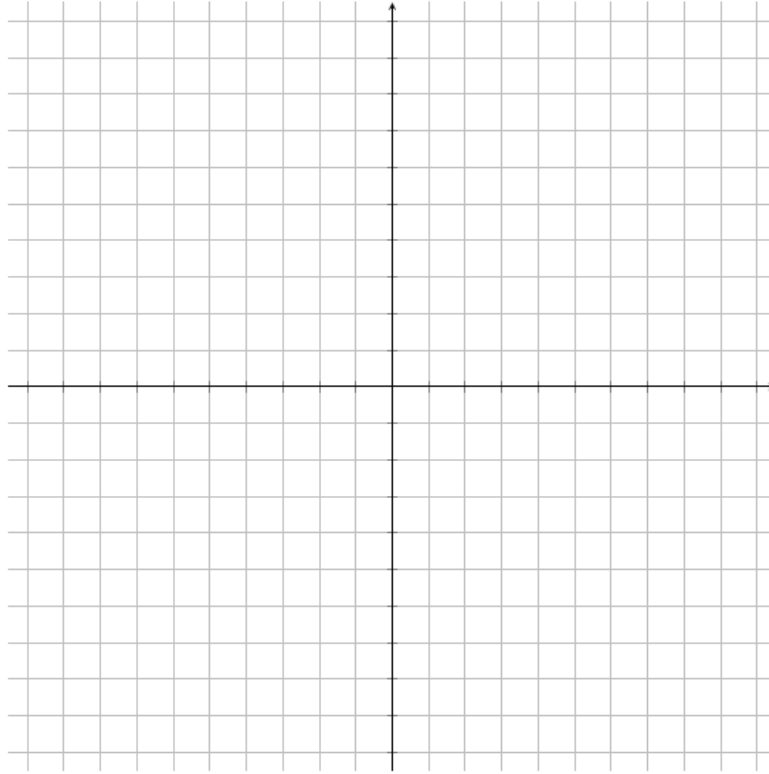


Parametric Equations

Polar Coordinates

1. (10 points) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is oriented.

a. (5 pts) $x = t^2 - 2t$, $y = t + 1$, $0 \leq t \leq 4$



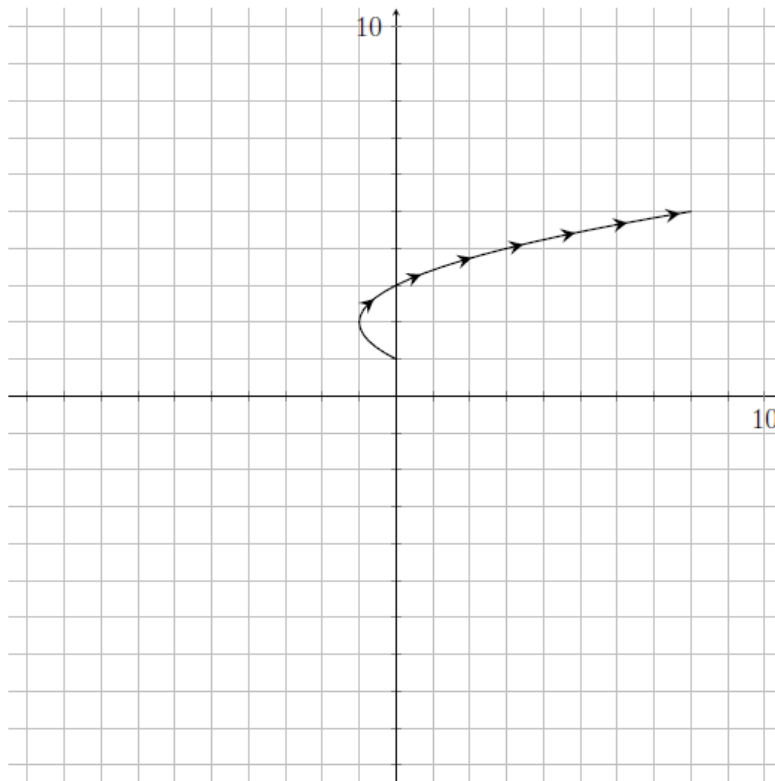
Parametric Equations

Polar Coordinates

Answers

1. (10 points) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is oriented.

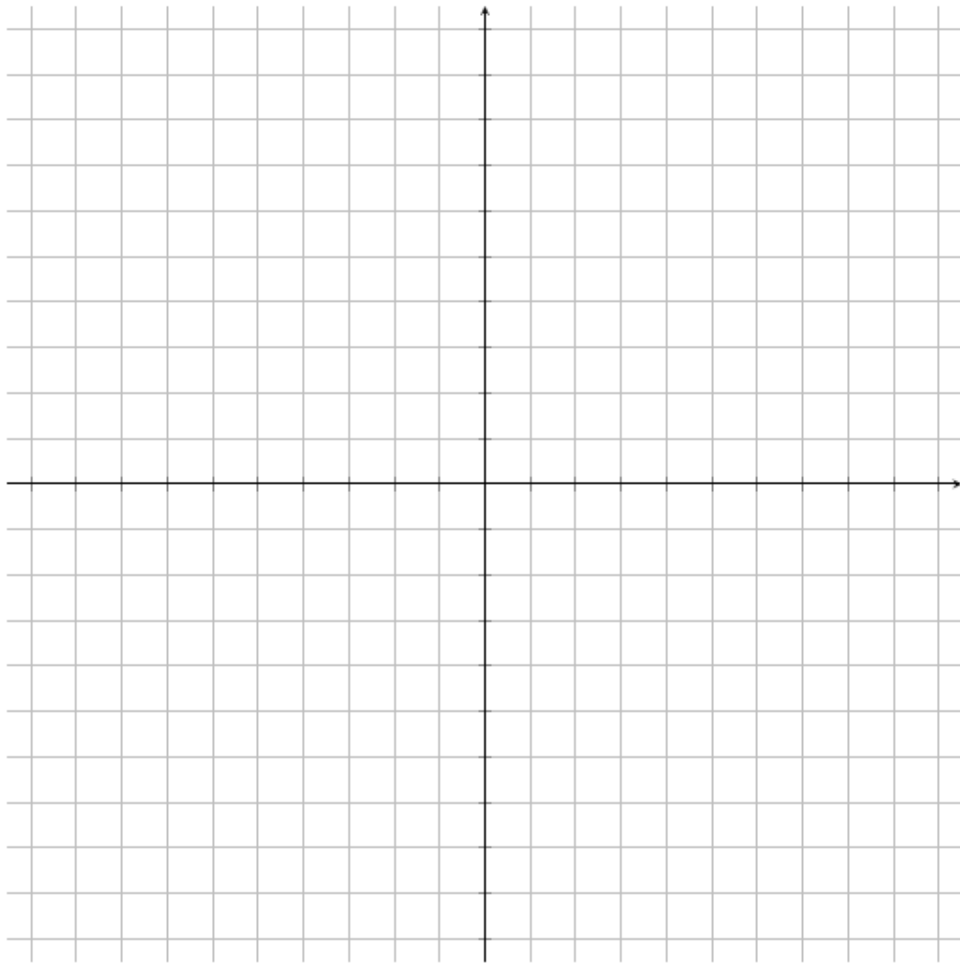
a. (5 pts) $x = t^2 - 2t$, $y = t + 1$, $0 \leq t \leq 4$



Parametric Equations

Polar Coordinates

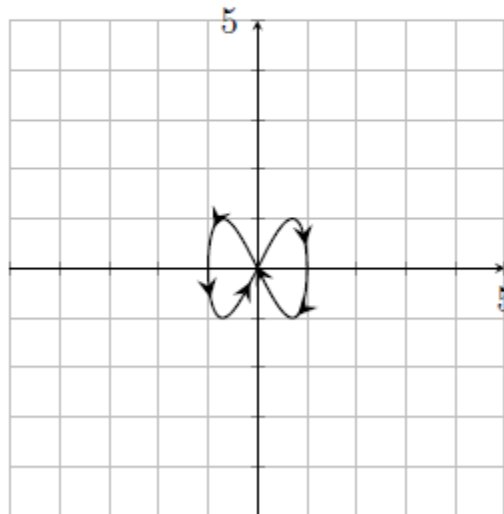
b. (5 pts) $x = \sin 2t$, $y = \sin 4t$



Parametric Equations Polar Coordinates

Answers

b. (5 pts) $x = \sin 2t$, $y = \sin 4t$

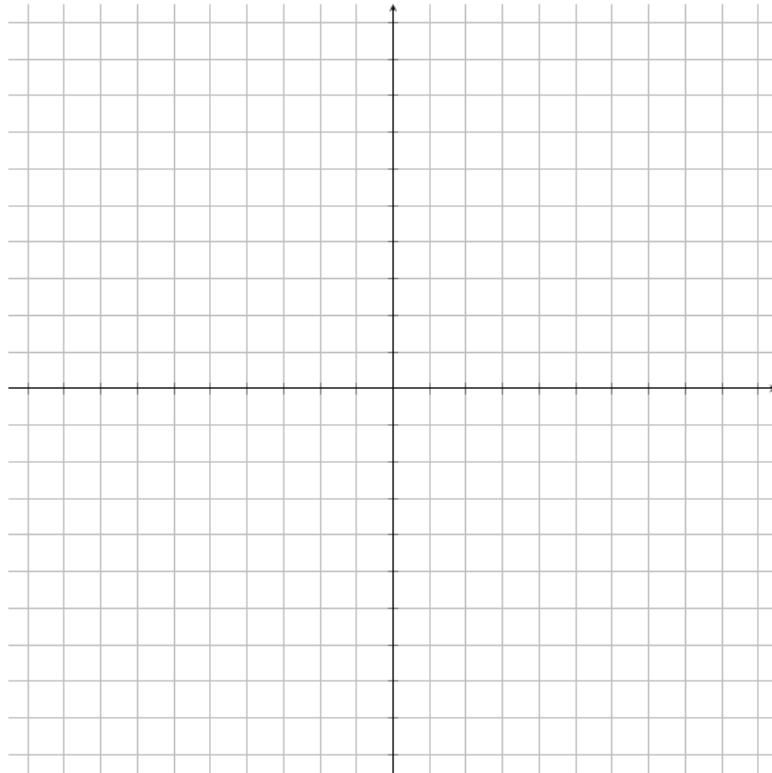


Parametric Equations

Polar Coordinates

2. (20 points) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is oriented. Then, eliminate the parameter to find a Cartesian equation of the curve.

a. (10 pts) $x = 3t + 2$, $y = 2t + 3$



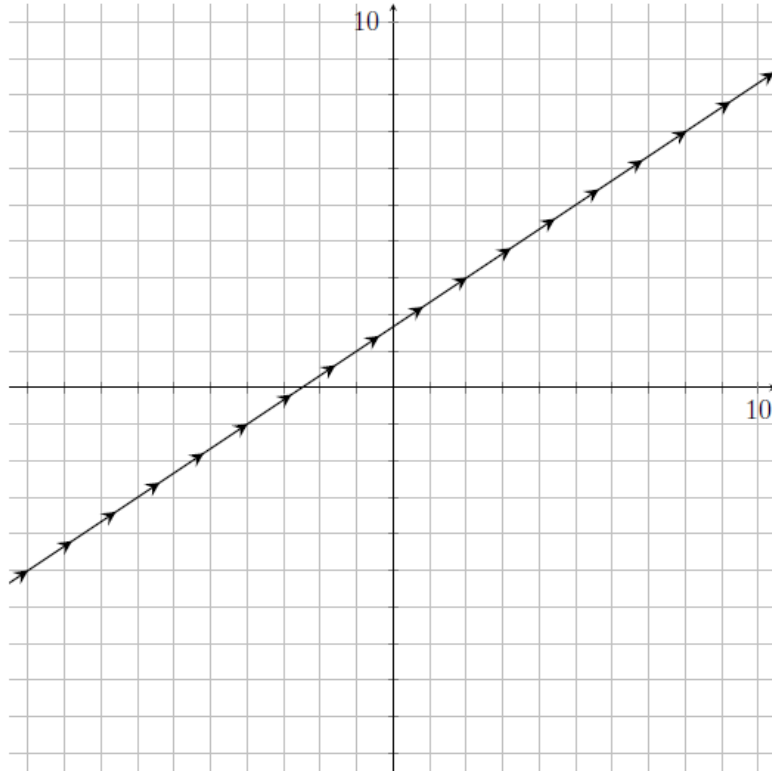
Parametric Equations

Polar Coordinates

Answers

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a. (10 pts) $x = 3t + 2$, $y = 2t + 3$

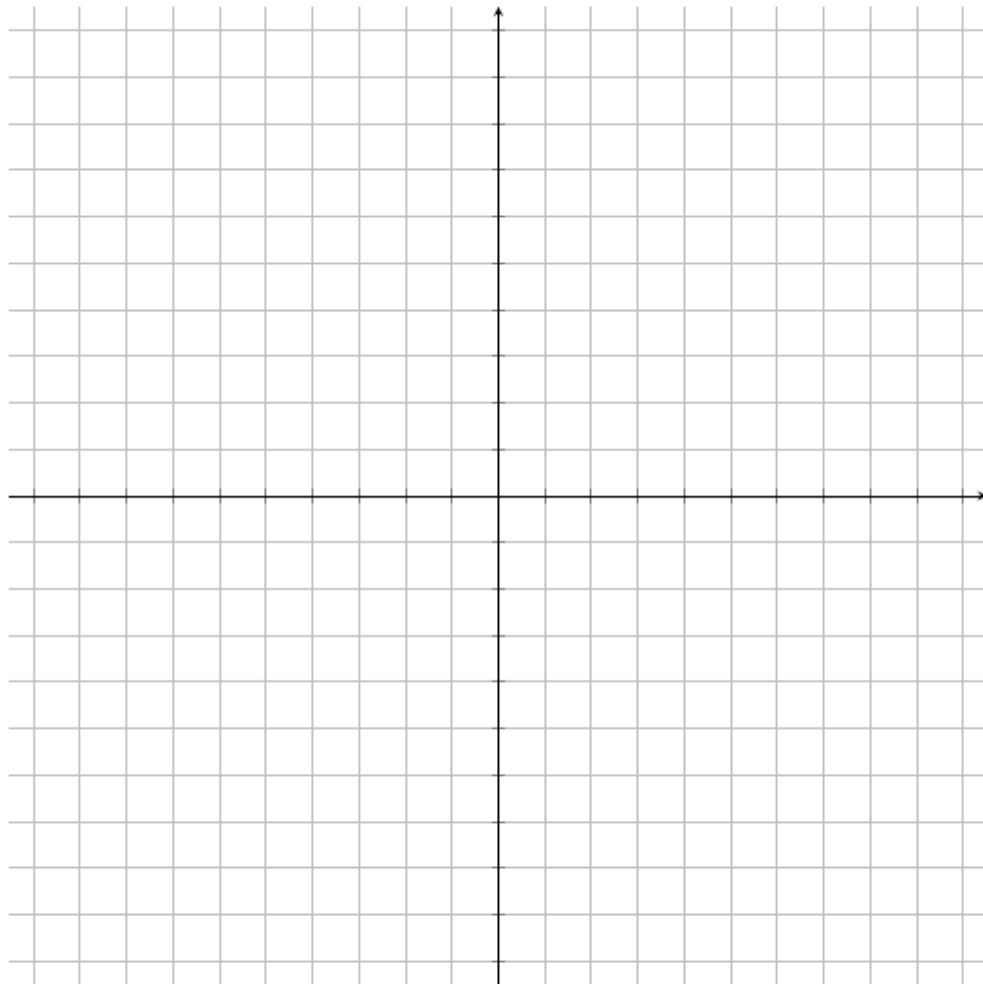


$$y = \frac{2}{3}x + \frac{5}{3}$$

Parametric Equations

Polar Coordinates

b. (10 pts) $x = \sqrt{t}$, $y = 1 - t$, $0 \leq t \leq 9$

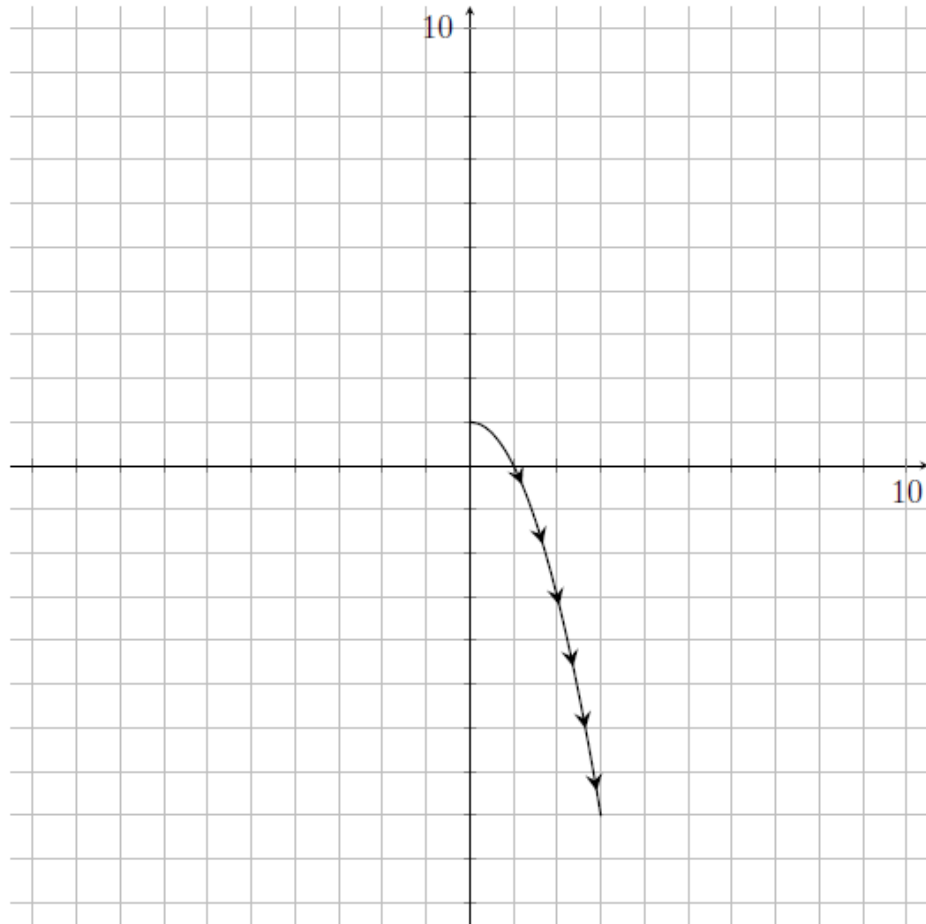


Parametric Equations

Polar Coordinates

Answers

b. (10 pts) $x = \sqrt{t}$, $y = 1 - t$, $0 \leq t \leq 9$



$$y = 1 - x^2$$

Parametric Equations Polar Coordinates

3. (12 points) Eliminate the parameter to find a Cartesian equation of the curve.

a. (4 pts) $x = \frac{1}{2} \sin \theta, y = \frac{1}{2} \cos \theta, 0 \leq \theta \leq \pi$

b. (4 pts) $x = \sin t, y = \csc t, 0 \leq t \leq \frac{\pi}{2}$

c. (4 pts) $x = t^2, y = \ln t$

d. (4 pts) $x = \tan^2 \theta, y = \sec \theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

Parametric Equations Polar Coordinates

Answers

3. (12 points) Eliminate the parameter to find a Cartesian equation of the curve.

a. (4 pts) $x = \frac{1}{2} \sin \theta$, $y = \frac{1}{2} \cos \theta$, $0 \leq \theta \leq \pi$

A: $x^2 + y^2 = \frac{1}{4}$

b. (4 pts) $x = \sin t$, $y = \csc t$, $0 \leq t \leq \frac{\pi}{2}$

A: $y = \frac{1}{x}$

c. (4 pts) $x = t^2$, $y = \ln t$

A: $x = e^{2y}$

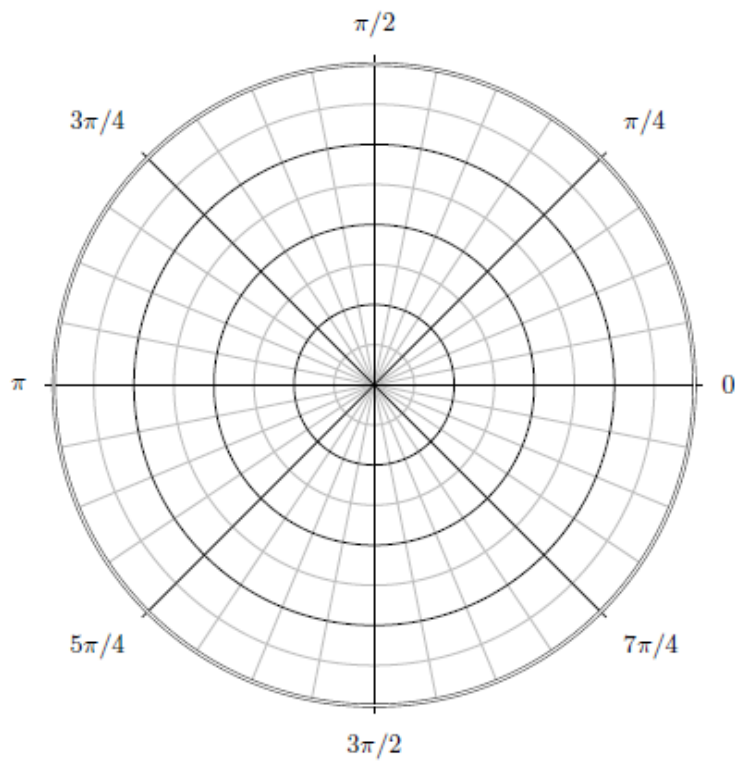
d. (4 pts) $x = \tan^2 \theta$, $y = \sec \theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

A: $x = y^2 - 1$

Parametric Equations

Polar Coordinates

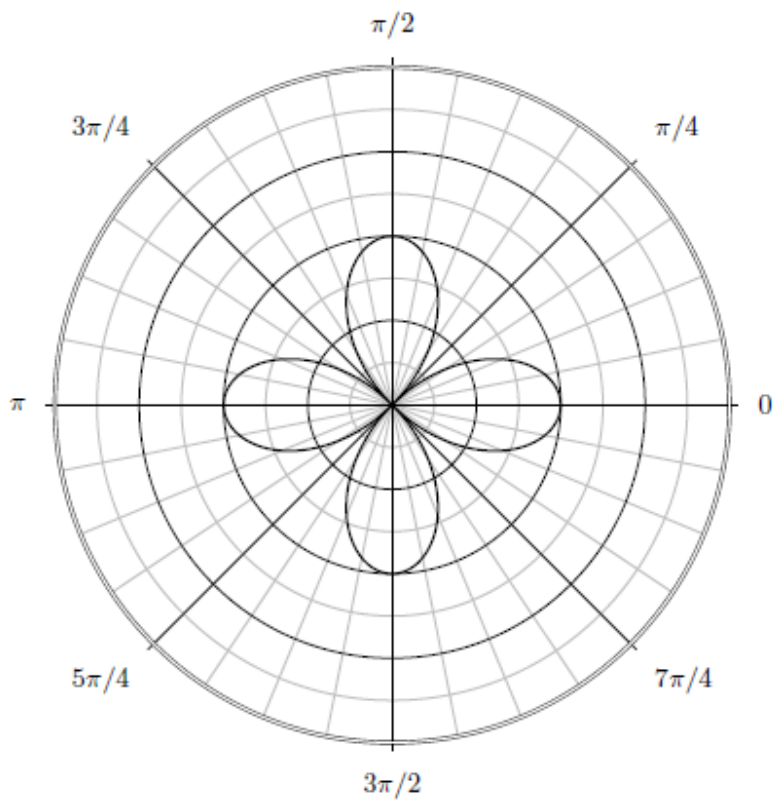
4. (5 points) Graph the planar curve $r = 2 \cos 2\theta$.



Parametric Equations Polar Coordinates

Answers

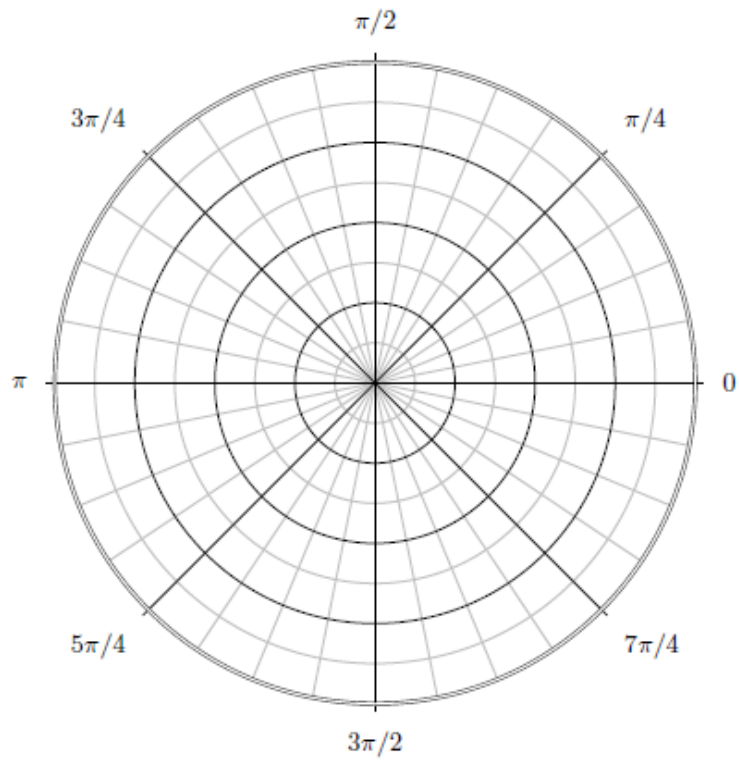
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Parametric Equations

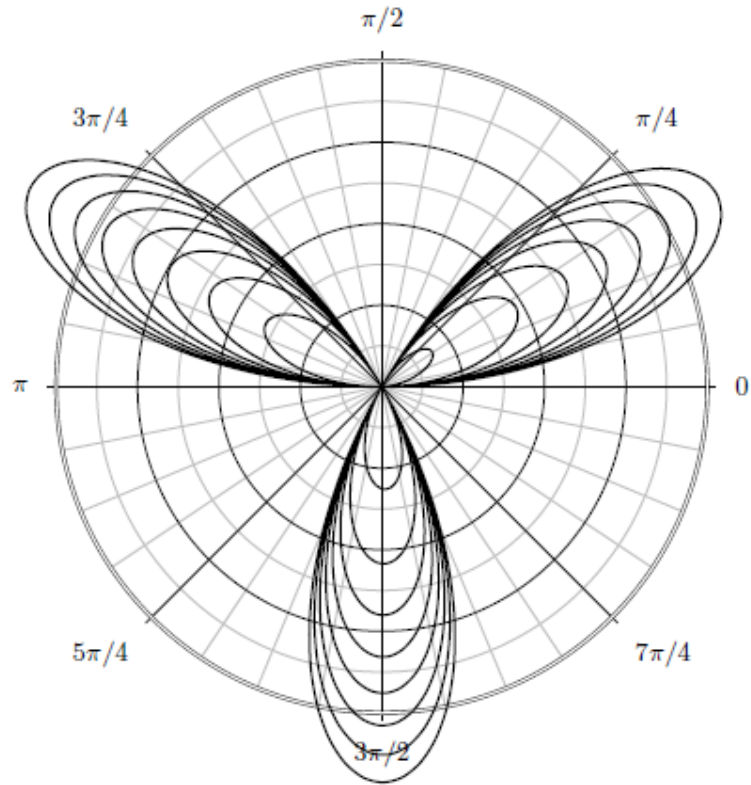
Polar Coordinates

5. (5 points) Graph the planar curve $r = \sqrt{\theta} \sin 3\theta$.



Parametric Equations
Polar Coordinates
Answers

5. (5 points) Graph the planar curve $r = \sqrt{\theta} \sin 3\theta$.



Parametric Equations Polar Coordinates

6. (16 points) Identify the curve by finding a Cartesian equation for the curve.

a. (4 pts) $r = 5 \cos \theta$

b. (4 pts) $r = 2 \csc \theta$

c. (4 pts) $\theta = \frac{\pi}{3}$

d. (4 pts) $r^2 \sin 2\theta = 1$

Parametric Equations Polar Coordinates

Answers

6. (16 points) Identify the curve by finding a Cartesian equation for the curve.

a. (4 pts) $r = 5 \cos \theta$

A: $x^2 + y^2 = 5x$

b. (4 pts) $r = 2 \csc \theta$

A: $y = 2$

c. (4 pts) $\theta = \frac{\pi}{3}$

A: $y = \sqrt{3}x$

d. (4 pts) $r^2 \sin 2\theta = 1$

A: $y = \frac{1}{2x}$

Parametric Equations

Polar Coordinates

7. (12 points) Find a polar equation for the curve represented by the given Cartesian equation.

a. (4 pts) $y = 2$

b. (4 pts) $4y^2 = x$

c. (4 pts) $x^2 - y^2 = 4$

Parametric Equations

Polar Coordinates

Answers

7. (12 points) Find a polar equation for the curve represented by the given Cartesian equation.

a. (4 pts) $y = 2$

$$r = 2 \csc \theta$$

b. (4 pts) $4y^2 = x$

$$r = \frac{1}{4} \cot \theta \csc \theta$$

c. (4 pts) $x^2 - y^2 = 4$

$$r = 2\sqrt{\sec 2\theta}$$

Parametric Equations Polar Coordinates

8. (8 points) Find $\frac{dy}{dx}$ by 1) not eliminating the parameter
and
2) by first eliminating the parameter.

$$x = 1 + \sqrt{t}, \quad y = e^{t^2}$$

Parametric Equations Polar Coordinates

Answers

8. (8 points) Find $\frac{dy}{dx}$ by 1) not eliminating the parameter and 2) by first eliminating the parameter.

$$x = 1 + \sqrt{t}, \quad y = e^{t^2}$$

$$A: 1) \frac{dy}{dx} = 4t^{3/2}e^{t^2}, 2) \frac{dy}{dx} = 4e^{(x-1)^4}(x-1)^3$$

Parametric Equations Polar Coordinates

9. (8 points) Find an equation of the tangent line to the curve
at the point corresponding to the given value of the parameter.

a. (4 pts) $x = \sqrt{t}$, $y = t^2 - 2t$, $t = 4$

b. (4 pts) $x = e^t \sin \pi t$, $y = e^{2t}$, $t = 0$

Parametric Equations

Polar Coordinates

Answers

9. (8 points) Find an equation of the tangent line to the curve at the point corresponding to the given value of the parameter.

a. (4 pts) $x = \sqrt{t}$, $y = t^2 - 2t$, $t = 4$

A: $y = 24x - 40$

b. (4 pts) $x = e^t \sin \pi t$, $y = e^{2t}$, $t = 0$

A: $y = \frac{2}{\pi}x + 1$

Parametric Equations Polar Coordinates

10. (8 points) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

a. (4 pts) $x = t^2 + 1, y = e^t - 1$

b. (4 pts) $x = \cos t, y = \sin 2t$

Parametric Equations Polar Coordinates

Answers

10. (8 points) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

a. (4 pts) $x = t^2 + 1$, $y = e^t - 1$

$$A: \frac{dy}{dx} = \frac{e^t}{2t}, \quad \frac{d^2y}{dx^2} = \frac{e^t(t-1)}{4t^3}$$

b. (4 pts) $x = \cos t$, $y = \sin 2t$

$$A: \frac{dy}{dx} = \frac{2 \cos 2t}{-\sin t}, \quad \frac{d^2y}{dx^2} = \frac{4 \sin t \sin 2t + 2 \cos t \cos 2t}{-\sin^3 t}$$

Parametric Equations Polar Coordinates

11. (12 points) Find the slope of the tangent line to the given polar curve at the point specified by the value of θ .

a. (4 pts) $r = 2 + \sin 3\theta, \theta = \pi/4$

b. (4 pts) $r = \cos(\theta/3), \theta = \pi$

c. (4 pts) $r = 1 + 2 \cos \theta, \theta = \pi/3$

Parametric Equations

Polar Coordinates

Answers

11. (12 points) Find the slope of the tangent line to the given polar curve at the point specified by the value of θ .

a. (4 pts) $r = 2 + \sin 3\theta$, $\theta = \pi/4$

A: $\frac{-1 + \sqrt{2}}{-2 - \sqrt{2}}$

b. (4 pts) $r = \cos(\theta/3)$, $\theta = \pi$

A: $-\sqrt{3}$

c. (4 pts) $r = 1 + 2 \cos \theta$, $\theta = \pi/3$

A: $\frac{\sqrt{3}}{9}$

Parametric Equations Polar Coordinates

12. (5 points) Find the arc length of the curve on the given interval.

$$x = 6t^2, y = 2t^3, 1 \leq t \leq 4$$

Parametric Equations
Polar Coordinates

Answers

12. (5 points) Find the arc length of the curve on the given interval.

$$x = 6t^2, y = 2t^3, 1 \leq t \leq 4$$

$$A: 70\sqrt{5}$$

Parametric Equations Polar Coordinates

- 13.** (5 points) Find the area of the surface generated by revolving the curve about the given axis.

$$x = a \cos \theta, y = a \sin \theta, 0 \leq \theta \leq \frac{\pi}{2} \text{ about the } x\text{-axis}$$

Parametric Equations

Polar Coordinates

Answers

13. (5 points) Find the area of the surface generated by revolving the curve about the given axis.

$$x = a \cos \theta, \quad y = a \sin \theta, \quad 0 \leq \theta \leq \frac{\pi}{2} \text{ about the } x\text{-axis}$$

$$A: S = 2\pi a^2$$

Parametric Equations
Polar Coordinates

14. (5 points) Find the area between the loops of $r = 3 - 6 \sin \theta$.

Parametric Equations
Polar Coordinates

Answers

14. (5 points) Find the area between the loops of $r = 3 - 6 \sin \theta$.

$$A : 9\pi + 27\sqrt{3}$$

Parametric Equations Polar Coordinates

15. (5 points) Find the slope of the tangent line to the polar curve

$$r = 2 \cos \theta + 3 \sin \theta \text{ where } \theta = \pi/4.$$

Parametric Equations
Polar Coordinates

Answers

15. (5 points) Find the slope of the tangent line to the polar curve

$$r = 2 \cos \theta + 3 \sin \theta \text{ where } \theta = \pi/4.$$

$$A: -\frac{3}{2}$$

Parametric Equations Polar Coordinates

16. (5 points) Find the arc length of the curve over the given interval.

$$r = 2a \cos \theta, \quad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

Parametric Equations
Polar Coordinates

Answers

16. (5 points) Find the arc length of the curve over the given interval.

$$r = 2a \cos \theta, \quad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

A: $a\pi$

Parametric Equations Polar Coordinates

17. (5 points) Find the area of the surface formed by revolving the polar equation about the given line.

$$r = 12 \sin \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}, \quad \theta = \frac{\pi}{2}$$

Parametric Equations Polar Coordinates

Answers

17. (5 points) Find the area of the surface formed by revolving the polar equation about the given line.

$$r = 12 \sin \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}, \quad \theta = \frac{\pi}{2}$$

A: 144π