divergent.

a. (*5 pts*) Divergent

b. (*5 pts*) Divergent

c. (*5 pts*) Convergent

d. (*5 pts*) Convergent

7. (*20 points*) Determine if the p-series is convergent or divergent.

a. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

b. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^{8/7}}$

c. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{e}{n^{\pi}}$

d. (*4 pts*) $1 + \dfrac{1}{2\sqrt{2}} + \dfrac{1}{3\sqrt{3}} + \dfrac{1}{4\sqrt{4}} + \dfrac{1}{5\sqrt{5}} + \cdots$

e. (*4 pts*) $1 + \dfrac{1}{\sqrt[4]{8}} + \dfrac{1}{\sqrt[4]{27}} + \dfrac{1}{\sqrt[4]{64}} + \dfrac{1}{\sqrt[4]{125}} + \cdots$

Sequences and Series

Answers

7. *(20 points)*

Determine if the p-series is convergent or divergent.

a. *(4 pts)* Divergent

b. *(4 pts)* Convergent

c. *(4 pts)* Convergent

d. *(4 pts)* Convergent

e. *(4 pts)* Divergent

8. (*20 points*) Test the series for convergence or divergence.

a. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

b. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n}}{e^{n}}$

c. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{\ln(n+1)}$

d. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} \cos n\pi$

e. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{1 \cdot 4 \cdot 7 \cdots (3n-2)}$

Sequences and Series

Answers

8. (*20 points*) Test the series for convergence or divergence.

a. (*4 pts*) Convergent

b. (*4 pts*) Convergent

c. (*4 pts*) Divergent

d. (*4 pts*) Convergent

e. (*4 pts*) Convergent

Sequences and Series

9. (*24 points*) Use the ratio test to determine convergence or divergence.

a. (*4 pts*) $\sum_{n=1}^{\infty} \frac{n^4}{4^n}$

b. (*4 pts*) $\sum_{n=0}^{\infty} \frac{(-1)^n}{e^n}$

c. (*4 pts*) $\sum_{n=0}^{\infty} \frac{2^n}{(n+2)!}$

d. (*4 pts*) $\sum_{n=0}^{\infty} \frac{e^{n+1}}{n!}$

e. (*4 pts*) $\sum_{n=0}^{\infty} \frac{(n!)^2}{(4n)!}$

f. (*4 pts*) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-1}}{n!}$

Answers

9. (*24 points*)

Use the ratio test to determine convergence or divergence.

a. (*4 pts*) Convergent

b. (*4 pts*) Convergent

c. (*4 pts*) Convergent

d. (*4 pts*) Convergent

e. (*4 pts*) Convergent

f. (*4 pts*) Convergent

Sequences and Series

Sequences and Series

10. (*20 points*) Use the root test to determine convergence or divergence.

a. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

b. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} (2\sqrt[n]{n}+1)^n$

c. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{500}\right)^n$

d. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n}-\frac{1}{n^2}\right)^n$

e. (*4 pts*) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

Sequences and Series

Answers

10. *(20 points)*

Use the root test to determine convergence or divergence.

a. *(4 pts)* Convergent

b. *(4 pts)* Divergent

c. *(4 pts)* Divergent

d. *(4 pts)* Convergent

e. *(4 pts)* Divergent

11. *(8 points)*

Find the nth Taylor polynomial for the function, centered at c.

a. *(4 pts)* $f(x) = \dfrac{2}{x},\ n = 3,\ c = 1$

b. *(4 pts)* $f(x) = \sqrt{x},\ n = 2,\ c = 9$

Sequences and Series

Answers

11. *(8 points)*

Find the nth Taylor polynomial for the function, centered at c.

a. *(4 pts)* $2 - 2(x-1) + 2(x-1)^2 - 2(x-1)^3$

b. *(4 pts)* $3 + \frac{1}{6}(x-9) - \frac{1}{216}(x-9)^2$

12. *(8 points)*

Find the nth Maclaurin polynomial for the function.

a. *(4 pts)* $f(x) = xe^x,\ n = 4$

b. *(4 pts)* $f(x) = \ln(1 - x),\ n = 2$

Sequences and Series

Answers

12. (*8 points*) Find the nth Maclaurin polynomial for the function.

a. (*4 pts*) $x + x^2 + \frac{x^3}{2} + \frac{x^4}{6}$

b. (*4 pts*) $-x - \frac{1}{2}x^2$

13. *(8 points)*

Approximate the given quantity using a Taylor polynomial with 3 nonzero terms.

a. *(4 pts)* $e^{0.12}$

b. *(4 pts)* $\sqrt{101}$

Sequences and Series

Answers

13. *(8 points)*

Approximate the given quantity using a Taylor polynomial with 3 nonzero terms.

a. *(4 pts)* 1.1272

b. *(4 pts)* 10.04988

14. *(20 points)*

Find the radius of convergence and interval of convergence for the series.

a. *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

b. *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$

c. *(5 pts)* $\displaystyle\sum_{n=1}^{\infty} (n+1)!(5x+3)^n$

Answers

14. *(20 points)*

Find the radius of convergence and interval of convergence for the series.

a. *(5 pts)* $1,\ (-1, 1]$

b. *(5 pts)* $\infty,\ (-\infty, \infty)$

c. *(5 pts)* $0,\ \left\{-\frac{3}{5}\right\}$

15. *(15 points)*

Find a power series for the function, centered at 0, and determine the interval of convergence.

a. *(5 pts)* $\dfrac{1}{1-3x}$

b. *(5 pts)* $\dfrac{2x^3}{1-x}$

c. *(5 pts)* $\dfrac{4x^{12}}{1-2x}$

Sequences and Series

Answers

15. *(15 points)*

Find a power series for the function, centered at 0, and determine the interval of convergence.

a. *(5 pts)* $\displaystyle\sum_{n=0}^{\infty}(3x)^n,\ \left(-\frac{1}{3},\frac{1}{3}\right)$

b. *(5 pts)* $\displaystyle\sum_{n=0}^{\infty}2x^{n+3},\ (-1,1)$

c. *(5 pts)* $\displaystyle\sum_{n=0}^{\infty}2^{n+2}x^{n+12},\ \left(-\frac{1}{2},\frac{1}{2}\right)$

16. *(12 points)*

Find the Taylor series, centered at c, for the given function.

a. *(4 pts)* $f(x) = \sin x,\ c = \dfrac{\pi}{2}$

b. *(4 pts)* $f(x) = \dfrac{1}{x},\ c = 1$

c. *(4 pts)* $f(x) = \ln x,\ c = 3$

Sequences and Series

Answers

16. (*12 points*) Find the Taylor series, centered at c, for the given function.

a. (*4 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(x-\frac{\pi}{2}\right)^{2n}$

b. (*4 pts*) $\displaystyle\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

c. (*4 pts*) $\displaystyle \ln 3+\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n3^n}(x-3)^{n+1}$

17. *(18 points)*

Find the Maclaurin series for the given function.
Then find the interval of convergence.

a. *(6 pts)* $f(x) = e^{-x}$

b. *(6 pts)* $f(x) = (1 + x^2)^{-1}$

c. *(6 pts)* $f(x) = \tan^{-1} x$

Sequences and Series

Answers

17. (*18 points*) Find the Maclaurin series for the given function. Then find the interval of convergence.

a. (*6 pts*) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n, \ (-\infty, \infty)$

b. (*6 pts*) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}, \ (-1, 1)$

c. (*6 pts*) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \ [-1, 1]$

18. *(12 points)*

Use the first 6 terms of the Maclaurin series to estimate the given integral.

a. *(6 pts)* $\displaystyle\int_{-1}^{1} e^{-x^3}\,dx$

b. *(6 pts)* $\displaystyle\int_{0}^{\pi/2} \sin(x^2)\,dx$

Answers

18. (*12 points*)

Use the first 6 terms of the Maclaurin series to estimate the given integral.

a. (*6 pts*) 2.149267

b. (*6 pts*) 0.8281